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ON THE ROLES OF EXPERIMENT AND THEORY IN COGNITION

(Comment on the article by Max Born)*

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I

In this issue of this journal there is published a translation of an article by Max Born on "Experiment and Theory in Physics." The author is well known as an outstanding theoretical physicist who participated directly in the development of present-day physics and has lived for over a half century in the creative atmosphere of the science, an atmosphere of great and abrupt changes in physical and philosophical ideas. For this reason in itself the present article, in which decisive events in the history of physics are explained by one who witnessed them and took part in them, is of decided interest. This interest is increased by the fact that the article is written not only on the historical but also on the epistemological level, as a discussion of the roles of experiment and theory in the development of contemporary physics.

As is well known, the part played by theory in modern physics has increased immensely. Complicated processes, not directly accessible to observation, are represented in the theory by highly abstract concepts and relations; but is is precisely these abstract representations that enable science to progress. Furthermore, theories that have taken on the greatest importance in physics are characterized by vast powers of generalization, as for example in thermodynamics and the theory of relativity. All this has engendered in some scientists the view that theoretical reasoning may by itself be able, without relying on experiment, to develop knowledge of natural processes. In the West, and in particular in England, where Born worked for many years, Eddington's number mysticism was widely accepted, and Jeans based on speculative arguments a calculation of the time since the "creation of the world."

The sharp edge of Born's article is directed against just such speculations, which have no genuine scientific significance.

We cannot fail to rejoice that such an important scientist as Max Born comes forward against mysticism in science.

Being himself a theorist, Born tries to find the proper role to be assigned to theory, and to emphasize its basic connections with experiment. He shows that every principle that has been confirmed and has achieved great generality, such a principle as that of least action, the laws of thermodynamics, or Pauli's exclusion principle, even though it may seem at first glance to be of an a priori nature, is in actual fact a generalization from experimental results, from "the results of long experience." The general theory of relativity, though in virtue of its mathematical perfection it has an a priori appearance, is also "a gigantic synthesis of a long series of empirical results, and not a spontaneous brain wave."

Born takes a somewhat skeptical attitude toward the later attempts of Einstein, and also of Weyl and of Eddington, to develop the general theory of relativity in the direction of including all the physical features of the world in the differential equations of a single geometrized field; since they have ignored the experimental material of nuclear and quantum physics, these efforts have not given definite results, and the general theory of relativity "did not help in understanding the nature of matter, the existence of different ultimate particles and fields," as Einstein had hoped.

In this "Princeton period" of his life Einstein looked for the further development of science only from the creative power of thought, and consequently took a course entirely different from that which Born defends in his article. In the report "Physics and Relativity" (1955) dedicated to the fiftieth anniversary of the theory of relativity, Born tells of the reaction this article of his received from Einstein, who perceived in it a reproach to himself: "I sent a copy to Einstein and received a very interesting reply which unfortunately has been lost; but I remember a phrase like this: 'Your thundering against the Hegelism is quite amusing, but I shall continue with my endeavors to guess God's ways.' A man of Einstein's greatness who has achieved so much by thinking has the right to go to the limit of the a priori method. Current physics has not followed him; it has continued to accumulate experimental facts, ... "*

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*Cambridge University Press, 1944.

Born regards experiment as the driving force of theory. Experiment constantly discovers in nature things not yet included in existing theory. The development of theories occurs in relation to this. This compulsion to the development of theory is repeatedly emphasized by Born in this article. All the major innovations in physics — whether we think of the relativization of time, or the transition to quantum concepts, or the statistical interpretation of the wave function — appear with a natural necessity, despite the psychological resistance to them of individual scientists. Born testifies that Planck's introduction of the idea of quanta of energy was an act of downright desperation, but that it had to be done because of "the failure of the classical laws to account for the properties of radiant heat."†

Born states correctly that concepts and theories are not free creations of the intelligence. Even the speculations of Eddington on the so-called E-numbers could occur only after the dimensionless numbers had been established experimentally.

Born's criticism of the idea of free creation is also directed against the views of Einstein, who all his life defended his view of theory as a free creation of the intelligence, and also against the conventionalism of H. Poincare. Somewhat later, in an article "Physics and Metaphysics," Born stated even more clearly: "Here is another point where I disagree with Einstein's philosophy. He accepts the doctrine of conventionalism which in my youth was powerfully advocated by the great French mathematician Henri Poincare. According to this view all human concepts are free inventions of the mind and conventions between different minds, justifiable only by their usefulness in ordinary experience. This may be right in a restricted sense, namely for the abstract parts of theories, but not for the connection of the theories with observations, with real things. It neglects the psychological fact that the building of language is not a conscious process. And even in the abstract part of science the use of concepts is often decided by facts, not by conventions."

We are pleased to note the agreement between our previously expressed criticism of the elements of conventionalism in Einstein's ideas* and these remarks of his very close friend Max Born.

The recognition that our concepts and theories are not free inventions of the mind, that they have a content independent of us, is a great step of the natural scientist on the path of progressive thought; this step is all the more valuable because it flows from the immediate professional experience of the scientist, and because it inescapably leads farther, to the question as to what is the source of the objective content of the theories. Consistent reasoning cannot fail to lead to the conclusion that the element of necessity in the development of physical theories is a consequence of the fact that they are the reflection, the image, of the objective world in consciousness. Precisely this path has been followed by many outstanding physical scientists. In particular this path was also travelled by Max Planck, who went from the Machism, to which he inclined in his youth, to materialism, "to the recognition of a self-existent world, independent of the experimenter and standing over against him, which imposes its laws on him whether he will or not."†

It is important that Born also comes to the question of this image, indeed not in connection with the establishment of the independent content of the theories, but in connection with the analysis of the basis of theoretical predictions. This analysis is also of interest in itself, since here an attempt is made to bring to light the nature of creation in theoretical work.

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*Max Born, Physics in My Generation, Pergamon Press, London 1956, page 205.

†It is interesting to note that Planck himself also concluded that a change in our ideas about the world was inevitable, being "a consequence of an irresistible compulsion." "This sort of change," wrote Planck, "becomes a bitter necessity each time experiment encounters a new fact in nature which the existing picture of the world cannot explain."

Cf. "Sinn und Grenzen der exakten Wissenschaft," in the collection: Max Planck, Vorträge und Errinnerungen, Stuttgart 1949, page 371.

*See the introductory article by the writer in the Russian translation of The Evolution of Physics by A. Einstein and L. Infeld, Gostekhizdat, Moscow, 1948 and 1956.

†Max Planck, Naturwissenschaft und reale Aussenwelt, 1940.

We pay a tribute of high respect to this outstanding scientist and passionate fighter for the scientific materialistic view of the world; the hundredth anniversary of his birth was recently observed by the scientific community of the entire world, among them Max Born, who was at one time his assistant.

In accordance with the main purpose of his article, Born examines theoretical predictions in order to show that they are based in the last analysis on experiment, and not on *a priori* principles. He divides theoretical predictions into analytical and synthetic predictions.

By an analytical prediction Born means the prediction of results that are the consequence of logical development of a theory from its original premises. As examples of such predictions Born discusses the prediction of the planet Neptune by Adams and Leverrier, the prediction of conical refraction by Hamilton, and the explanations of line and band spectra and of the nature of the metallic state and of chemical valence, and the prediction of para- and orthohydrogen, based on quantum mechanics. Since the original premises of the theory are based on experiment, here the position is clear from the beginning: there is nothing of the *a priori* about these predictions.

Synthetic predictions, according to Born, are of a more complex nature, but also they open up wider prospects. Let us recall some of the examples he gives of synthetic predictions: Maxwell's addition of the term $\frac{1}{c}\frac{\partial E}{\partial t}$ to the field equation curl $H = 0$; Einstein's development of the ideas of the general theory of relativity, from which followed, in particular, the prediction of the deflection of light rays passing near the Sun; the discovery of non-Euclidean geometry by Lobachevskiĭ and by Gauss; the discovery of noncommutative algebra by Hamilton; de Broglie's association of waves with corpuscles; the discovery of the matrix calculus; obviously, here we should also include the addition by the Japanese physicist Yukawa of a term $\frac{\Phi}{a^2}$ to the wave equation $\Delta\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = 0$.

Born declares that the characteristic feature of synthetic predictions is that they are made "without direct experimental basis", and that the intuition of the scientist plays an essential part in them. Also they are not logical consequences of an existing theory, but on the contrary are themselves the foundation for, or an important part of, a new theory. But perhaps they also rest on *a priori* principles? Born shows that even in the case of synthetic predictions there is nothing of the *a priori*. Here he has made use of the concept of a "shape."

Born says that he takes the concept of a shape from Gestalt psychology, and emphasizes that it has established the "experimental fact" that simultaneous sense impressions are not independent of each other like the elements of a mosaic but form a "psychic unit." This idea of the unity of a shape plays an important part in his further arguments. Born goes on to transfer the idea of the unity of a shape to the external world, and speaks of the "shape of physical things". Here he mentions his favorite idea, as he calls it, that the shapes of physical things are the invariants of the equations.* We must not worry about the fact that they are described by the formulas of theoretical physics, says Born: "These have the same kind of reality — I mean: objective reality in the external world, as any shape of familiar things, for instance that of the human body."

Just so, at some stage of cognition the shape conceived may be incomplete, "rough." Because of the fact that the elements of the shape form a unit, and not a mosaic, the scientist can intuitively feel this incompleteness of the shape and complete it by synthetic predictions. "Maxwell's addition of the missing term is just such a smoothing out of a roughness of a shape, though this shape is here a mathematical structure of a more refined type than a sphere (in the previous example about the moon — S. S.)." Furthermore the synthetic prediction is a hypothesis, which Born calls a mathematical one. "If confirmed by experiment it produces new knowledge, and although hypothetical it is a legitimate method. But its success depends in a high degree on intuition. . . ." In any case a synthetic prediction is also based not on *a priori* or innate ideas, but on a partial knowledge of the shape from experiment and a further hypothetical completing of the shape, with subsequent verification by experiment.

Such are Born's ideas about theory and its relation to experiment. We shall now take the liberty of rearranging his statements in a certain logical order.

The external objective world has effects on us (the possibility of this is not doubted, since we are a part of it and a product of its development); in the complex interaction with the world that begins with our contact with it, that is with elementary experience, we form concepts and theories; the content of these concepts and theories is neither an *a priori* matter, nor a free invention of thought, since, as the whole history of science shows, they are not arbitrary, but form a single and unified physical shape. This is indeed the shape of "objective reality in the external world." The unity of the shape in our thinking finds its explanation in the fact that it is a consequence of the unity of the object it reflects. But this shape is not revealed to us all at once, in a sudden single act; analysis of specific relations that have already been discovered enables us to make hypotheses about relations of an entirely new nature between the various categories of the object ("synthetic prediction"). The correctness of these hypotheses is tested by practice (experiment). From the new relations that have been found logical consequences are deduced ("analytical predictions"). It is thus that a theory — a coherent shape of the object — is formed and developed.

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*For more detail see: M. Born, (Physical Reality), Uspekhi Fiz. Nauk 46, No. 2 (1957), and also: S. G. Suvorov, The Problem of "Physical Reality" in the Copenhagen School, ibid.

In summarizing Born's statements I have tried to explain the process of cognition in complete agreement with Born's ideas of this process. I have allowed myself only certain "liberties," namely: I have interpreted the frequently used concept "experiment" as the result of our direct interaction (encounter) with the external world, doing this on the basis of a number of antipositivistic pronouncements of the author; and further I have combined Born's statements about the compulsory nature of the development of theories and about the "shapes of physical things", which are regarded by him as the "objective reality in the physical world". This combination of the statements, however, has been only an advantage, giving us features of a unified and consistent concept; and this concept is indeed just the materialistic theory of cognition, which many foreign scientists, including Born himself, have not brought themselves to acknowledge openly.

The advantage gained does not lie in the external form of the theoretical structure, but in the fact that the unified concept enables us to look at various problems under discussion from a more general point of view, and owing to this to reach opinions about them that are somewhat different from those of the author.

II

The main subject of the article by Max Born is a criticism of the doctrine of a priori and innate ideas, a struggle against dogmatism and metaphysical speculations, a defense of the experimental origin of knowledge.

It is true that criticism of a priori ideas is an urgent necessity in contemporary natural science. The grandiose system created by Kant dominated natural science for a long time and hindered its development. In fact, if such general categories as space, time, and causality are a priori categories, forms of our understanding, connected with the nature of our intelligence, then consequently they are given once for all, unchangeable, and independent both of the objects toward which cognition is directed and of the depth of our knowledge of them. It is easy to understand how Kant's a priori doctrine put shackles on cognition. Sooner or later it had to fall under the pressure of the actual facts of natural science itself. The discovery of Lobachevskiĭ's non-Euclidean geometry already gave it a crushing blow: the a priori forms exclude any lack of uniqueness. Furthermore the inevitable appearance in physics of the idea of the field, the analysis of the electrodynamics of moving bodies and the development on this basis of the special theory of relativity, the generalization of the concept of the gravitational field in the general theory of relativity — all these theories, developed under the pressure of undeniable facts, lead to changed ideas about space and time and about their genetic connection with matter and the laws of its motion, and left no room for the idea that they are a priori forms. In just the same way physics has been forced to overcome the Kantian ideas about the a priori origin of the category of causality, because under the pressure of experimental facts in the atomic domain ideas about the nature of this category have had to be changed. Not only in the article we are discussing, but in many papers by Born one sees the necessity of breaking free from the confining doctrine of the a priori. "After relativity has changed the ideas of space and time," Born wrote in one of his articles,* "another of Kant's categories, causality, has to be modified. The a priori character of these categories cannot be maintained." Naturally, if the idea of the a priori origin of such general categories as space, time, and causality had been overthrown, the roots of the belief in the a priori nature of any other principles of physics were destroyed. The primary source of knowledge is experience; this is the conclusion, the opposite of the a priori doctrine, to which contemporary natural science leads with compelling force.

A problem of no less importance is the struggle against dogmatism. We do not have in mind those ideas that arise from prejudice, from the dogmas of religion, or are imported into science from other nonscientific fields. On the contrary, we are speaking of ideas that arise in science itself, on the basis of definite experience. They become dogmatic because of poor methodology; it is not hard to expose the key feature of this methodology: having arisen on the basis of limited experience, ideas that are correct in a relative sense are then absolutized and transferred without being tested to a new and wider range of experience. This process of the petrifaction of ideas and their illegitimate transfer from one domain to another is the essence of dogmatization.

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*"Some Philosophical Aspects of Modern Physics," 1936; see the collection: Max Born, Physics in My Generation, Pergamon Press, London, 1956.

The struggle against dogmatization is not less — perhaps in our time it is even more — important than the struggle against the doctrine of a priori ideas. When our knowledge takes in a new domain of nature, when it makes great strides into the depths of things, it is especially important to have the strength of mind not to shrink from the necessity for a critical reexamination of the general concepts that arose in theories that reflect a different domain of nature or a different level of knowledge.

This critical reexamination of general concepts is a legitimate process of the development of science, which occurs continuously in contemporary natural science. It is only a pity that for many active scientists this process of reexamination of general ideas, like the struggle against the doctrine of a priori ideas, has gone on and is going on without awareness of the development of philosophical thought and of the struggle in philosophy. Knowledge of the development of philosophical thought could accelerate and facilitate this entire process, make clear its limits, and give it a direction appropriate for the description of nature. In particular, it would be clear that reexamination of concepts during a time of abrupt changes can proceed, and historically has proceeded, from different positions. In his own time Mach received great recognition from many natural scientists for his declared war against fetishism in science and against the periodic ossification of concepts. This was at the end of the last century and the beginning of the present one, when changes were occurring in ideas about absolute space and time, about the finite structure of the atom, and so on. But Mach made his attack on dogmatism from positivistic positions; he believed that a guarantee against ossification of concepts in science is to be found in the recognition that concepts are only conventional, and just for this reason very changeable, designations for series of sense impressions. This position for criticism of dogmatism is not tenable for natural scientists, and Born also admits the inacceptability of positivism.

The very first attempts to understand the regularities of the atomic domain led to the conclusion that a further break with classical ideas is necessary. This was a deep break and affected such categories as causality, contingency and necessity, possibility and actuality, physical reality, and so on, and this sharpened the conflict against the a priori doctrine and against dogmatism. In this period theorists tried to discard everything that in one way or another hampered their thought. Without much risk of being mistaken we can say that many scientists in the West came out against materialism only because they mistakenly associated with it ossification of concepts and attempts to impose on nature final and unchangeable laws, which were characteristic only of early, predialectic, materialism.

In this period of sharp change of ideas Heisenberg took the so-called "principle of observability" as a guiding principle for avoiding dogmatic transfer of old and spurious concepts into the new domain. The theory of the processes in question must be constructed by the use of only those quantities that are observable in the given domain of phenomena. If the orbits of the electrons in the atom are not observed, then they are spurious concepts that must be excluded, and in the theory of the atom one must rely only on the observed energy levels. At first glance it seems that this is a reasonable idea; at least it is true that quantum theory made progress only by renouncing the direct attack on the atom with the arsenal of classical ideas — orbits, continuity of radiation, and so on.

In principle Born supports this general proposition of Heisenberg's. He writes that with this principle Heisenberg "wished to found the new mechanics as directly as possible on experience. If this is a 'metaphysical' principle, well, I cannot contradict; I only wish to say that it is exactly the fundamental principle of modern science as a whole, that which distinguishes it from scholasticism and dogmatic systems of philosophy. But if it is taken (as many have taken it) to mean the elimination of all non-observables from theory, it leads to nonsense. For instance, Schrödinger's wave function ψ is such a non-observable quantity, but it was of course later accepted by Heisenberg as a useful concept. He stated not a dogmatic, but a heuristic principle."

But the meaning of introducing the principle of observability is not that it is to save physicists from spurious concepts by the simple recipe: avoid non-observables. Born has remarked that in physics one does not succeed in avoiding quantities that are not directly observable. This is undoubtedly true, and it is not unexpected, because theories operate with categories which are by their nature abstractions, and therefore in the general case are not directly observable. But as soon as it is admitted that non-observables can also legitimately appear in the theory, then nothing remains of the principle of observability, because the general idea of the necessity of developing science on the basis of experience only is by no means an exclusive prerogative of this principle, even if we interpret it in the most favorable sense. Moreover, any influence on science is exerted not by the unexpressed intention of the author of a recipe, but by the actual content of the recipe itself, which reduces just to the exclusion of non-observables. Therefore Born's correct statement that the exclusion from the theory of all non-observables leads to nonsense cannot be taken as anything else than an admission of the incompetence of the "principle of observability."

The question just considered is very instructive from the epistemological point of view. In recommending the principle of observability, Heisenberg makes a double mistake: a factual mistake, which Born also notes (non-observables cannot be excluded from the theory), and a methodological mistake. The methodological mistake consists in the unescapable assumption that a purely empirical recipe could be found as a guarantee against spurious concepts.

But recipes are useless weapons in the field of thought. Furthermore, in reasoning a recipe inevitably becomes at once the very thing against which Heisenberg was trying to aim his "principle of observability" — a dogma, divorced from the actual process of cognition. Guarantees against the introduction of spurious concepts are provided not by some recipe or other, but by the unity of the correct doctrine, which examines as a whole the process of the reflection of the object in cognition, and consequently considers the source and the laws of development of concepts and theories, and finds the general conditions under which concepts developed in one domain can be transferred to another domain.

This is indeed the conception that was spoken of above — the theory of cognition that is based on the recognition of an objective world which is approximately reflected in our theories.

Let us examine how this recognition requires scientists to deal with general categories.

The results of the development of science, especially during the last century and a half, show that the objective world is a unit and at the same time manifests specific properties in its parts. The unity of the objective world is the basis of its capacity for being known; it manifests itself, in particular, in the existence of a single logic and of general categories and laws, such as motion (in the broad sense), space-time forms, causal interrelations, and other equally general or less (but still sufficiently) general categories. In our time, when the possibility has been proved of unlimited reciprocal transformations not only of forms of motion, but also of elementary particles and fields, it would be naive to think that the categories and laws that reflect some objects are not connected with the categories and laws of other objects, though perhaps through complicated intermediate steps. And this connection in fact manifests itself at every point. The general categories discovered in one domain are not entirely discarded, but only changed in form within the range of new experience. In contemporary physics this situation finds its reflection in the fact that new experience is expressed with the same mathematical apparatus, except that it contains characteristic parameters that vanish for the old domain and take definite values for the new. Thus the new theories in physics are generalizations of the old ones.[*]

In seeking out the outlines of the new theory of atomic processes, physicists relied on the Correspondence Principle, which directly expresses the fact that quantum theory is a generalization of classical theory and goes over into it in cases in which one can neglect a characteristic parameter, the so-called Planck constant or "quantum of action" (1.05×10^{-27} erg sec). As has been stated by Niels Bohr, at a certain stage of the development the Correspondence Principle was the "only guiding principle" in the new quantum theory that gave relations between the unexpected experimental results in the microscopic domain and macroscopic processes.[†] In this article Born also refers repeatedly to the Correspondence Principle, noting the fact that during the period of the development of quantum mechanics this principle guided the process of derivation of quantum formulas from classical formulas.

In just the same way, W. Heisenberg bases his very latest attempts to create a theory of elementary particles on this same principle: he tries to construct a generalized theory in which there is to appear as a characteristic parameter a new constant, the so-called elementary or minimal length. The idea is very simple, and at the same time interesting: in regions in which the new constant can be neglected processes follow the laws already known, but in regions comparable with this constant ($\sim 10^{-13}$ cm) laws specifically applying to the elementary particles should manifest themselves, and this should do away with the difficulties with infinities that appear in crude attempts to apply the present theory to elementary particles.* Thus, as in the previous revolutionary stage, when quantum theory was being developed, Heisenberg is trying to construct a generalized theory that is to have as a special case, for a certain value of a new characteristic constant, the presently existing quantum theory. The search for such theories is reasonable only in virtue of the existence of general categories in the external world, in virtue of the unity of the world.

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[*] This conclusion applies with compelling force to contemporary physical science. It is not an accident that it is reached independently by specialists in different fields who study the crucial stages of the development of science, for example the outstanding Russian geometer and famous student of the development of non-Euclidean geometries, V. F. Kagan (cf. V. F. Kagan, *Lobachevskiĭ*, Moscow 1944, page 328) and also the famous German physicist who discovered light quanta—the first window into the microscopic world—Max Planck (cf. Max Planck, *Vorträge und Erinnerungen*, article "Sinn und Grenzen der exakten Wissenschaft").

[†] Cf. N. Bohr, "Discussion with Einstein" in the collection: *Albert Einstein: Philosopher-Scientist*, 1949.

For more detail on the Correspondence Principle cf. I. V. Kuznetsov, Принцип соответствия в современной физике и его философское значение, (The Correspondence Principle in Contemporary Physics and Its Philosophical Significance), Gostekhizdat, Moscow 1948.

The specific character of structures in the objective world, on the other hand, is manifested particularly in the fact that concepts developed within the limits of old experience cannot be applied unchanged in the domain of new experience. On becoming categories of the new, generalized, theory they themselves change their nature to some extent; thus the momenta and coordinates of microscopic objects, though they have something in common with classical momenta and coordinates, are still different in their fundamental nature; they are quasi-momenta and quasi-coordinates, with a different relation of a specific sort between them: for example, unlike the classical quantities, they do not commute.

The above statements about categories are essentially the two sides of the same proposition: the general categories are altered in a specific way in the changed object, just as in the theories that reflect it. I repeat: this proposition is not the demand of any philosophical dogmas, but a conclusion from the development of natural science, and also of the historical sciences and of philosophy. It is actually by this proposition that Max Born was also guided when, for example, he quite correctly expressed the idea that in quantum physics the principle of causality, as properly understood, is not discarded, but only takes a new form as compared with Laplacian determinism.†

This is the correct doctrine, which regards concepts and theories as reflections of the properties of the material world. For the exclusion of dogmatism and doctrines of the a priori it has no need of special recipes, like Heisenberg's "principle of observability," which have been shown to be unacceptable. It is incompatible with dogmatism simply because it admits the development of the forms of categories and laws in accordance with their transfer to new objects. It is clear that these new forms are not given a priori, but are found out by experiment. In this struggle against a priori ideas and dogmatism, and in this acceptance of objective experience as the source of knowledge, the conception we have described has much in common with the views of Max Born.

Its advantages, however, which are due to its completeness, manifest themselves in the fact that, as compared with Born's discussion, it makes a different evaluation of the role of experiment, and also of the significance of epistemological principles.

Born unconditionally sets experiment in opposition to tradition; he writes that "essential distinction between our time and the middle ages consists in the renunciation of tradition and the establishment of experience as the true source of knowledge." He enters on the path of denying philosophy as a science and rejecting its guiding influence, because he asserts that "as soon as they (principles, i.e., discoveries of natural science — S. S.) have become a part of a philosophical system there begins a process of dogmatization and petrification." And this proposition is regarded as an immutable law of knowledge, manifesting itself in any philosophy. In the concluding words of his article Born thus describes the position of the scientist: "But I believe that there is no philosophical highroad in science, with epistemological signposts. No, we are in a jungle and find our way by trial and error, building our road behind us as we proceed."*

But this attitude is contradicted by the actual history of the development of science, and especially by that of quantum physics. Born is of course right when he states that the development of the new quantum physics represents a deep revolution in ideas. Nevertheless the new ideas did not spring up on bare soil. Born himself notes in his article that "both aspects of quantum mechanics are in a large degree based on the work of Hamilton," that Hamilton prepared the way for the wave form of the theory by explaining the connection between geometrical optics and the wave theory and demonstrating the close analogy between Fermat's principle in optics and his own formulation of the principle of least action in dynamics. But also the other form of quantum mechanics, which is characterized by the use of matrices and operators, can be traced to fundamental ideas of Hamilton, to his noncommutative algebra. And we know by the admission of Bohr, Born, and other scientists that the Correspondence Principle, of which we spoke earlier, served as a kind of epistemological signpost at this difficult turning-point in the development of physics. The very statement of the new problems was evidence of great progress in science, and the inability to find the solutions at once only says that the results that had led to the statement of the new problems had not yet been subjected to sufficient analysis and generalization. Whatever novelty may characterize the problems confronting a science, the scientist must not believe that he is in a jungle and is feeling his way blindly.

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*Cf.: "Die Plancksche Entdeckung und die philosophischen Grundfragen der Atomlehre," report by Heisenberg on the occasion of the centenary of the birth of Max Planck, April 1958. Russian translation published in Uspekhi Fiz. Nauk 66, No. 2 (1958).

†M. Born, Natural Philosophy of Cause and Chance, Oxford 1949.

*These views are held not only by Max Born, but also by Niels Bohr. Thus Bohr relates that when Einstein expressed to him a feeling of dissatisfaction because of "the apparent lack of firmly laid down principles for the explanation of nature, in which all could agree," Bohr replied, "in dealing with the task of bringing order into an entirely new field of experience we could hardly trust in any accustomed principles, however broad ....." Cf. N. Bohr, "Discussion with Einstein."

If we had agreed with the statements quoted above, this would mean that in the name of the struggle against dogmatism we were forgetting an important historical fact, namely that our achievements are the result of the fact that we stand on the shoulders of previous generations, and not at all of nihilism regarding previously attained knowledge; philosophically this would mean we were reducing experience to the given single experiment, that is, were taking the position of pure empiricism. But in putting forward experience as the source of knowledge we understand it to mean not isolated experience, but accumulated experience.

We have grounds for concerning ourselves with accumulated experience precisely because it relates to a lawfully developing objective world, and not to a chaos of random flashes, "elements of sensation." Every experience that reflects the world genuinely, even though not completely, is a step toward deeper knowledge of the world.

It is in this that the materialistic interpretation of experience differs from the positivistic interpretation. Just because it has its source in the external world, experience accumulates and, as Born states correctly, has for us a compelling character. The acceptance of just isolated experiences does not distinguish science from mysticism, because every sort of rubbish, such as spirits of the dead, ghosts, and goblins, can exist in the isolated experiences of the superstitious, those led to believe in mysticism, or the mentally unbalanced.

But experience accumulates not in the form of a sum of separate facts, not in the form of a mosaic. It becomes generalized, takes form at first in a theory of the motion of a definite type of objects, and finally in a single image or shape of the external world, composed of many and various mutually related objects, an image which with the advance of knowledge becomes ever more exact and more refined, going from the primitive and concrete to ever more complex and abstract meanings. If this process of generalization occurs in the domain of sense impressions, as Born declares, it is no less true that it also goes on in the domain of cognition, forming what is called a world view.

The scientist is always guided by his world view, weighs new facts in the light of it, and at the same time enriches and develops his world view in the light of new facts. It is precisely experience, accumulated and generalized in a world view, that led to Born's rejection of Einstein's attempts to construct a theory of a geometrized field which has as its singularities particles with finite rest masses; led him to reject the idea with such conviction that, as he admitted himself in his report "Physics and Relativity" (1955), he did not even take the time to study Einstein's last work in detail.*

In this connection we would also like to point out the following. In this article, devoted to the roles of experiment and theory in physics, one thought is emphasized explicitly and with great force: theories are based on experiment. This is of course correct, but this is only one side of the problem. In the process of cognition theories play a no less important epistemological role. Though cognition begins from experience and experiment, and therefore this is a necessary element of cognition, still theory is just as essential an element of cognition, since it generalizes experiment and gives a coherent image of the object studied, which is the final goal of cognition, the giving of a richer idea of the object than that obtained from direct perception. At the same time, as has already been noted, having first appeared as a generalization of previous experiments, a theory that has been confirmed in practice serves as a guiding principle in the domain of subsequent new experiments. Furthermore, the results of contemporary experiments have such an abstract appearance and depend on such a complicated organization of suitable conditions that they can be obtained and given meaning only on the basis of a deeply developed theory; but this last role of theory in the treatment of experiments has also been pointed out by Born.

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*In this report Born said: ". . . . right from the beginning I just did not believe in their success and therefore did not study his difficult papers with sufficient care." (Cf. "Physics and Relativity" in the collection: M. Born, Physics in My Generation, 1956.

Thus if we are to understand experience as objective, generalized, historical experience, and not as subjective, isolated, empirical experience, we come to the conclusion that the scientist is by no means in a jungle, that he always has epistemological signposts by which he is guided. The struggle against dogmatism by no means requires the exclusion of the experience accumulated and generalized in theories and in our view of the world, it requires only that new experience has to be taken into account and must enter as a component part into the new image of the objects studied; that it must be taken into account in the search for the particular forms of laws and categories that are inherent in a given domain. Contemporary scientific materialism satisfies this requirement.

The dogmatism that has done harm to science arises not because use is made of experience accumulated and generalized in a unified world view, but because use is made of that world view and those traditions in which the categories are regarded as unchangeable and independent of new experience. Of course, many theories are discarded into the trash can of history. It would be incorrect, however, to follow a priori principles and treat as dogmatism those theories and that world view which truly correspond to generalized experience and have been tested and confirmed by practice.

III

It gives the writer especial pleasure to note the arguments of Born against the operational method of introducing concepts into science by identifying them with operations of measurement (P. W. Bridgman).

Born finds that this method is a reasonable one for defining concepts in classical physics, since here one has to do with quantities accessible to direct measurement. In the quantum theory, however, according to Born the operational method is "rather out of place" and "comes to grief," because in this theory an essential part is played by concepts that cannot be associated with any operation of direct measurement. "I cannot see," Born writes, "what experimental 'operation' could be devised in order to define a mathematical operator? Moreover, I have already mentioned that there are concepts used in wave mechanics which are not observable, for instance, Schrödinger's wave function; there are in principle no means to observe it, hence no 'operational' definition."

Born's criticism of operationalism shows that he is a subtle thinker, who reflects on the processes occurring in the theoretical thinking of physicists, and notices the dangers arising from them. It is all the more essential to point out that the operationalists, like Born, start from a high opinion of the importance of experiment; but in identifying concepts with operations of measurement they have arrived at an apologetics of pure empiricism. In coming out against the operational method of introducing concepts into science, Born has gone against a trend which has had a broad appeal for physicists, to whom it has seemed that this method saves science from uncontrolled speculations. But empiricism has never yet saved anyone from speculations, as Engels showed long ago; and indeed Milne's speculations about the date of the creation of the world, which Born criticizes, arise out of Milne's extreme empiricism.

The writer of these lines has had repeated occasion to speak out against operationalism as a method of defining concepts.* At that time I could not refer to any important physicist who had come out against operationalism; unfortunately it is only now that I have become acquainted with Born's article. Though I understand the importance of this unexpected support for my views, I still wish to record some considerations about Born's remarks on operationalism.

Born's criticism of operationalism consists of a direct reference to the fact that in quantum mechanics there are concepts with which one cannot associate any operation of direct measurement. The essence of this criticism is accordingly that there is empirically established a domain in which operationalism is obviously bankrupt. At the same time a domain is pointed out in which the operationalistic rule is useful; this is classical physics, in which operations of measurement can be directly assigned to concepts.

Here, however, there is still no analysis of the epistemological tendencies of operationalism, and this is necessary, especially in view of the fact that this doctrine is attractive by its appearance of definiteness, which is regarded as a reaction against "verbalism and word fetishism."

Let us inquire what is the meaning of the basic demand of operationalism, that concepts be introduced into science only through a description of an operation of measurement, even if only an imagined one. It means an understanding of the process of cognition, according to which one first defines concepts, by adopting appropriate procedures of measurement, and afterwards one looks for relations between the concepts (of course such as do not contradict experiment), that is, one develops a theory. But only standardized houses are built from prepared bricks and timbers. The development of new theories, especially those of contemporary science, occurs in more complicated ways. We shall not enter at once upon a discussion of these ways, but merely note, what Born also asserts, that the entire development of quantum mechanics shows how at first abstract formulas are gradually established for the compact description of sets of observations and measurements, and "understanding of their meaning follows afterwards." But this way is in obvious contradiction with the method of operationalism. The actual process illustrates the epistemological fact that a concept receives its content only through a verified and substantiated theory, through the useful part that it plays in a theory that reflects an objective process. There is nothing unusual in this, for science has shown that this is just the nature of the relation between the unified whole of any object and the categories subordinated to it, and theories are only the images or shapes of the unified objects.

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*Cf. e. g.: a) Uspekhi Fiz. Nauk 39, No. 1 (1949); b) Great Soviet Encyclopedia, 2d. ed. article "Operationalism," 1955.

The great merit of quantum mechanics lies precisely in this, that it taught us to understand the atom, and, indeed, any physical system, not as a mechanical conglomerate of component parts, but as a unity, in which there arises a law characteristic of the definite object (a "specific" law); this law is obeyed by the components, which in doing so change their own nature. Only in this way have explanations been found for such properties of atoms as their stability, saturation, and so on, which prequantum physics was unable to explain.

This sort of connection between a theory and the categories reflected in it manifests itself most prominently in quantum physics precisely because here it is most clearly seen that some of the categories of the theory do not exist at all outside the unity of the theory.

But in the present context it is essential to emphasize that this connection between theory and categories is a general law of cognition, which is manifested in any theory, and consequently also in classical physics. To convince ourselves of this, we examine the concept of temperature, for which Born readily admits the operational method of definition: "it is reasonable to introduce temperature by describing the thermometric operations," he writes. But for the concept of temperature to have physical meaning, it must satisfy a number of conditions. It must be shown that in thermodynamic equilibrium there exists a certain single-valued, monotonically varying function of the state of the system (for example, increasing monotonically with increasing energy of the system). The possibility of measuring the values of this function is determined, firstly by the fact that it must have the property of transitivity, whereas the energy of systems interacting by the exchange of heat has the property of additivity; secondly, by the fact that when the system passes to a new thermodynamic equilibrium there must be monotonic change not only of the required function, but also of at least one of the other parameters of the state of the system. The existence of a function satisfying these requirements is established in thermodynamics.* This function is the temperature, a concept organically connected with all the other categories of thermodynamics. This extremely indirect definition of temperature also makes clear the limits on the use of this concept. Thus when there is no thermodynamic equilibrium, for example in an electric discharge, the concept of temperature cannot be applied; this limit on the domain of application of the concept of temperature cannot be perceived when it is defined in terms of a thermometric operation.

It follows from what has been said that although before the development of the theory it was possible to measure temperature and even have a primitive idea about its meaning (for example, as the "degree of hotness of a body"), the definition of the scientific concept of temperature is given not by a procedure of measurement but by the theory in which the concept appears as one of its features, as a category connected by the theory with other categories. On the other hand, the possibility of measuring the temperature and the very procedure of the measurement bare based on the presence of definite objective properties that provide the conditions for the definiteness and uniqueness of the measurement and define the limits of the usefulness of the concept; consequently this possibility is based on a certain theory that reflects specific objective processes, in the present case the theory of thermodynamics. Similar arguments apply to all other concepts.

We thus see that operationalism is not able to reflect the genuine process of cognition because of its basic shortcoming: being an empirical conception, it ignores the part played by theory in the process of the formation and development of concepts.

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*Cf. e. g., M. A. Leontovich, Введение в термодинамику, (Introduction to Thermodynamics), Gostekhizdat, Moscow 1952.

One more point must be emphasized: operationalism attaches to each concept, once and for all, a definite procedure of measurement; by so doing it restricts the content of a concept that historically has been discovered before others. Every object, however, that is reflected by a concept (or every category inherent in an object) has a multitude of interrelationships with other objects (or categories), which may turn out to be of greater importance than the relation already adopted as the definition of the concept, though they are discovered only afterwards. For example, the concept of mass is not exhausted in its meaning by Mach's operationally motivated fixation of the role it plays in the mutual accelerations of bodies; in fact, mass also has other essential properties and relationships that were discovered later, for instance its connection with energy, or its dependence on relative velocity, and it is scarcely legitimate to suppose that these relations have less significance than the one Mach took as the basis for the definition of mass. With the advance of knowledge it is found that the definition of a concept in turns of that one of the possible measurement procedures that had become known is a limited one from the historical point of view. Such a definition either turns the concept into an extra-historical category given once for all, or else comes to be regarded as purely conventional. It is well known, for example, that Mach, who also identified each concept with a definite operation of measurement, regarded these definitions as no more than conventional stipulations; he based this view on the fact that the historical sequence of discoveries is accidental and by no means uniquely determined. Let us recall the course of one of these arguments given by Mach. Asserting that physics regards heat as motion and electricity as a substance, Mach shows that this difference in the ideas is determined by the methods of measuring them that developed historically. "In studying the discharges of a Leiden jar," Mach writes, "we can use two different operations of measurement: one with the Coulomb balance, constructed in 1785, and the other with the Riess thermometer, invented in 1838; Since the time of Coulomb the result of the first measurement has been called quantity of electricity, and we call the result of the second the potential." "When the electrical discharge of a Leiden jar produces heat, its potential changes, and, as the Riess thermometer shows, the value decreases. But the quantity of electricity, according to Coulomb's measurement, remains unchanged. Now let us imagine that the Riess thermometer was discovered earlier than Coulomb's torsion balance. It is not hard to imagine this, because these inventions do not depend on each other in any way. Would it then not have been more natural if the quantity of electricity contained in a Leiden jar was evaluated in terms of the heat produced in the thermometer? But then the so-called quantity of electricity would have decreased on the production of the heat, whereas now it remains unchanged. Consequently electricity would in that case not have been a substance, but would have been a motion, whereas now it is still a substance. From this it is clear that if we think about electricity otherwise than about heat, this fact has a purely historical and quite accidental conventional basis." In Mach's general conception this conventional quality of concepts is justified by the fact that concepts, according to Mach, are not the reflections of objective categories, to which they must correspond. "In the study of nature," he writes in connection with the arguments given above, "all that is of importance is a knowledge of the relations of phenomena. Whatever we imagine as behind the phenomena exists only in our minds, and has for us only the significance of a mnemonic device or formula, whose form, being arbitrary and immaterial, changes very easily with the state of our culture."*

Mach's position was purely positivistic. A scientist who starts from the existence of an objective world outside ourselves is of course unable to accept this position.

All these considerations lead us to the conclusion that the operationalistic method of introducing concepts into science does not assure the objectivity of knowledge, and therefore is unacceptable; it is unacceptable not only in quantum physics, but also in general, as an epistemological method.

Let us summarize briefly.

Born is right in holding that there are no other ways of knowing the external objective world except by interacting with it. New experience changes old ideas about the world in a compelling way, deepening them and making them correspond more truly to the world. The fact that the development of our ideas is forced on us shows the existence of an objective content in our experience. From this arises the high value given to experience, to experiment, as the source of cognition.

But a reflection of the external world is not confined to a single isolated experience; the single experience is generalized and given meaning in a theory; relying on experience, man is free to represent the most subtle and complicated relations in the world in terms of abstract theories, which therefore give a deeper knowledge of the world than the single direct experiment. From this comes the value given to theory, as a deeper representation of the external world, in which experience is generalized and given meaning. Practical activity on the basis of the theory thus developed verifies its correspondence to the external world.

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*Cf. E. Mach, History and Root of the Principle of the Conservation of Energy (Russian translation from German original, S. Peterburg 1909).

Experience, accumulated and generalized in theory, and thereupon in a unified world view, starting from the recognition of an external world and its reflection in human consciousness, enables the scientist to overcome blind expiricism and is his guide in the progressive advance of knowledge. The justified struggle against dogmatism cannot exclude the guiding role of theory and the world view, provided only that they accurately reflect the regularities of the objective world.

These are the inescapable conclusions that must be reached by a scientist thinking about the question of the roles of experiment and theory in cognition.

Translated by W. H. Furry

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RESEARCH ON CONTROLLED THERMONUCLEAR REACTIONS IN THE U.S.S.R.

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INTRODUCTION

1. The idea of using controlled thermonuclear reactions for power generation by exploiting the fusion of light nuclei has been proposed many times in the past. However, it is only after many years of preliminary work, the results of which were not immediately obvious, that a basis had been laid for justifying the hope that a successful solution for this problem could be realized. There has been a rapid expansion of the scale on which research in this field is being carried on and at the present time the development of methods of obtaining controlled thermonuclear reactions is one of the most important problems in nuclear technology.

An important factor in the rapid development of scientific research in this field was the removal of rigid secrecy restrictions which had earlier isolated physicists working on this problem in different countries. As long as the prevailing opinion was that the danger of publishing scientific results was greater than any advantage which could be obtained by exchanging scientific information, the research work proceeded slowly. A significant change took place in 1956 when the results of work carried on in the U.S.S.R. were communicated for the first time. These disclosures were followed by the gradual publication of the results of research carried on in England and the U.S.A. It then became possible to carry on an exchange of ideas and experiences and this was of incalculable value, especially for a project of such scientific and technical difficulty as that of attaining a controlled thermonuclear reaction.

In spite of the wide range of research on controlled thermonuclear reactions, at the present time all approaches to this problem are more or less in the exploratory stage. No one approach has been definitively evaluated. The one thing that is apparently agreed upon by all concerned is that the solution of the problem will require a method in which the confinement of a hot plasma is realized by means of a strong magnetic field.

2. The methods of using a magnetic field for confinement and heating of a plasma can be divided into two basic classes. One class consists of methods in which the plasma is accelerated by electrodynamic forces; the other consists of methods of producing equilibrium plasma configurations, i.e., states in which the pressure of the plasma is balanced by magnetic pressure. The difference between these two general methods can be seen more clearly if we express these ideas in terms of magnetohydrodynamics. As is well-known, magnetohydrodynamics is concerned with the general behavior of a conducting fluid in a magnetic field. Under certain conditions, which we shall assume satisfied, a plasma may be considered, from the macroscopic point of view, to be a conducting fluid. The equation that describes the behavior of a plasma subjected to electrodynamic forces is of the following form:

$$\rho \frac{d\mathbf{v}}{dt} = \frac{1}{c}\,\mathbf{j} \times \mathbf{H} - \operatorname{grad} p. \qquad (1)$$

The quantities $\mathbf{v}$ and ρ denote respectively the velocity and density of an elementary volume of the plasma, which moves under the action of the electrodynamic forces and the pressure differential. All terms in the equation refer to a unit volume of the plasma. The electrodynamic force operating on a unit volume of the plasma is represented by the first term on the right side of the equation. This term is due to the interaction between the magnetic field and the currents that flow in the plasma ($\mathbf{H}$ is the field intensity and $\mathbf{j}$ is the current density).

Examination of this equation reveals that two limiting cases can be considered, each characterizing a large class of confinement methods.

If the gas kinetic pressure is relatively small, the electrodynamic force is balanced by "inertial forces":

$$\rho \frac{d\mathbf{v}}{dt} = \frac{1}{c}\,\mathbf{j} \times \mathbf{H}.$$

Under these conditions, the electrodynamic forces can impart to the plasma as a whole a directed velocity that can exceed significantly the random thermal velocities of the ions. The kinetic energy of the directed motion acquired by the plasma in its acceleration in the magnetic field can be used for subsequent heating in processes such as compression, impact of accelerated plasmoids on a target, etc. Specific applications of these techniques will be described below.

A feature of this type of interaction between the plasma and a magnetic field is the short duration of the process. In order-of-magnitude terms this quantity is a/v, where a is the distance traversed by the plasma under the action of the accelerating forces and v is the velocity achieved. In cases of practical interest the duration of the acceleration process is on the order of $10^{-5} - 10^{-6}$ sec. It is apparent that pulsed processes of such short duration are of interest only if they can be used in the primary phases of plasma heating. These processes must result in the transformation of kinetic energy into heat and a transition to some quasi-stationary state in which the rapid inertial motions which continue after the first phase are damped out rapidly.

The other limiting case is that in which the acceleration of the plasma is small and the "inertial term" on the left side of the equation can be neglected compared with the pressure gradient. In this case the gas kinetic pressure and the magnetic pressure balance each other at all times:

$$\text{grad } p = \frac{1}{c} \mathbf{j} \times \mathbf{H}.$$

It is possible to think of a number of different ways of realizing equilibrium plasma configurations, characteristic of a quasi-stationary state of the plasma in the magnetic field. At the present time the following more-or-less clearly defined trends seem to be developing:

(a) methods for confinement and heating of a plasma in systems with high discharge currents maintained by external voltages and stabilized by external magnetic fields;

(b) magnetic-trap methods, in which high-temperature plasmas are produced by the accumulation of fast particles injected into a trap.

This classification of the methods of obtaining and maintaining high temperatures in a plasma is obviously arbitrary; however, it is rather well suited to the purposes of the present paper since the divisions indicated here correspond to the basic approaches being used in the U.S.S.R. for research into controlled thermonuclear reactions.

3. Before describing some actual results obtained in the theoretical and experimental research carried out along the lines indicated above, we shall consider certain general characteristics that relate to the future of all thermonuclear reactors. Obviously any discussion of this kind, at the present stage of our knowledge, is based on our faith in the ultimate triumph of human ingenuity in this problem.

First of all, it must be noted that any magnetic-confinement system proposed for a solution to the problem of obtaining thermonuclear power must satisfy one basic requirement: the energy evolved in nuclear fusion must exceed the energy consumed from other sources used to maintain the high temperature in the plasma.

A simple analysis will show that this requirement can be written in terms of the following relation between the basic parameters which describe the performance of a thermonuclear generator:

$$H^2\tau > A(1-\eta). \tag{2}$$

In this formula H is the intensity of the magnetic field that confines the plasma, and τ is the time during which the high temperature is maintained in the plasma. The quantity η denotes that fraction of the thermal energy of the plasma which is transformed into electrical energy over the cycle. The constant A depends on the nuclear fuel. Under most optimistic assumptions concerning the processes which take place in a thermonuclear generator this constant may be taken as 10^{10} if pure deuterium is used and 10^8 if a mixture of D and T, in equal proportions, is used. These values are based on the assumption that the thermonuclear reactions in the plasma occur at the "optimum" temperature; in deuterium this temperature is 50 kev (5×10^8 degrees) and in a D-T mixture, 15 kev (1.5×10^8 degrees).

In applying (2) with the indicated values for A one must keep in mind the fact that, strictly speaking, this relation describes only the ideal case, in which particles do not escape from the plasma under high-temperature conditions. In turn, this implies that the particle lifetimes are the same as the time during which the high temperature is maintained in the plasma.

As follows from Eq. (2), the shorter the time interval during which the high-temperature is maintained, the higher the required intensity of the magnetic field. In order for these requirements to be satisfied by present-day electrical facilities, it is necessary to use a method which permits confinement of fast particles in a plasma for periods of time that may be several seconds or even tens of seconds. Thus, if we assume that $\tau = 10$ sec the field intensity required in a pure-deuterium generator is on the order of 30,000 oersteds. A field intensity of this kind is within the capabilities of stationary apparatus. It should be noted, however, that in this case the power generated per unit volume of the generator would be small; for a complex machine of this kind to be of technological usefulness it would have to be of enormous dimensions.

4. We now consider the problem of direct conversion of thermonuclear energy into electrical power. The energy evolved as a result of the fusion process consists of two parts, which play roles of greatly differing importance in the operation of a thermonuclear generator. The fraction of the energy carried away by neutrons has no effect on the processes that take place in the plasma. In the computation of the electrical energy balance for the generator this factor is of the order of 0.3 (i.e., the same as that which applies for the energy generated in ordinary nuclear power stations that use fission of heavy nuclei). The other part of the nuclear fusion energy, which appears in the formation of high-energy charged particles, is released directly into the plasma, causing a temperature rise that can be converted into electrical energy with an efficiency close to unity.

The possibility of direct conversion is due to the fact that in magnetic confinement the high-temperature nuclear fuel is surrounded by a strong magnetic field, which acts very much like an elastic shell in compressing the plasma. If the high-temperature plasma expands, its thermal energy is used to do work against the magnetic pressure and is thus converted into electrical energy. If the maximum temperature of the plasma at the onset of expansion is T_1 and the minimum temperature to which the plasma is cooled at the end of each working cycle is T_2, the maximum value of η is given by the well-known formula

$$\eta = 1 - \frac{T_2}{T_1}.$$

From this it follows that in principle the quantity η can be very close to unity, inasmuch as the upper temperature in the thermal cycle is very high, making the ratio T_2/T_1 extremely small.

It should be kept in mind, however, that a substantial reduction in temperature during expansion can be realized only at the expense of an increase in the volume occupied by the plasma. In adiabatic expansion the temperature is inversely proportional to $V^{2/3}$, where V is the volume occupied by the plasma. Hence, to convert a significant part of the thermal energy of the plasma into electrical energy the volume occupied by the plasma must be increased by a large factor. This means that during the time at which the plasma is at maximum temperature, and is a source of thermonuclear power, it must occupy a very small part of the volume of the vacuum chamber in the thermonuclear generator; the remaining volume must be filled solely by a strong magnetic field. Thus, when η is close to unity only a small part of the generator volume is used efficiently; for this reason a compromise value of η, probably less than 0.75, must be used.

The basic advantage of the direct conversion of thermal energy of the plasma into electrical energy lies in the fact that in this method the utilization of the thermal energy of the plasma is characterized by a huge reduction in the irreversible thermal losses (which are proportional to $1-\eta$). In addition, the operating conditions in the system are more favorable from the thermal point of view because the thermal load on the walls of the plasma chamber is reduced.

The total amount of energy produced by a thermonuclear generator should not depend on whether direct conversion of thermal energy to electrical energy is used or whether this conversion takes place by means of an intermediate heat transfer agent. This is easily seen if it is recalled that a large part of the energy released in nuclear fusion is carried away by the neutrons, and cannot therefore be directly converted into electrical energy.

5. The development of methods of launching controlled thermonuclear reactions are based on the supposition that the cource of energy must be deuterium or a mixture of deuterium with tritium. Of these two nuclear fuels, it would seem that in the future the D-T mixture will be the important one. The chief advantage of this mixture is the high effective cross section for the reaction. Over the entire temperature range of practical interest, the reaction yield for a D-T mixture (equal amounts) exceeds the reaction yield for pure deuterium by two orders of magnitude. Although the tritium obtained from conventional nuclear reactors is very expensive at the present time, this factor will only be a temporary barrier to its utilization, since there are methods that can be used to compensate for the tritium losses in a thermonuclear generator.

In each elementary D-T reaction event one tritium nucleus disappears and a fast neutron (14.1 mev) is produced and subsequently leaves the plasma. If the thermonuclear generator is surrounded with a thick layer of material in which the (n, 2n) reaction is induced by fast neutrons, it is possible to increase the primary neutron flux significantly. Neutron multiplication via the (n, 2n) reaction can be accomplished through the use of beryllium or heavy elements such as lead or bismuth. In these elements the cross section for the (n, 2n) reaction for neutrons at 14.1 mev is much greater than the cross sections for competing nuclear reactions. If any one of these materials is used as a blanket for the reactor the number of neutrons should increase by a factor of 1.5 or 2. This increase in the neutron flux can be used for breeding tritium by disintegration of Li^6. An analysis of the data referring to the (n, 2n) reactions and the tritium producing reactions indicates that even for very conservative estimates the tritium breeding ratio in a thermonuclear reactor can easily be made greater than unity.

Thus, in working with a D-T mixture it should be possible to realize conditions for which the supply of tritium increases in the course of time. Hence, as long as there is no danger of exhausting the available supplies of Li^6 thermonuclear reactors can be operated with a D-T mixture and also used to regenerate tritium. It should be added that even if for some reason it becomes necessary to give up the regeneration of tritium and work with deuterium the principle energy effect will still be obtained from the tritium formed in the D-T reaction.

At this point we turn to an examination of the results of the experimental and theoretical research carried on in the U.S.S.R. on the problem of controlled thermonuclear reactions in the past few years. Our attention will be directed chiefly to the results of new work which has not yet been discussed widely. The material will be considered in accordance with the research classifications indicated above.

I. FAST PULSED PROCESSES

6. Up to this time, the major effort in the study of fast pulsed processes has been concentrated on the features of intense pulse discharges in deuterium at low pressure. The main idea here is to obtain a high temperature and a high density in a compressed plasma column for a short period of time. Investigation of pulsed discharges with a very high rate of rise (from 10^{10} to 10^{11} amp/sec) has shown that whether or not such discharges take place in straight tubes or in toroidal chambers, the important factor is the acceleration of the plasma by electrodynamic forces. In the initial phases of a pulsed discharge the plasma contracts to the axis of the discharge tube; this contraction is the first stage in the rapid oscillations of the plasma pinch. The maximum temperature and density are achieved at the time at which the plasma pinch is of minimum radius.

The theoretical analysis of effects that take place in a plasma in pulse discharges has passed through two states. At first a very simple model was used for a qualitative explanation of the dynamics of a plasma pinch. The motion of the mass of ionized gas was analyzed as a whole under the effect of the given electrodynamic forces. In the next stage of development an attempt was made to carry out a more rigorous quantitative analysis of the contraction and oscillation of the plasma column, taking into account the formation of shock waves in the plasma and the time variation in the mass of the moving gas. This calculation was carried out in the magnetohydrodynamic approximation, in which the plasma is considered a monatomic gas with constant conductivity. Even in this idealized form the problem leads to a complex system of partial differential equations, which can only be solved by numerical integration in large electronic computers.

The solution of this problem yields data on the density, temperature, and drift motion for each segment of the plasma column at various moments of time as well as data on the distribution of current density and magnetic and electric field. The amount of information of this kind is considerably greater than that which can be obtained from experimental studies of plasma properties. However, it is possible to check the theoretical results at a number of important points by making comparisons with the experimental data. Comparisons of this kind can be made for the distribution of current density and radial plasma velocity since these features of the process are amenable to experimental measurement. Comparisons of this type between experiment and theory, which have been carried out for a number of particular cases have, generally indicate satisfactory agreement at least as regards the initial stages of the pulsed process (up to the second contraction). For this reason there is some justification in using the results of the theoretical calculations to estimate quantities which are difficult to measure experimentally with the required accuracy. The most important of these plasma characteristics are the temperature under the maximum implosion conditions of the shock wave and the distribution of matter at this instant of time.

The numerical calculations indicate that the following equation can be used to estimate the lower limit of the temperature at the time of the first contraction:

$$\overline{T} = 4.6 \cdot 10^{12} \frac{I^2}{N}, \qquad (3)$$

where I is the current strength in kiloamperes at this instant of time and N is the number of particles of a given sign per unit length of pinch. This equation gives the mean plasma temperature value (in kev) on the assumption that there is ideal heat exchange between the ions and electrons. The radial temperature distribution for a pinch is shown schematically in Fig. 1. The dashed line represents the mean value of T. Because of the cumulative effect produced by the incident shock wave, at small distances from the axis the temperature

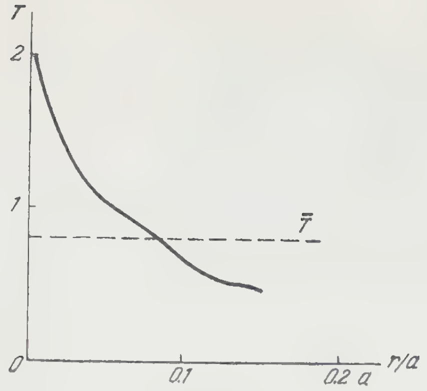


FIG. 1

increases sharply and exceeds the mean value by a large factor. If we also note that under typical experimental conditions thermal equilibrium between the ions and electrons is achieved at maximum contraction only when $T < 0.1$ kev, it becomes apparent that the formula given above corresponds to a very conservative estimate of ion temperature in an actual pinch. Therefore we can use it with confidence in estimating the lower limit of the temperature achieved in the latest experiments with high-power pulsed discharges.

Figure 2 shows the density distribution of matter at the implosion maximum. The maximum density at this instant of time exceeds the initial gas density by a factor of 30 or 40. One of the interesting features of the theoretically determined density distribution is the diminution near the axis. This type of density variation is directly related to the temperature rise near the axis (the pressure, which is proportional to pT, should level off in this region).

7. Experiments carried out in recent years on high-power pulsed discharges have generally been following the trends of earlier research, the results of which were published in 1956 and are well known. In these recent experiments much attention has been devoted to improving the various discharge parameters with the purpose of achieving higher temperatures.

Improvements in the construction of spark-gaps and in the power systems have made it possible to increase the voltage per unit length of discharge tube and to reduce the stray inductance of the circuit. As a result it has been possible to increase the rate of rise of the current and the current during the first contraction (500 kiloamperes in a discharge tube 50 cm long and 40 cm in diameter). In Fig. 3 is shown the dependence of temperature in a plasma pinch on the initial voltage applied to the discharge tube for a tube filled with deuterium at an initial pressure of 0.05 mm Hg. The temperature has been computed from Eq. (3) which, as has been indicated above, yields values which are too low. For this reason we can assert with confidence that in these experiments temperatures exceeding 3 or 4 million degrees were certainly attained. In particular, it may be assumed that the neutron radiation obtained under these conditions is due, in large degree, to thermonuclear reactions.

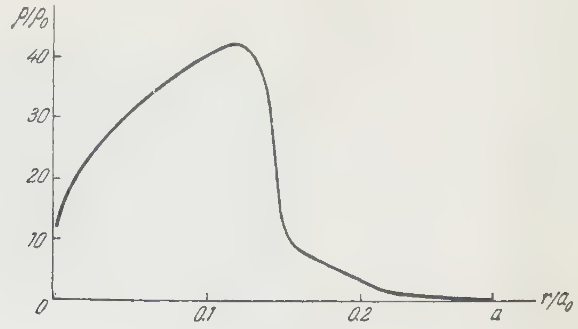


FIG. 2

In this connection it should be noted that in these enhanced power reactions neutrons appear immediately after the first contraction phase, i.e., at the time at which both the temperature and density are at a maximum; the neutron pulse extends over a period on the order of a microsecond.

At the present time, however, the proof or disproof of the thermonuclear origin of a small burst of neutron radiation in a pulse discharge is not important enough to warrant special attention. For this reason I do not think it is necessary to insist that thermonuclear reactions have actually been observed in the experiments mentioned above. The question of whether or not a given neutron belongs to the noble race descended from thermonuclear reactions or whether it is the dubious offspring of some disreputable acceleration process is something which may upset the representatives of the press, but at the present stages of the problem, should not disturb scientists. When the number of neutrons produced in one discharge pulse becomes greater than 10^{12}, any doubts as to the origin of this effect will be dispelled.

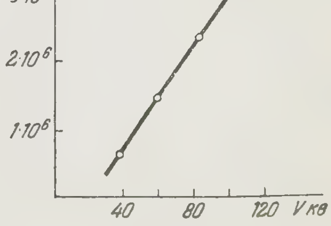


FIG. 3

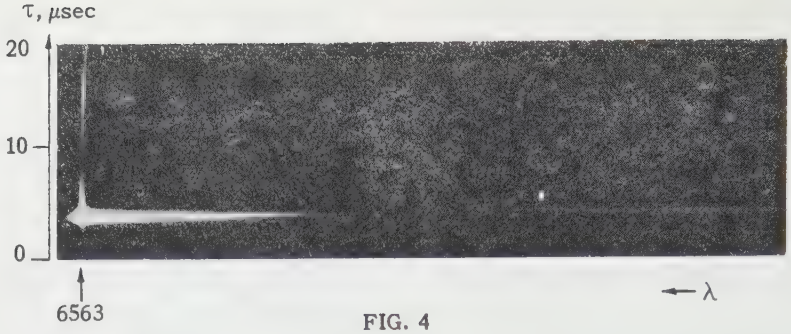


FIG. 4

However, to attain intensities of this kind in thermonuclear neutron radiation in pulsed discharges, it will be necessary to carry on experiments with circuits in which the values of the parameters are much higher than has been the case heretofore. The chief difficulty in this respect is that any further increase in the power of a pulsed discharge is limited by heating of the chamber walls. To a certain degree this difficulty can be overcome by using chambers with sectionalized metal walls. However, attempts should be made to investigate other methods: for instance, magnetic shielding of the walls and use of vacuum chambers with local injection of directed gas streams.

In addition to the aforementioned series of experiments with high-power discharges, a number of other investigations have been carried out to study the various properties of the high-temperature plasma in a pulsed discharge. There have been many advances in spectroscopic investigations of plasma. After a method was developed for obtaining streak photographs of discharge spectra it was found that at the time of the first contraction there is a sharp increase in the continuum over the entire range. This flash is shown clearly in the photograph in Fig. 4, which is a streak photograph of the spectrum of a discharge in hydrogen with an initial pressure of 0.1 mm Hg and an initial voltage of 35 kv. The continuum flash is explained by the fact that at maximum contraction there is a discontinuous rise in the plasma ionization and this results in an increase in the concentration of free electrons. In turn these cause intense bremsstrahlung and recombination radiation. By measuring the intensity of the continuum in a given spectral range it is possible to make a fairly accurate measurement of plasma density (under the assumption that total ionization obtains).

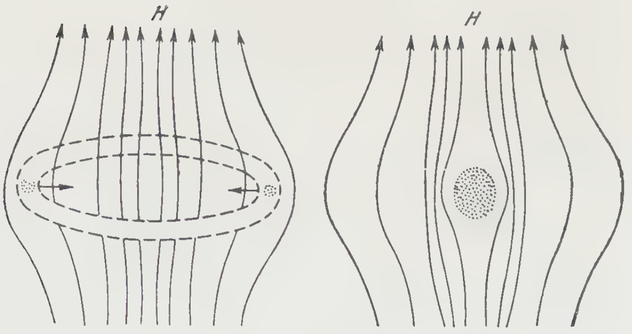
The calculations indicate that, because of fortuitous circumstances in the numerical values, the intensity of the radiation is very insensitive to electron temperature. For this reason any arbitrariness in the choice of electron temperature has essentially no effect on the determination of density by means of measurements of the spectral continuum. Density measurements carried out in this way yield results which are in good agreement with the values obtained from magnetohydrodynamic theory.

Spectral observations also make it possible to evaluate the plasma temperature from the Doppler broadening of impurity lines. As yet this method has been applied only to discharges under standard conditions (initial voltage 35 kev, hydrogen pressure 0.05 mm Hg, tube length 90 cm). Under these conditions the plasma temperature at maximum contraction as determined from the width of the nitrogen line (λ 3479 A) is found to be approximately 100 ev, as compared with the 65 ev predicted from Eq. (3).

The aim of a number of investigations has been the determination of the properties of the hard radiation produced in the plasma and the mechanism responsible for this radiation. Cloud chambers have been used to study the spectrum of electrons which are produced by the hard x-rays from pulsed discharges. These studies have confirmed the maximum x-ray energies which had been estimated earlier (350 — 400 kev).

Mass-spectroscopic analyses of the fast particles produced in the discharge have also been carried out successfully. The parabola method has been used to measure the e/m ratios and the energies of ions which are extracted from the discharge chambers through apertures in the side walls or in the electrodes. It has been established that the neutron energy is as high as 200 kev.

In discussions of the possible mechanisms which lead to the appearance of hard radiation it has frequently been asserted that an important role is played by "sausage" type instabilities which enhance the implosion of the shock wave. To verify these suggestions, experiments have been carried out in discharge geometries in which the conditions have been such as to make the contracting plasma assume an essentially spherical shape. These experiments have yielded interest-



a) FIG. 5 b)

ing results and indicate that the artificial creation of spherical implosions in a contracting plasma changes the characteristics of the hard radiation to a marked degree.

We may dwell briefly on the question of the future of methods of generating thermonuclear reactions in which powerful pulsed discharges are used. From the purely physically point of view, in the final analysis everything depends on two basic characteristics of the pulsed discharge:

(a) the degree of contraction of the pinch, which is characterized by the minimum value of the pinch radius a;

(b) the time duration of the contraction state τ.

It is easy to see that the power efficiency of a pulsed discharge, viewed as a thermonuclear reaction source, is proportional to the square of the current strength and the quantity τ/a^2. Theoretical estimates of the numerical values of τ/a^2 for very intense pulsed devices differ by more than two orders of magnitude, depending on the degree of optimism which implicitly or explicitly enters into the initial stages of the calculation. However, even with optimistic assumptions it turns out that the efficiency of a pulsed thermonuclear reactor can approach unity only if an enormous amount of energy is concentrated in the system (the order of 10^{10} joules for operation with a D-T mixture). This energy is initially stored in the power sources and then, in a brief time interval, is converted to a considerable degree into thermal and mechanical energy of the expanding plasma. This stage of the process must have the properties of a powerful explosion (at least equivalent to the explosion of 10 tons of TNT). At the present level of technological development we do not have available methods of making use of explosive energy of this kind and do not know of means of protecting the cumbersome and extremely expensive apparatus from possible destruction after each pulse. Nevertheless, it is still believed that research on pulsed discharges should be carried out at a higher technological level than has been achieved so far; it will be of interest to reach current rates of rise of the order of 10^{13} amp/sec under conditions in which the gases evolved from the walls of the discharge chamber have no effect on the initial stages of contraction. Experiments of this kind are not beyond the limits of present-day technological feasibility. Moreover, they may lead unexpectedly to the discovery of new factors which may have important effects on the general development of research in this field. Work on powerful fast discharges has been carried out mainly at the Institute for Atomic Energy of the U.S.S.R. Academy of Sciences. Certain problems in this field have also been studied at the Physics Department of Moscow State University. Experimental research on other types of pulsed processes in which a plasma interacts with a strong magnetic field have also been carried out at the Institute for Atomic Energy of the U.S.S.R. Academy of Sciences, the Ukrainian Physico-Technical Institute, and the Sukhumi Institute of Electron Physics.

Plasma acceleration by electrodynamic forces is realized in its purest form in accelerators such as the electrodynamic gun. The simple principle underlying devices of this kind is the following. A plasmoid, produced by the passage of a high current through a fine wire or a gas cloud, is accelerated along metal rails by virtue of forces due to the interaction of the current with the magnetic field produced by the conducting feeders (or, in another version, with an external magnetic field). Experiments have shown that in systems of this kind it is possible to achieve plasma velocities up to 5×10^7 mm/sec without any great effort. It is more than likely that even higher velocities will be achieved.

Another method of obtaining plasmoids has been studied at the Institute for Atomic Energy. This method is as follows: first a circular plasma loop containing a current is formed in an alternating external magnetic field, the lines of force of which are perpendicular to the plane of the loop (Fig. 5). This loop is produced by virtue of the breakdown of the gas in the induced electric field which accompanies the buildup of the H field. After a certain time interval, following the formation of the loop, the loop starts to contract rapidly toward the axis and changes into a plasmoid. Experiments indicate that with this method it is possible to obtain particle densities of 10^{16} and initial temperatures greater than 100 — 200 ev. One of the practical consequences of these experiments may be the development of a method of injecting hot plasma into a magnetic trap.

It may also be possible to realize a pulsed thermonuclear reaction under conditions in which the high temperature is reached by contraction and implosion produced not by electromagnetic forces, but by a charge of conventional explosive (such as TNT or something more powerful) which surrounds a capsule containing deuterium or a mixture of deuterium and tritium. Without dwelling on the experimental details, we may note that conditions have been found under which the generation of neutrons in D-D and D-T reactions have been established reliably and reproducibly. The detection apparatus is destroyed in these experiments. However, the signal from the neutron pulse reaches the buildings housing the detection system (which were located at a distance) before the buildings are destroyed. In experiments carried out in 1952 recordings were made of fast neutrons which passed through the charge without any great loss of energy and neutrons which were slowed down in the explosive and then entered the recording apparatus, forming a pulse which extended over several tens of microseconds. Obviously, in this case the notorious question of whether these neutrons are thermonuclear or not does not enter. There is no doubt that in this case we have observed neutrons which are formed as a result of the heating of matter to extremely high temperatures.

The chief difference between this process and electromagnetic contraction is that the former takes place under conditions in which the density of matter is very high (significantly exceeding the normal density of solids). The brief duration of the heating process makes it possible to dispense with the use of magnetic confinement. It is apparent that this process can be of economic interest only if the release of thermonuclear energy can balance the cost of the expensive explosive materials.

Just as in the case of large pulses of electrical power [with power yields up to ten tons of TNT (cf. above)], the practical exploitation of explosive heating involves difficulties associated with the explosive nature of the process.

II. SLOW PULSED DISCHARGES IN TOROIDAL CHAMBERS

8. It is to be expected that there should be fundamental differences when the current in a plasma builds up at a slow rate as opposed to the case in which the current rises at a fast rate. A quantitative criterion which can be used to distinguish between "slow" and "fast" discharges is the ratio of the current rise time to the period of the inertial radial oscillations of the pinch. In rarified gas discharges with maximum current strengths of the order of $10^5 - 10^6$ amp, hundreds of inertial oscillations can take place during the first half-period (of the order of 10^{-3} sec). Such discharges may be called slow, in contrast with fast discharges in which only two or three radial oscillations take place in the time the current reaches its peak value.

In slow discharges one may expect the electrodynamic forces to balance the gas kinetic pressure of the plasma with the plasma temperature being increased by virtue of Joule heating. In order for this equilibrium state to be useful for the heating of plasma to very high temperatures the following conditions must be satisfied:

(a) the pinch must not come in contact with the walls;

(b) the equilibrium state must be a stable one.

Theoretical investigations of equilibrium conditions and of the stability of pinches and their heating characteristics have been carried out at the Institute for Atomic Energy under the direction of M. A. Leontovich. It was first shown in this work that stability of a plasma pinch with more or less sharply defined boundaries can be obtained only when the discharge chamber is enclosed in a conducting sheet (close to the chamber walls) and when, in addition to the field set up by the plasma current, there is also a stabilizing magnetic field in the direction of the pinch (which must be set up by external coils). Two different stability modes are found. The first obtains when the contracting plasma column entrains a large part of the magnetic flux of the longitudinal field which exists initially in the discharge chamber ("paramagnetic pinch"). In this case the intensity of the longitudinal field inside the pinch is approximately $H_0(b/a)^2$, where H_0 is the initial intensity of the field and the quantities b and a denote respectively the inner radius of the discharge chamber and the radius of the pinch.

As is well known, this mode has been investigated in detail by British physicists working with the Zeta system. From the theoretical point of view this approach seems to be most promising as far as stabilization of the shape and dimensions of the pinch are concerned. In the magnetohydrodynamic approximation the main requirement for stabilization of all dangerous perturbations of a highly conducting pinch is the requirement that H_z inside the pinch be rather close to the field intensity of the current at the boundary of the pinch and that it be several times greater than the strength of the longitudinal field beyond this point (cf. Fig. 6, in which is shown schematically the distribution of longitudinal field H_z and current field H_φ). Another requirement is that the pinch radius a must not be too small as compared with

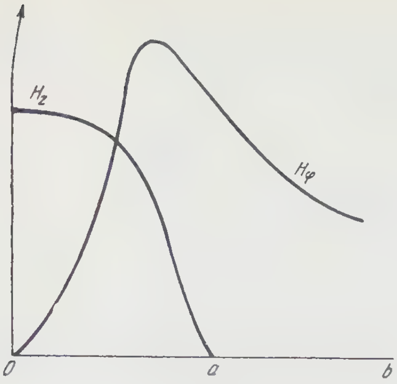


FIG. 6

the chamber radius; if this is not the case the stabilizing effect of the conducting sheet is no longer realized.

A relatively small initial value for the initial longitudinal field H_0 is sufficient for realizing this mode of operation. Another advantage is the fact that this mode of operation is one which arises naturally during the course of a pulsed discharge which is initially spread over the whole cross section of the chamber; later, as H_φ becomes comparable with H_0 the discharge contracts, entraining the longitudinal force lines. From the theoretical point of view, however, there are certain disadvantages associated with this mode of operation. To achieve stability of a paramagnetic pinch the quantities H_z and $H_\varphi(a)$ must be approximately the same. Hence, only a small part $(0.3-0.4)$ of the electrodynamic pressure produced by the current is useful for counteracting the gas kinetic pressure. This means that at equal values of I and N the temperature of a paramagnetic pinch will be several times smaller than that which obtains in the case in which the longitudinal field is distributed uniformly over the cross section of the discharge chamber. It should also be noted that in order to obtain good heat exchange between electrons and ions in the paramagnetic mode the minimum value of N must be at least an order of magnitude greater than in the case in which a field is not captured.

The magnetohydrodynamic analysis also predicts the possibility of another stable discharge mode in which stabilization of the pinch is achieved by virtue of a strong longitudinal magnetic field of strength H_z which is everywhere larger than H_φ. In this case the theory indicates that the criterion for stability is the satisfaction of the inequality

$$\frac{H_z}{H_\varphi} > \frac{L}{2\pi a}, \tag{4}$$

where L is the length of the pinch (for a circular pinch in a toroidal chamber of radius R this quantity is $2\pi R$).

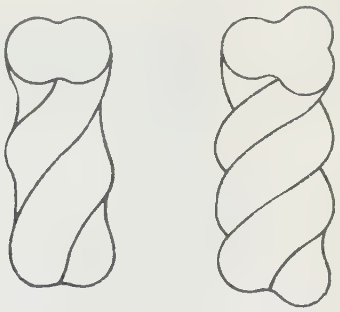


FIG. 7

However, even when this requirement is satisfied certain types of perturbations can still lead to instabilities. The nature of these perturbations is shown schematically in Fig. 7. It would seem that these are not too dangerous since they do not involve a displacement of the pinch axis.

In the case being considered here the longitudinal magnetic field is distributed over the entire cross section of the chamber and does not change appreciably near the pinch boundary (cf. Fig. 8). An advantage of this method of stabilization is the fact that the time during which it is realized does not depend on the skin-effect period but only on the time during which H_z and I are maintained. Another advantage is the possibility of obtaining a much higher ion temperature for a given current strength than is the case in the "paramagnetic" mode. However these advantages must be viewed in terms of the high cost involved in the need for operation at very strong magnetic fields. In order to approach the threshold of the temperature region of interest it is necessary to produce magnetic fields of the order of 3×10^4 to 5×10^4 oersteds over very large volumes and very considerable technological difficulties are involved in producing such fields. Difficulties are also encountered in bringing the system up to this level of operation.

The results of the magnetohydrodynamic analy-

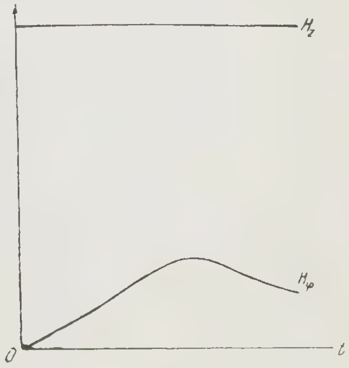


FIG. 8

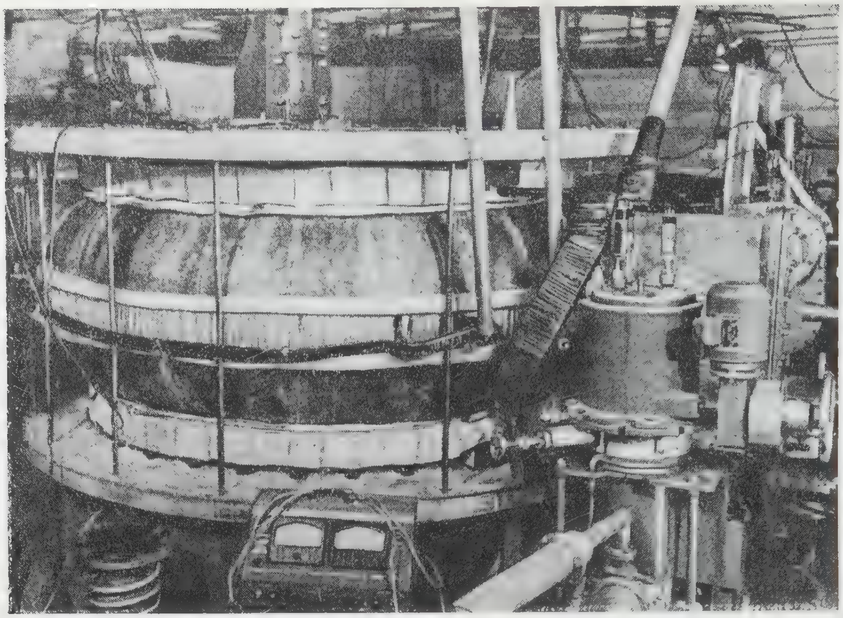


FIG. 9

sis, which have been presented here briefly, are a serious argument for believing that this approach represents a promising avenue of success. It should not be forgotten, however, that these analyses are based on a simplified physical picture of a discharge in which many important details have been neglected. This theory does not consider such displacements from the equilibrium state as are characterized by wavelengths smaller than the thickness of the skin layer and which evidently cannot be stabilized by methods which apply for long-wave perturbations.

It should also be noted that by its very nature the magnetohydrodynamic analysis is not suitable for analyzing such kinetic effects as the transition of electrons into a continuous acceleration mode in an electric field. Such processes can occur near the boundary of the pinch as a consequence of the low density of matter and the reduction in the retarding forces that the ions exert on the electrons. These processes, however, also occur inside the pinch since the energy spectrum of the electrons must include particles of energies considerably greater than the mean energy for which a transition to a continuous acceleration mode can take place, even with high values of N. As is well known, an accelerated beam of electrons can excite various kinds of plasma oscillations and thus change the character of the process. Other forms of disturbances are also possible which do not fit into the simple magnetohydrodynamic picture but which are a source of potential danger as far as maintenance of pinch stability is concerned.

9. We now turn to a consideration of the experimental results. In the first stages of the experimental work all the basic investigations were carried out in chambers made from insulating materials (glass, quartz, and porcelain). Because of the strong outgassing from these walls reliable results could be obtained with these chambers only when relatively high gas densities were used in the chamber and when the discharges were not too strong.

A study of the effects of a fixed longitudinal magnetic field on the behavior of slow discharges has shown that with a slow current rate of rise there is a paramagnetic effect in the plasma, such as has been discovered earlier by Soviet physicists in fast pulsed discharges. At low values of H_z this effect leads to the contraction of the pinch. The effect of a longitudinal magnetic field on the conductivity of the plasma is found to be weak. Regardless of the initial pressure, the intensity of the induced electric field, and the H_z, the conductivity is found to be approximately 1 to 3×10^{14} esu; this indicates a low plasma temperature in chambers with insulating walls. A study of the shape of the pinch by high speed movie photography and analysis of the oscillograms of the electrical and magnetic parameters in the discharge confirms the assumption of the pronounced instability of the process at small values of the ratio of H_z to H_φ. As the external field is increased, the discharge becomes more stable but the conductivity does not exhibit a significant rise. To ascertain the stability conditions for a pinch that is isolated from the walls, a number of experiments have been carried out with straight discharge tubes in which the discharge is started close to the axis and then expands towards the walls. These experiments con-

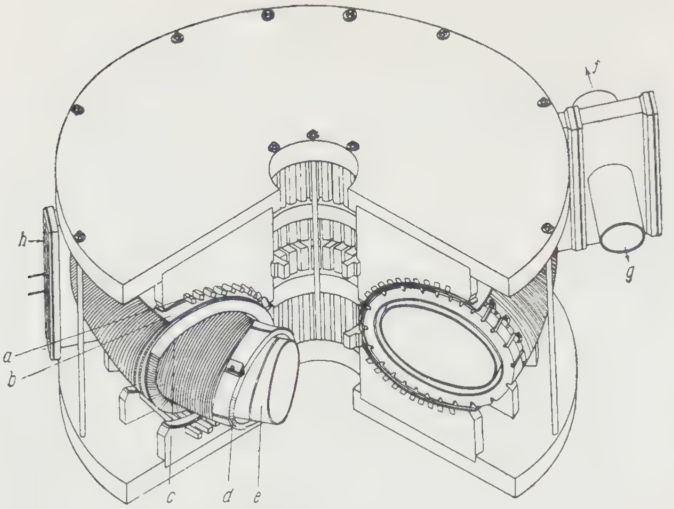


FIG. 10

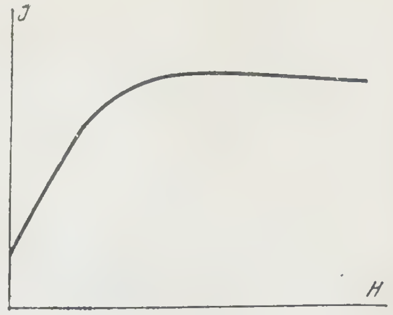


FIG. 11

firm qualitatively the theoretical conclusion that the pinch is stable if the inequality in (4) is satisfied. However, in these experiments, because of the high losses at the electrodes, there could be no hope of obtaining high plasma conductivity.

A further step was the transition to chambers with metal walls. It was expected that in chambers of this type it would be possible to maintain the purity of the gas during powerful discharges. Several large assemblies with metal toroidal chambers have been built at the Institute for Atomic Energy. A photograph of one of these systems, designed for investigation of high-current discharges over a wide range of external magnetic field, is shown in Fig. 9. In Fig. 10 is shown a cutaway diagram of the apparatus. The discharge takes place in a closed toroidal chamber made from stainless steel 0.2 mm thick. This is located inside a toroidal copper casing with walls 20 mm thick. The copper casing has two insulated cuts in a plane parallel to the axis of the toroid and one insulated cut along the generating line. The inside thin-walled chamber and the space between it and the casing are evacuated by separate vacuum systems. The diameter of the inner cross section of the discharge chamber is 0.5 m and the mean diameter of the toroid is 1.25 m. The chamber is the secondary winding of an air-core transformer. The primary winding is formed by 20 turns of thick copper strip wound on the casing. A shield, which encloses the casing, is used to eliminate stray magnetic fields. The coils which produce the longitudinal field are wound directly on the surface of the copper casing. The intensity of this field can be made as high as 12,000 oersteds. The electrical energy for the discharge circuit and the windings which produce the longitudinal field are capacitor banks (at peak voltage the total energy storage in these banks is 1.2 million joules). In the experiments carried out with the device described here the maximum current in the gas was 400 kiloamperes (at a discharge voltage of 0.45 kv and a longitudinal field of 12,000 oersteds).

The basic results obtained in the first stage of experimental work of discharge processes in metal toroidal chambers may be summarized as follows:

1. In hydrogen and deuterium at initial pressures ranging from 3×10^{-4} to 5×10^{-3} mm Hg the peak current of a discharge whose half period lies between 300 and 1200 μsec is approximately proportional to the initial discharge voltage and is only weakly dependent on gas density. As H_z increases, the current strength increases and then remains essentially constant. The general nature of the dependence of I_{max} on H_z is shown in Fig. 11.

2. At small values of H_z the pinch separates from the walls. As H_z increases the diameter of the pinch also increases. If $H_z \gg 0.2\, I_{max}/b$, where b is the chamber radius, the plasma column always remains in contact with the walls.

3. For small values of H_z the conductivity of the plasma (computed under the assumption that the current distribution is uniform over the cross section of the pinch) approaches the value 1×10^{15} esu when the pinch remains separated from the walls. In discharges in strong longitudinal fields, in which the plasma fills the entire cross section of the chamber, the conductivity falls to 1 or 2×10^{14} esu.

These results would seem to indicate that in the present experiments the electron temperature (as determined from the conductivity) was probably no higher than 15 — 25 ev.

It follows that even under conditions for which the pinch remains completely isolated from the walls the energy losses can still be very large; it is precisely for this reason that the plasma temperature does not reach the values predicted by theory. At the present time the mechanism for these energy losses is still not clear. Possible explanations are that the pinch is not sufficiently stable or that the impurity atoms evaporated from the chamber walls contaminate the plasma. These conclusions are merely preliminary ones since the experimental work with metal chambers is still in progress.

In the near future our main efforts will be directed toward the creation of conditions which reduce sharply any deleterious effects due to the presence of impurities in the plasma.

III. MAGNETIC TRAPS

10. Generally speaking the term "magnetic trap" can be applied to any device used to obtain extremely high temperatures in which magnetic confinement is used. However we will find it expedient to narrow the meaning of this terminology, applying it only to a definite type of system.

In the devices considered above the main agent responsible for confining the heated matter was the current maintained in the plasma by virtue of external voltage sources. This means that in such devices particles "hold on to each other" by means of the mutual self-consistent fields produced by the particles themselves. In this case the external magnetic fields fulfill the auxiliary role of a medicine used to treat the instability. In systems with high plasma currents the hydrodynamic pressure p is of order $H^2/8\pi$; hence the plasma is capable of exerting strong counter effects on the magnetic field distribution and this results in a number of characteristic instabilities.

In contrast to the above, we shall use the term magnetic trap to apply to systems in which the plasma confinement function role is played exclusively by the external field, with the plasma conduction currents remaining unimportant. Hence, in magnetic traps the quantity $8\pi p/H^2$ can be small compared with unity, i.e., the rarefied plasma may not have an important effect on the external magnetic field.

Various means can be used to obtain matter heated to high temperatures in magnetic traps. One method is to fill the magnetic system with fast ions injected from a powerful accelerator. Another method is to fill the trap with plasma and then heat this plasma by means of dynamic magnetic fields or high-frequency electromagnetic fields. Still another possibility is that of obtaining fast ions directly in the trap itself by using constant or alternating electric fields to accelerate ions derived from the plasma itself.

The most natural approach to the problem of magnetic traps would be to analyze the problem of the bounded motion of a single particle in an external field; then, having found a satisfactory solution to this problem, the next step would be to investigate the behavior of a large number of particles which comprise the plasma. In spite of its obvious shortcomings, this approach is very straightforward and, at the first stage, makes it possible to circumvent the difficulties associated with the formulation of a rigorous theory for the behavior of a plasma in complex fields.

The problem of devising such a magnetic trap which, at least in principle, would seem to allow of practical realization, has occupied many Soviet physicists. A stimulant to the discussion of this problem was the work of A. D. Sakharov and I. E. Tamm. In 1950 these investigators proposed the first concrete model for a magnetic thermonuclear reactor. In this version it was suggested that rarified deuterium be isolated and heated in a toroidal chamber in which a strong longitudinal magnetic field was produced by means of a coil wound on the outer surface of the chamber. However, this system has a fundamental defect when considered as a magnetic trap. Because the field inside the toroidal chamber is inhomogeneous, each particle executes a drift motion perpendicular to the field lines; this motion is terminated when the particles strike the chamber walls. The authors directed attention to this situation and proposed later that the drift motion could be eliminated through the use of the magnetic field of the plasma current.

Further work in this field confirmed the fruitfulness of the original idea and several types of magnetic systems were developed with which it was possible to confine particles to limited regions of space. As a result the experimental work on the development of magnetic traps became extremely important and promising. It should be noted, however, that the most important goal, the development of a trap which confines all particles, regardless of their direction or velocity, has still not been achieved and the question of whether such a system can be devised at all is still open.

11. The actual types of magnetic traps which have been studied up to this time can be divided into two basic classes:

(1) Traps with magnetic "stoppers" (mirror systems);

(2) Traps with limited drift.

A theoretical investigation of systems of the first type was initiated by G. I. Budker in 1953. Particles whose velocity directions form angles which are not too small with respect to the lines of force are reflected when they approach the strong field region and are thus trapped in the

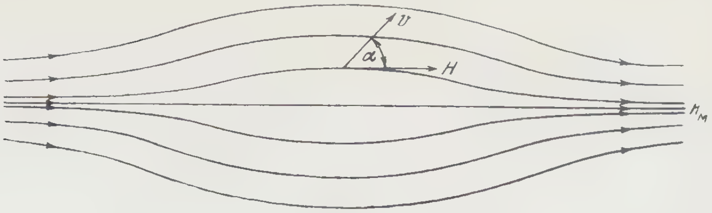


FIG. 12

magnetic system. For simplicity we shall assume that the field in the median plane is homogeneous. Suppose that the strength of this field is H_1 while the maximum intensity at the points of concentration of the force lines ("magnetic stoppers") is H_m. Under these conditions, if a particle in the region of homogeneous field is characterized by a velocity vector which lies within one of the cones defined by the condition

$$\sin\alpha < \sqrt{\frac{H_1}{H_m}},$$

in its subsequent motion the particle will pass through the region of maximum field and escape; however, if the initial velocity lies outside these cones the particle is trapped.

Let us assume now that a thermonuclear generator is to be built using a magnetic trap of the kind we have considered. An important question which immediately rises is that concerning the role of collisions between particles. It is apparent that even one collision will be sufficient to cause the velocity vector of the particle to fall into the escape zone. Hence the lifetime of particles in the plasma in a magnetic stopper trap will, in order of magnitude, be equal to the time between two Coulomb collisions between ions. In order that the energy generated in a system of this kind as a result of thermonuclear reactions be sufficient to compensate for the energy losses due to the escape of particles by virtue of Coulomb collisions the following condition must be satisfied:

$$\sigma_n W_n \sim \sigma_C kT.$$

In this expression σ_n is the effective cross section for the nuclear reaction, σ_C is the effective cross section for Coulomb collisions, W_n is the energy generated in each elementary nuclear fusion event.

Making use of this condition it is possible to estimate the minimum temperature at which the thermonuclear generator should begin to produce excess energy. For the D-T case this figure is on the order of tens of kilovolts; for pure deuterium it is on the order of 1 Mev. Thus, the use of pure deuterium in systems of these kinds is essentially hopeless as far as practical purposes are concerned. This conclusion is verified by more rigorous calculations, in which other kinds of losses are taken into account (bremsstrahlung and betatron radiation from fast electrons).

Theoretical investigations of the stability of a plasma in magnetic traps with plugs have been carried out at the Institute for Atomic Energy and indicate that there are perturbations of the equilibrium state which, themselves, are not stable. Pressure anisotropy in the plasma and pronounced deviations from the Maxwellian distribution for the particle velocity components along the lines of force constitute additional sources of instability in systems of this kind.

The mechanism for the formation of certain kinds of instabilities is related in an important way to the geometry of the magnetic field in the trap; it is therefore possible to have cases in which perturbations of a given kind can be stabilized in one kind of a trap and grow in other traps. For example, one kind of dangerous instability can arise in systems in which the plasma boundary has a negative curvature (Fig. 13). At points of negative curvature the plasma can leak out in the form of individual "tongues" which are perpendicular to the force lines. However this form of instability

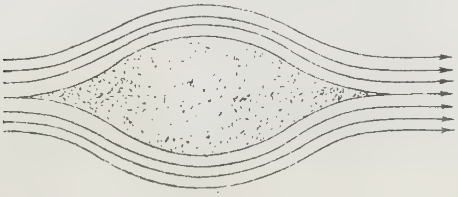


FIG. 13

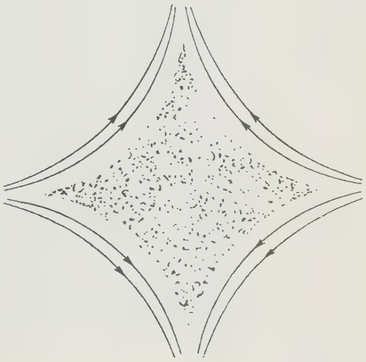


FIG. 14

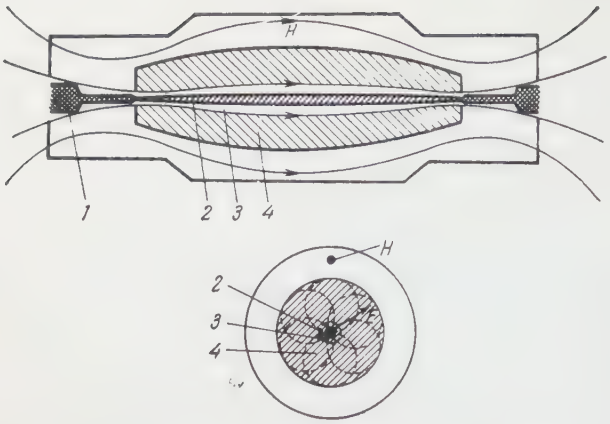


FIG. 15. 1) plasma source; 2) plasma beam; 3) region of strong electric field; 4) region of fast ion motion.

is probably not characteristic of systems in forced lines with positive curvature (Fig. 14). With an increase in the ratio p/H^2 there is a marked increase in the danger associated with all kinds of instabilities. Even if traps with magnetic stoppers turn out to be unfeasible from the practical point of view, it is possible that they can still be used to demonstrate the feasibility of a thermonuclear reaction at small values of $8\pi p/H^2$.

An experimental investigation of traps with magnetic stoppers was carried out at the Institute for Atomic Energy several years ago. The first experimental system was designed for studying the storage of fast ions extracted from a cylindrical plasma pinch which was produced along the axis of the magnetic system (Fig. 15). The pinch, with an ion concentration of 10^{13}, was produced by means of an arc discharge in a longitudinal field. In Fig. 16 is shown a general view of the apparatus with which experiments of this kind are being carried out at the present time. The length of the vacuum chamber is 2 meters; the magnetic field strength at the center is 8500 oersteds while the magnetic field at the stoppers is 12,000 oersteds. During operation of the ion source a pressure of 10^{-7} mm Hg is maintained by means of titanium pumps. A voltage of 40 kv is used to accelerate the ions between the pinch and the side walls of the chamber. The main purpose of these experiments is the determination of the lifetime of fast ions for various ion densities in the magnetic system. The mean lifetime of these ions can be determined from the rate at which the density decreases after the accelerating voltage is switched off.

The results of the first experiments indicate that in these systems the lifetime of a plasma containing fast ions is of the order of several milliseconds. This lifetime increases with improved vacuum conditions and with an increase in the value of H.

In the summer of the past year construction was completed at the Institute for Atomic Energy of a large experimental device with magnetic stoppers. This device will be used to study external injection of particles. The D^+ ions are accumulated by dissociation of D_2^+ molecular ions which are injected into the central section of the trap. The molecular ions are dissociated by collisions with molecules or atoms of the residual gas or by interactions with the ions of the rarified deuterium plasma which has previously been produced in the chamber. In Fig. 17 is shown a schematic diagram

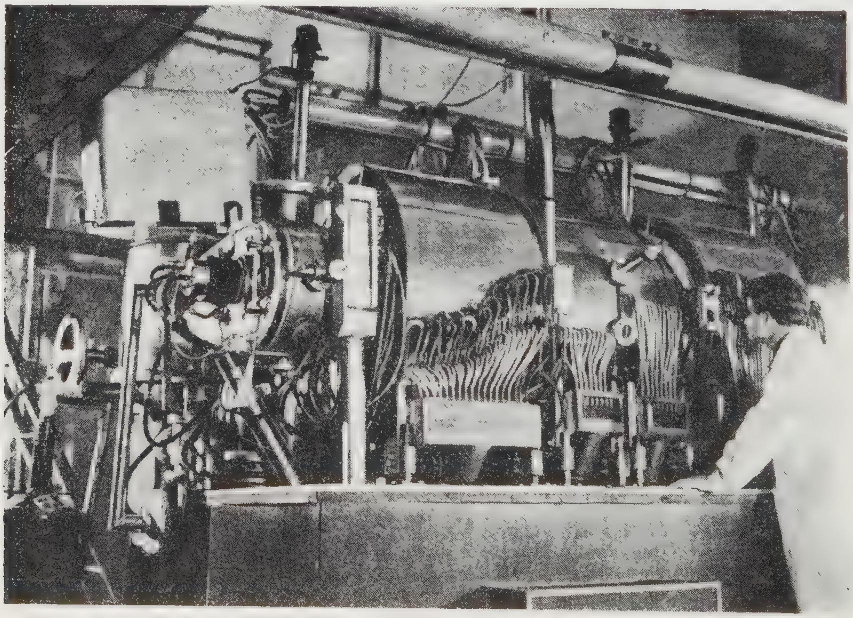


FIG. 16

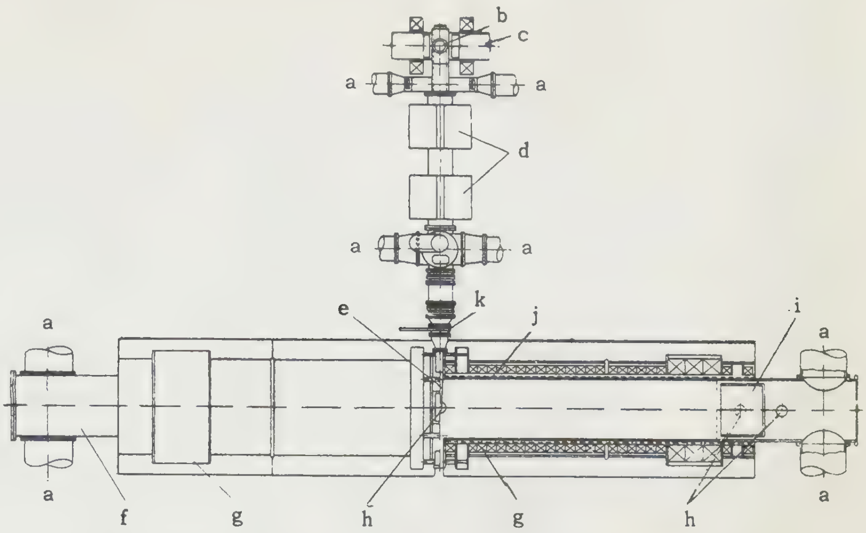


FIG. 17. a) to evacuation system; b) ion source (+ 220 kv); c) source magnet; d) quadrupole lenses; e) magnetic channel; f) chamber; g) winding; h) titanium evaporator; i) electrode (-15 kv); j) heating chamber; k) valve.

of this apparatus. The distance between the centers of the stoppers is 12 meters; the inner diameter of the vacuum chamber is 1.4 meters. The field intensity in the central region can be as high as 5,000 oersteds and the field in the stoppers can be raised to 8,000 oersteds. An intense beam of D_2^+ ions is obtained from an arc source in a transverse magnetic field. The D_2^+ ions, accelerated by a voltage of 200 kv, are focused and injected into the magnetic trap by means of a system of electric and magnetic fields. It is expected that after the system is tuned up that the current of D_2^+ ions entering the vacuum chamber will be several hundred milliamperes. It is hoped that with this intensity for the injected ion beam the density of D^+ ions in the trap will be as high as 10^{12}.

An experimental study has also been started on systems with geometries such as that shown in Fig. 14. A field of this kind is interesting for several reasons: in particular, as has been indicated above, it is hoped that in this case a higher degree of plasma stability can be obtained with respect to certain kinds of perturbations.

12. There are several other ways of evaluating magnetic systems which can be used as magnetic traps. The main problem is to ascertain the conditions under which the drift velocity of the particles in the inhomogeneous field does not lead to their escape from a restricted region of space. In Figs. 18 and 19 are shown magnetic systems which illustrate two possible approaches to the solution of this problem. In the system shown in Fig. 18, the magnetic field along the straight sections is "corrugated" (by using non-uniform windings along the chamber surface). Particles moving in an inhomogeneous field of this kind are subject to rotational drift motion so that their trajectories rotate about the axis of the field.

Thus, in spite of the presence of the curved end sections a particle which makes a complete circuit does not approach the walls since the rotation of the trajectories in the linear sections balances out the effect of drift in the curved end sections.

An example of another method in which the drift is limited through the use of rotation about the force lines is the system shown in Fig. 19. This is a torus which is deformed into the form of a figure eight.

It is easy to see that both systems described here suffer from the same shortcomings as a trap with stoppers. These systems are capable of confining only those particles whose velocity vectors lie within a definite region of allowed directions. Particles with small longitudinal velocities which enter the curved spaces will strike the walls as a

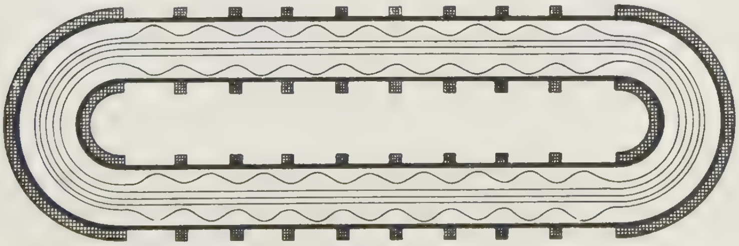


FIG. 18

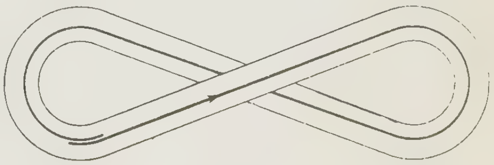


FIG. 19

result of drift motion before escaping from this space.

However, it appears that future developments of these principles of compensating drift (corrugation of the field and involution of the field lines) may make it possible to reduce the range of directions for particle escape so that the properties of these systems will begin to approximate those of ideal traps.

The methods described above for magnetic traps are based on the application of constant or slowly varying (in time) magnetic fields. Theoretical studies indicate the broad possibility of confining and isolating a plasma by means of high-frequency electromagnetic fields. In the present case the term "high-frequency" means that the oscillation period of the field is considerably less than the times characteristic of the behavior of the plasma. The frequencies in question are tens of megacycles and higher. The force which confines the plasma in this kind of field is determined by the mean square value of H rather than the instantaneous value.

It would seem that high-frequency fields can be used most effectively in conjunction with fixed magnetic fields used as auxiliary fields for eliminating energy losses due to corpuscular beams. For example, high-frequency electromagnetic fields can be used as a "stopper" to confine particles which have escaped along the field lines from the region of strong fixed magnetic field.

The feasibility of confining plasmas by means of field combinations of this type has been verified experimentally in a small device at the Institute for Atomic Energy. The practical prospects for the use of high-frequency fields for isolating a heated plasma seem to be rather unfavorable at the present time, however, because of the high energy consumption required to maintain these fields. Nevertheless, investigation of various versions of high-frequency confinement are of considerable interest.

High-frequency electromagnetic fields can also be used as a means of heating plasma. Greatest interest in this connection now attaches to an investigation of the possibility of using ion cyclotron resonance effects. Research in this field, which has been carried out in the U.S.S.R., is described in a report presented by the Ukrainian Physico-Technical Institute at the Geneva Conference in 1958.

CONCLUSION

In his speech at the first Geneva Conference on the Peaceful Uses of Atomic Energy the President of the Conference, Dr. Homi Bhabha, said:

"The historical period we are just entering, in which atomic energy released by the fission process will supply some of the power requirements of the world, may well be regarded one day as the primitive period of the atomic era. It is well-known that atomic energy can be obtained by the fusion process as in the H-bomb and there is no basic scientific knowledge in our possession today to show that is is impossible for us to obtain this energy from the fusion process in a controlled manner. The technical problems are formidable, but one should remember that it is not yet fifteen years since atomic energy was released in an atomic pile for the first time by Fermi. I venture to preduct that a method will be found for liberating fusion energy in a controlled manner within the next two decades. When that happens the energy problems of the world will truly have been solved forever for the fuel will be as plentiful as the heavy hydrogen in the oceans."

Three years have passed since this prediction was made and now, before our eyes, we begin to see the first rough outlines of the scientific foundation on which the methods of solving the problem of controlled fusion will probably be based. This foundation has been laid as a result of the experimental and theoretical work which has been carried on in recent years in the U.S.S.R., U.S.A., England, and other countries. For the first time this work has been the subject of open discussion on an international scale; in this respect it probably represents the most important step which has been made in the solution of this problem. The importance of this step is greater than that of the results of any of the individual research work which, as yet, have not brought us very much closer to the final goal.

We do not wish to appear pessimistic in evaluating the future of our work but, at the same time, we must not underestimate the difficulties which must be overcome along the path to the achievement of controlled nuclear fusion. In the final analysis the chief difficulty is the fact that in a light material such as rarified plasma any instability effect develops at a tremendous rate. It is difficult to build any automatic monitoring device which could rapidly compensate for deviations from the equilibrium state. Hence it would appear that the most rational approach to the problem would be a system in which all forms of instability are eliminated beforehand.

It is probable that the solution of the problem will be easier in the case in which the reaction of the plasma on the magnetic field is small, i.e., when $p \ll H^2/8\pi$.

Turning now to a more general appraisal of the situation, it can be said that no one of the presently-known ideas has been shown to be decisively better than any other as far as the production of a controlled fusion reaction is concerned. For this reason, in the near future research should be carried out along the widest possible front.

An important factor in ensuring the success of this research will be the continuation and further development of the international scientific cooperation which has been started by the Geneva Conference. The solution of the problem of thermonuclear fusion will require a maximum concentration of intellectual effort and the mobilization of very sizeable material and technological facilities. This problem would seem to have been specially created to stimulate cooperation between scientists and engineers of various countries, working in accordance with a general program and continuously exchaning the results of their calculations, experiments and technical developments.

The international fusion of research work on the problem of controlled fusion will undoubtedly lead to a considerable reduction in the time required for us to reach the desired goal.

Translated by H. Lashinsky

SOVIET PHYSICS USPEHKI VOLUME 66 (1), NUMBER 2 NOVEMBER-DECEMBER, 1958

REMARKS ON BOHR'S ARTICLE ON HIS DISCUSSIONS WITH EINSTEIN

V. A. FOCK

Usp. Fiz. Nauk **66**, 599-602 (December, 1958)

THE paper by Niels Bohr about his discussions with Albert Einstein on the basic problems of atomic physics, which is printed in this issue of "Uspekhi,"* gives a remarkably clear exposition of the physical foundations on which a correct interpretation of quantum mechanics must be based, and Bohr's paper is of exceptionally great interest in this connection.

With extraordinary cogency, using many simple examples, and without any mathematics except the Heisenberg relations, he demonstrates the limitations of the "classical" way of describing phenomena, i.e., the description in which phenomena are dealt with "by themselves," apart from the means of observation. The possibility of a description of atomic phenomena "by themselves" is analyzed by Bohr on the basis of the Heisenberg relations, in virtue of which the manifestation of different properties of an atomic object in general depends on the use of different experimental conditions, which may be mutually exclusive. Bohr denotes this state of affairs by the term "complementarity"; in explaining this term Bohr says (page 210*) that he understands it "in the sense that only the totality of the phenomena exhausts the possible information about the objects."

An extremely important and indisputably correct point made by Bohr is that classically described apparata play an indispensable part in our knowledge about microscopic objects. Bohr formulates this position in the following words (page 209): "however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms."

This deeply dialectical proposition of Bohr's must unquestionably lie at the foundation of every interpretation of quantum mechanics.

In reading the works of Bohr (in particular the paper printed here) one nevertheless gets the impression that his point of view is in some ways a onesided one. In fact, all of Bohr's efforts are directed to the explanation of the limitations of the old classical concepts, and not to the explanation of the new concepts introduced by quantum mechanics. A formal sign of this onesided approach is that out of the whole apparatus of quantum mechanics Bohr uses only the Heisenberg relations. This cannot simply be due to the effort to make the exposition elementary: even in the most elementary exposition one can and should say something also about the new concepts, and not only show the limitations of the old ones. It seems to us that the point here is the role that Bohr ascribes to the apparatus of quantum mechanics. Bohr mentions it only in passing, and mainly in order to emphasize its supposedly exclusively symbolic character. According to Bohr the mathematical symbols of quantum mechanics, unlike the mathematical symbols of classical physics, do not in themselves possess physical meaning, but serve only as an "adequate tool for the complementary mode of description," as Bohr says in another of his papers.[1] Actually it is scarcely possible that Bohr believes that the mathematical apparatus of quantum mechanics serves only for the coordination of instrument readings obtained in measurements, but this point inevitably arises when one peruses his writings. At any rate, such a point of view would be incorrect. It is unquestionable that quantum mechanics (like any other physical theory) provides, along with other things, a way of correlating the readings of instruments involved in measurements. But this is not its basic significance. The task of a physical theory is always to describe the properties of physical objects in their relations to the external world. Certainly neither Bohr himself nor any other physicist denies the objectivity of such properties of atomic bodies as the charge, the mass, the spin, the degrees of freedom, the form of the wave equation in a given field, the law of interaction with other particles, and so on. Not only are these properties objective, but also they can be abstracted from the apparatus and ascribed to the objects themselves.

The main efforts of Bohr are directed, as we have said, to the explanation of the state of affairs in atomic physics which he denotes by the term "complementarity." Admitting all the necessity and importance of this explanation, we can still regret that Bohr does not show the way out of this situation, does not say what the new primary concepts (physical, intuitive, and not merely symbolical, concepts) are that must take the place of the classical concepts, and does not emphasize the unlimited possibilities of improving the accuracy of our description of atomic objects by means of new concepts. Not only the limitation inherent in the description of phenomena "by themselves," in abstraction from the means of observation (complementarity) is of philosophical importance, but also the constructive features of quantum mechanics and the new primary concepts associated with it.

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*Page references in the translation are to Albert Einstein: Philosopher-Scientist, edited by P. A. Schilpp, New York 1951. The Bohr article referred to begins on p. 201. (Translator).

In our opinion, these primary concepts, on which atomic physics can be constructed, are the following: relativity to the means of observation, the difference between the potentially possible and the actually observed (or between prognosis and fact), and, finally, the concept of probability as a numerical measure of the potentially possible. The apparatus of quantum mechanics, which has as its direct task to serve for the calculation of this numerical measure, is at the same time a means for the introduction of new abstractions and new, more refined physical concepts, and for the more accurate description of the properties of atomic objects on the basis of these new concepts. In connection with the introduction of the new primary concepts the concept of causality also receives a new formulation.

We shall not enter here upon an exposition of our point of view on these questions, since we have done so elsewhere.[4] We wish only to explain what we have in mind when we speak of new primary physical concepts.

In his writings Bohr repeatedly emphasizes the necessity of considering an experiment as a whole, without dividing it into stages. Bohr assumes that we can speak of a definite phenomenon only when we have a completed experiment, and therefore proposes to consider only such experiments. Of course a definite answer to the question for which the particular experiment has been set up can be obtained only when this experiment has been completed. Nevertheless Bohr's demand that we consider only completed experiments seems to us too categorical, since it leaves no room for quantum-mechanical abstractions. Only the division of an experiment into stages,* and only an arrangement of the experiment in such a way that the choice of the last stage, the stage of measurement, remains free, actually makes it possible to introduce the concept of the state of the object, which is a fundamental concept for the quantum mechanics. The situation that occurs with such an arrangement of an experiment at the time when just the choice of the last stage remains free can be called a "conceptually interrupted experiment." In our opinion only the discussion of an uncompleted, conceptually interrupted experiment enables us to introduce the concept of the wave function.*

In fact the key points in the discussion between Bohr and Einstein arise in the discussion of just such conceptually interrupted experiments, and this is understandable, since it is just in these cases that the peculiarities of quantum physics that distinguish it from classical physics are most strikingly manifested. The category of conceptually interrupted experiments includes also both the experiments proposed by Einstein, namely the experiment in which one could measure either the energy of a quantum or the time of its emergence, and the experiment in which one could measure either the coordinate of a particle or its momentum (cf. pp. 219 and 229). Thus Bohr himself does not strictly observe his requirement that one consider only a completed experiment. The pronouncements made by Bohr on this point are to be understood more correctly not in the sense of a compulsory requirement, but rather in a broader sense, as a reminder of the fact that one cannot always carry out a division of an experiment into stages, and that in cases in which this cannot be done the concept of the object does not have a definite meaning.

Thus our main comments bear on a certain incompleteness of the article, on the fact that Bohr's brilliant demonstration of the limitations of classical concepts is not accompanied by at least a brief indication of what must replace them. To this main point we may add a few remarks, which are in part, however, only concerned with terminology,

We feel that Bohr has adopted a most unfortunate use of the word "causality," which as it were allows causality and complementarity to be placed in opposition to each other, and thus leads toward a denial of causality. One must introduce two terms, for example, "Laplacian determinism," which means the belief in the possibility in principle of prognoses of unlimited precision, and the more general term "causality," in the sense of the existence of laws of nature. Laplacian determinism is in fact overthrown by quantum mechanics, but causality is altogether maintained, and one has only to express it in new forms.

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*On the division of an experiment into stages see previous papers by the writer,[2,3]

*If we use the concept of "preparation of the object," then we can say that in an "uncompleted" experiment we have to do with an object prepared in a definite way. The concept of the wave function has indeed always been associated with such an object.

Another unfortunate term used by Bohr is "uncontrollable interaction." Essentially one is speaking here not about an interaction in the proper sense of the word, but about the logical interconnection between the quantum and classical modes of description at the nexus between the part of the system that is described qunatum-mechanically (the object) and the part that is described classically (the instrument). In the passage from the quantum language to the classical there is as it were a loss of precision; when Bohr says that an uncontrollable interaction occurs between the object and the instrument, he undoubtedly has just this in mind. Taken literally, however, the term "uncontrollable interaction" leads to misunderstanding: surely every physical process is knowable, and consequently is also accessible to control. It may be supposed that this term arose from the attempt to use classical concepts outside their domain of validity.[3]

One also finds in Bohr's article imprecise expressions like the following: "the knowledge of the position of the diaphragm" (page 217) or "our knowledge of the adjustment of the clock", (page 227), and so on, whereas in actual fact the meaning is not concerned with our knowledge, but with objective facts, for example, with the accuracy to which we can establish a correspondence between the position of the diaphragm and a scale fixed in the laboratory, or between the readings of a given clock and those of the laboratory clock. In such expressions "we" seem to be identifying ourselves with the "laboratory". Therefore it is not to be supposed that the use of such expressions reflects any subjectivity in Bohr's point of view; without question this is simply carelessness, and there is no real need to comment on such imprecise expressions.

In conclusion we would like to point out that all, or nearly all, of the remarks made here were the subject of personal discussions between Bohr and the writer at the time of the writer's visit to Copenhagen in February and March of 1957. As a result of the series of discussions (during which we always had before us the original of the paper of Bohr which is republished here) Bohr evidently agreed with many of the writer's comments, and declared from the very beginning that he is an opponent of positivism. Bohr's emendations to his original statements have been expressed in a paper "Quantum Physics and Philosophy," which will be published in the next (January) issue of Uspekhi Fizicheskikh Nauk.

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[1] N. Bohr, Dialectica **1**, 312 (1948).

[2] V. A. Fock, Usp. Fiz. Nauk **45**, 3 (1951).

[3] V. A. Fock, **Чехословацкий физический журнал** (Czechoslov. Phys. J.) **5**, 436 (1955).

[4] V. A. Fock, Usp. Fiz. Nauk **62**, 461 (1957).

Translated by W. H. Furry

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FORBIDDEN LINES IN ATOMIC SPECTRA

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Uspekhi Fiz. Nauk. **66**, 603-652 (December, 1958)

INTRODUCTION

THE problem of forbidden lines in atomic spectra arose comparatively recently, 30 or 35 years ago. Prior to this time atomic spectroscopy had achieved a relatively high level of development. Atomic spectra of all the elements known at that time had been investigated in fairly great detail and both classical and quantum theories of radiation had been formulated, although the quantum theory was far from complete.

According to the classical theory, the electric dipole radiation was always produced simultaneously with the multiple radiation of all orders, although it was considerably more intense than this multipole radiation. This description was carried over completely into the quantum formulation. Hence, in both the classical and quantum theories, only the spontaneous electric dipole radiation was investigated at the beginning. It was only for this type of radiation that transition probabilities were computed and selection rules formulated. Transitions which did not obey these rules were considered impossible and were called forbidden. States from which atomic optical transitions to lower levels are impossible were called metastable. It was thought that atoms could remain in these states for rather long periods of time and usually made transitions to other states by means of collisions of the first or second kind, by absorption of radiation, or by virtue of various fields which disturb the selection rules.

A comparison of the theory of radiation with the experimental data indicated a completely satisfactory agreement. At the beginning of the 1920's almost all the lines in atomic spectra produced in the laboratory had been identified; that is to say, these lines were found to be in agreement with the Ritz combination rule, according to which lines are to be associated with differences between definite atomic energy levels. Among the lines which had not been identified were certain lines which were observed in the spectra of celestial bodies — in planetary nebulae, the solar corona and the Northern lights.

Since none of these lines could be associated with elements which were then known it was believed that they were produced by elements which had not yet been identified. Because of the circumstances under which these mysterious spectral lines were discovered, these elements were called respectively "nebulium," "coronium," and "geocoronium." In 1924, however, one of these lines was produced under laboratory conditions. In a helium discharge at a pressure of 1 mm Hg which contained a small admixture of oxygen (as an impurity) a green emission line was observed which was later identified as a line from geocoronium (λ 5577.34 A). In time it was found that there was no place for these elements in the periodic table and these mysterious lines became one of the more interesting problems in spectroscopy. An explanation was first given in 1928 by Bowen, who noted that the lines from "nebulium" were actually forbidden lines for the atoms O II, N II, and O III, while the lines in geocoronium were forbidden lines for O I. The wave numbers for the lines in nebulium and geocoronium coincided exactly with the difference in the terms corresponding to low-lying metastable states and ground states of these atoms. It was more difficult to identify the lines in coronium, but these too were found to be forbidden lines for highly ionized atoms of well-known elements.

Immediately after the identification of the forbidden lines, a new question arose — were these lines due to spontaneous or induced radiation? The second alternative was rejected because the density of the gas in the non-terrestrial bodies was obviously very small (for example in planetary nebulae this density is of the order of 1000 atoms per cubic centimeter) and the existence of a field strong enough to disturb the selection rules was excluded. An argument in support of this interpretation was the fact that the forbidden lines in O II were observed in interstellar space, where the gas density is less than 20 atoms per cubic centimeter. Thus the forbidden lines for O II, N II, and other elements were interpreted as arising from spontaneous radiation and were not bound by the selection rule for electric dipole radiation. In this connection it became necessary to reformulate the quantum theory of radiation. This was first done in 1928 by Rubinowicz, who showed that the selection rules for multipole radiation are different from those for electric dipole radiation and that these rules are different for different order multipoles; thus the multipole radiation could not always be expected when electric dipole radiation was observed. It was found that the selection rule for second-order multipole radiation* (taking account of the various types of electron coupling) allowed for transitions corresponding to spectral lines which formerly had been assigned to nebulium, coronium and geocoronium.

When it was established that the majority of forbidden lines were to be associated with spontaneous multipole radiation, and that as far as "forbiddenness" was concerned these lines were equivalent to electric dipole radiation, the notion of forbiddenness was reconsidered. In 1936, Bowen had already directed attention to the provisional nature of the concept of forbidden lines and had, as a convenience, divided all lines (whether or not they obeyed the selection rules for electric dipole radiation) into two groups: (1) those characterized by high transition probabilities (10^{-3} — 10^9 sec^{-1}), and (2) those characterized by small transition probabilities (10^2 sec^{-1} or less).[1] A more sophisticated definition of forbiddenness was given by Mrozowski,[2] who defined as forbidden all lines which, for unexcited atoms, have a very small transition probability as compared with the usual transition probabilities for the given atoms; in this formulation the transitions being compared must refer to levels with approximately the same quantum numbers. The last provision is necessary in order to exclude remote terms in a series, since these have small transition probabilities as compared with the first lines in the series. In some cases the notion of "very small" was interpreted as 10^{-3} and in others 10^{-4}. It is apparent that the notion of forbidden lines is well taken in the Mrozowski formulation but that it leads to a classification system which involves certain difficulties. Although the old idea of forbidden lines is somewhat inconvenient, and actually reflects a particular stage in the development of atomic spectroscopy, we shall use it hereinafter (the old concept of "forbiddenness" is the generally accepted one).

Forbidden lines are observed in different regions of atomic, molecular, and nuclear spectra and encompass the frequency range from gamma rays to radio waves. A study of these lines leads to a more fundamental understanding of the properties of matter in various states. Forbidden lines are especially important in astrophysics. The intensity of these lines yields information on the density, temperature, chemical composition, and other important properties of various non-terrestrial bodies. Forbidden lines are also important in nuclear spectroscopy. This importance stems from the fact that the forbidden lines in gamma spectra are generally more intense than the allowed lines.

In the present review we shall consider the important results of theoretical and experimental research on forbidden lines in atomic spectra. Sections 1 and 2 are devoted to the general features and classification of spontaneous forbidden lines while Secs. 3 and 4 deal with the forbidden radiation of celestial bodies and lines which are induced by various fields. In conclusion we consider briefly the problem of obtaining forbidden lines in the laboratory and present a brief survey of the important unsolved problems pertaining to forbidden radiation in atoms.

1. GENERAL CHARACTERISTICS OF SPONTANEOUS FORBIDDEN LINES

As has been indicated in the introduction, the existence of intense forbidden lines in the spectra of celestial bodies made it necessary to formulate a quantum theory of multiple radiation. This theory was formulated in the 1930's. At first electric quadrupole and magnetic dipole radiation were studied, since these encompass almost all the multipole lines encountered in atomic spectra. Somewhat later an investigation was made of third-order multipoles and it was only relatively recently (in the 1940's), in connection with the rapid development of nuclear spectroscopy, that a need arose for the development of a theory for multipole radiation of any order. The simplest derivation of the formulas for the fields and for the probability of multipole radiation of any order in the nonrelativistic case is that due to Berestetskiĭ.[3] Because the present paper is concerned with atomic spectra, we deal chiefly with second-order multipole radiation. Quantum theory gives the following formulas for the radiation intensity of multipole lines of this kind[4] (for natural excitation):

$$I_q(\mathrm{A}, \mathrm{B}) = N(a)\, h\nu A_q,$$

$$A_q = \frac{32\pi^6\nu^5}{5hc^5} \sum_{a,\, b} |(a\,|\,Q\,|\,b)|^2, \qquad (1.1)$$

$$I_m(\mathrm{A}, \mathrm{B}) = N(a)\, h\nu A_m,$$

$$A_m = \frac{64\pi^4\nu^3}{3hc^3} \sum_{a,\, b} |(a\,|\,\mathbf{M}\,|\,b)|^2, \qquad (1.2)$$

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*By second-order multipole radiation we mean electric quadrupole and magnetic dipole radiation; third-order radiation refers to electric octupole and magnetic quadrupole, etc.

where A_q and A_m are the probabilities for electric quadrupole and magnetic dipole transitions, Q is the atomic electric quadrupole moment, a tensor with components

$$Q_{\alpha\beta} = \frac{1}{3}\sum_i e_i (3x_{i\alpha}x_{i\beta} - r_i^2\delta_{\alpha\beta});$$

$M = \frac{e}{2m_0c}\sum(\mathbf{L}_i + 2\mathbf{S}_i)$ is the magnetic dipole moment of the atom, $\mathbf{L}_i$ and $\mathbf{S}_i$ are the orbital moment and spin of the i-th electron; N(a) is the number of atoms in the initial state a at a given instant of time; b refers to the final state; A and B are the states associated with the upper and lower energy levels between which the transition takes place; $\sum_{a,b}$ indicates that the intensity of the line (A, B) is equal to the sum of the intensities of its components, ν is the frequency.

Computing the matrix elements in Eqs. (1.1) and (1.2) we can establish the selection rules for various atomic transitions. These rules were first established by Rubinowicz, Brinkman and Blaton[5-7] who showed that these rules depend on the type of radiation and on the nature of the electron coupling in the atoms. For example, in the case of Russell-Saunders coupling, i.e., normal coupling, these rules are as follows:

Electric quadrupole radiation

$$\text{A)}\quad \begin{array}{c} \Delta J = 0,\ \pm 1,\ \pm 2;\ J_1 + J_2 \geqslant 2; \\ \Delta m = 0,\ \pm 1,\ \pm 2; \end{array} \left\{ \begin{array}{l} \text{even terms — even terms,} \\ \text{odd terms — odd terms} \end{array} \right.$$

$$\text{B)}\quad \Delta L = 0,\ \pm 1,\ \pm 2;\ \Delta S = 0.$$

Magnetic dipole radiation

$$\text{A)}\quad \begin{array}{c} \Delta J = 0,\ \pm 1;\ J_1 + J_2 \geqslant 1;\ \Delta n = 0; \\ \Delta m = 0,\ \pm 1; \end{array} \left\{ \begin{array}{l} \text{even terms — even terms,} \\ \text{odd terms — odd terms} \end{array} \right.$$

$$\text{B)}\quad \Delta L = 0,\ \pm 1;\ \Delta S = 0,$$

where ΔJ is the change in the quantum number J, the total angular momentum of the atom; Δm is the change in the magnetic quantum number (the projection of the total moment on the z axis); ΔL and ΔS are the changes in the quantum numbers for the orbital and spin moments of the atom and Δn is the change in the principle quantum number. The selection rule is easily generalized to the case of 2^k-pole radiation where k is the order of the multipole (k = 1 denotes dipole radiation, k = 2 denotes quadrupole radiation, k = 3 denotes octupole radiation and so on):

2^k-pole electric radiation

$$\text{A)}\quad \begin{array}{c} \Delta J = 0,\ \pm 1,\ \pm 2,\ \ldots,\ \pm k; \\ J_1 + J_2 \geqslant k; \\ \\ \Delta m = 0,\ \pm 1,\ \pm 2,\ \ldots,\ \pm k; \end{array} \left\{ \begin{array}{l} \left. \begin{array}{l} \text{even terms — even terms,} \\ \text{odd terms — odd terms,} \end{array} \right\} \text{for } k \text{ even} \\ \left. \begin{array}{l} \text{even terms — odd terms,} \\ \text{odd terms — even terms} \end{array} \right\} \text{for } k \text{ odd} \end{array} \right.$$

$$\text{B)}\quad \Delta L = 0,\ \pm 1,\ \pm 2,\ \ldots,\ \pm k;\ \Delta S = 0;$$

2^k-pole magnetic radiation

$$\text{A)}\quad \begin{array}{c} \Delta J = 0,\ \pm 1,\ \pm 2,\ \ldots,\ \pm k; \\ J_1 + J_2 \geqslant k; \\ \\ \Delta m = 0,\ \pm 1,\ \pm 2,\ \ldots,\ \pm k; \end{array} \left\{ \begin{array}{l} \left. \begin{array}{l} \text{even terms — even terms,} \\ \text{odd terms — odd terms} \end{array} \right\} \text{for } k \text{ odd} \\ \left. \begin{array}{l} \text{even terms — odd terms,} \\ \text{odd terms — even terms} \end{array} \right\} \text{for } k \text{ even} \end{array} \right.$$

$$\text{B)}\quad \Delta L = 0,\ \pm 1,\ \pm 2,\ \ldots\ \pm k;\ \Delta S = 0.$$

The selection rules given under B are satisfied exactly only in the case of normal coupling. If there is a departure from normal coupling these rules are violated to a greater or lesser extent. Thus, for light atoms, which exhibit normal coupling (in particular, helium), the rule ΔS = 0, i.e., the so-called intercombination rule, is strictly observed (transitions of the type $1sns\,{}^1L - 1sns\,{}^3L'$ are strictly forbidden); on the other hand, in the heavy elements (in particular, mercury) the rule ΔS = 0 does not hold at all. This and the violation of the rule ΔL = 0, ±1, ±2, . . . , ±k are explained as follows. Departures from normal coupling occur as a result of the spin-orbit interaction in the atom; this interaction is very small for the light elements but increases with atomic number. This interaction changes the wave functions. If the change is small the modified wave function for any term is not very different from the pure Russell-Saunders wave function for the same term. A calculation shows[8] that in this case there should be a small but increasing (with atomic number) admixture of wave functions of the other Russell-Saunders terms of different multiplicity and different L; however, all terms (including the given term) are characterized by the same value of J and are of the same configuration. Thus, a given state can have properties which apply to states with different values of L and multiplicity; for this reason it is possible to have weak transitions which are not compatible with rule B. For large spin-orbit interactions, i.e., j-j coupling, L and S are no longer good quantum numbers and selection rule B loses its meaning completely.

The rule $\Delta n = 0$ is satisfied strictly only in the nonrelativistic approximation. Thus, for example, a calculation of the probability for the $1s\,^2S_{1/2} - 2s\,^2S_{1/2}$, transition in H I carried out with relativistic wave functions shows that it is to be associated with magnetic dipole radiation although the transition violates the rule $\Delta n = 0$.[9]

It is apparent that the majority of the selection rules are the same for magnetic dipole and electric quadrupole radiation. Hence it is only in certain particular cases that these two effects can be distinguished by means of the selection rules. Characteristic rules are, for example, $J_1 + J_2 = 1$ for magnetic dipole radiation and $\Delta J = \pm 2$ for electric quadrupole radiation. The transitions

$$J = 0 \rightleftarrows J = 1 \quad \text{and} \quad J \rightleftarrows J + 2$$

refer respectively to pure magnetic dipole and pure electric quadrupole transitions if all the remaining rules for these multipoles are satisfied.

Having numerical values for the nonvanishing matrix elements $(a|Q|b)$ and $(a|M|b)$, it is possible to determine, as is apparent from Eqs. (1.1) and (1.2), the probabilities for various atomic transitions. A knowledge of the absolute or relative probabilities for these transitions is of considerable help in investigating multipole lines. In particular, from a knowledge of the relative probabilities of the components of the multipole lines it is possible to get some idea of the assignment to various multiplicities. A knowledge of these probabilities is especially important in work on forbidden lines in the spectra of non-terrestrial bodies. A knowledge of the absolute probability for transitions and intensities of the corresponding forbidden lines, determined from observed data, can be used to find the concentration of metastable atoms of various elements. If the electron temperature and density are known, using the quantum theory of radiation the astrophysicist can establish the density of atoms in the ground state and, in the final analysis, can determine the chemical composition of a non-terrestrial body.[10–13]

A calculation of the probabilities for multipole transitions has been carried out by a number of investigators; the most important work (aside from that cited[8]) is that of Condon, Shortley, Garstang and Pasternak.[14–17] The transitions which have been investigated in greatest detail (relative and absolute probabilities for the electric quadrupole and magnetic dipole cases) are those between levels characterized by the following electron configurations p^2, p^3, p^4, d^2, d^3, d^4, and d^5. The calculations for the p^2, p^3, and p^4 configurations have been carried out for the most general case, i.e., intermediate electronic coupling.[8] The results of the calculations are tabulated as functions of a parameter χ,* which varies from the value zero, in the case of normal coupling, to infinity, for j-j coupling. Rather good estimates of this parameter are available for various atoms.[18] The electric quadrupole transition probability (in contrast with that for magnetic dipole transitions) contains the square of the radial integral s_q:

$$s_q = \frac{2}{5} e \int_0^\infty r^2 R^2(n,\ p)\, dr, \tag{1.3}$$

which is of order unity (in an atomic system of units). As has been mentioned above, at small deviations from normal coupling the wave functions that correspond to definite terms in intermediate coupling can be given as a linear combination of the wave functions for Russell-Saunders terms of the same J, for the same configuration. The coefficients are functions of the parameter χ. For example, in the p^2 configuration

$$\psi(^1D_2') = a\psi(^1D_2) + b\psi(^3P_2);\ \psi(^3P_2') = a\psi(^3P_2) - b\psi(^1D_2), \tag{1.4}$$

where the primed terms denote intermediate coupling while the unprimed terms denote normal coupling. The coefficients a and b are:[8]

$$\left.\begin{aligned} a &= 1 - \frac{25}{144}\chi^2 - \frac{125}{864}\chi^3 + \ldots, \\ b &= \frac{5\sqrt{2}}{12}\chi\left(1 + \frac{5}{12}\chi - \frac{25}{72}\chi^2 + \ldots\right). \end{aligned}\right\} \tag{1.5}$$

These linear combinations mean that certain of the transitions that are forbidden in the case of pure normal coupling are allowed in intermediate coupling. Typical examples are the magnetic dipole transitions between different terms, the electric quadrupole transitions between the term systems $(^3P_2' - {}^1D_2')$, as is apparent from Eqs. (1.4) and (1.5) and so on. The probabilities for these transitions are proportional to χ^2 and χ^4 and are considerably smaller than the probabilities for the corresponding multipole transitions for normal coupling (for example, χ^2 for O III $2p^2$ is 10^{-4}). However, for wave lengths in the optical region (λ 4580 A), according to Shortley[8]

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*$\chi = \frac{\xi}{5F_2}$, where ξ and F_2 are the energies of the spin-orbit and electric interactions respectively.

$$A_m = 3\cdot 10^2 \text{ sec}^{-1}, \tag{1.6}$$
$$A_q = 8\cdot 10^{-1} \text{ sec}^{-1}, \tag{1.7}$$

and are considerably smaller than the probability for the corresponding electric dipole transition, which is

$$A_d = 2\cdot 10^7 \text{ sec}^{-1}. \tag{1.8}$$

The foregoing indicates how small the probabilities for these forbidden transitions are as compared with those for electric dipole transitions. The order of magnitude of these probabilities ranges from 10^{-2} sec^{-1} to 10^{-9} sec^{-1}, i.e., they are smaller than A_d by factors of $10^9 - 10^{16}$.* The same considerations apply for the probabilities for higher-order multipole transitions since the ratio of two successive multipole electric or magnetic transitions is $(2\pi a/\lambda)^2$ (a is the radius of the atom, λ is the wave length); for the optical region the numerical value of this ratio is approximately 10^{-6}.[5] This ratio increases considerably in the x-ray region and exhibits a strong dependence on atomic number:†

$$\left(\frac{2\pi a}{\lambda}\right)^2 = 10^{-5}Z^2.$$

Because of the extremely small probabilities for the forbidden transitions as compared with the allowed transitions, the forbidden lines are only weakly allowed. However there are special conditions under which both types of lines become comparable in intensity; indeed, in come cases the forbidden lines become stronger than the allowed lines. In this connection if is of interest to consider the dependence of line intensity on the conditions of excitation. In general, the intensity of any line is given by the expression[1]

$$I = \frac{N'h\nu A_i}{A_1 + A_2 + \ldots + A_i + \ldots + B + C}, \tag{1.9}$$

where N' is the number of atoms excited to the given state per unit time;* B is the probability of energy loss by the atom in collisions of the first or second kind (this depends strongly on the conditions within the radiation source); C is the probability for transition to a higher state by absorption of radiation; A_1, A_2, A_3, . . . are the probabilities for radiation of various lines and A_1 is the probability for radiation of the line being considered.

The transition probability A_i between two states is usually defined as the reciprocal of the mean lifetime of the atom in the higher state if the transition being considered is the only transition available (A_i should contain terms which depend on the radiation density but these are almost always small). The probability A_i can be given in the form of a series $A_i = A_{id} + A_{iq} + A_{im} + \ldots$ the first term of which is the probability for the electric dipole transition; the other terms represent the probabilities for various multipole transitions. Even where simultaneous electric dipole and multipole radiation is possible (where both are spontaneous and obey appropriate selection rules) it is impossible to detect the multipole lines in practice. Hence, great interest attaches to forbidden lines which arise in transitions from metastable levels. As has been noted in the introduction, transitions from such levels to lower levels are usually forbidden by the selection rules for electric dipole radiation. Since most of these rules are derived on the basis of approximations some are usually discarded (usually two are retained[19]). In this case in the denominator of Eq. (1.9) we are generally left with only B and C (all the A_i are of order 10^2 sec^{-1} or smaller) and this expression becomes

$$I = \frac{N'h\nu A_{ik}}{B + C}, \tag{1.10}$$

where A_{ik} is the probability for one of the multipole transitions.

The quantity B depends on the number of collisions of the first and second kind which the metastable atom experiences with other atoms and electrons. According to the kinetic theory of gases, in air under normal conditions an atom undergoes 10^{10} collisions (elastic and inelastic) per second. At lower pressures the number of collisions is reduced in proportion to the pressure. At the lowest pressure (approximately 10^{-3} mm Hg) for which it is possible still to have radiation in discharge tubes the number of collisions per second is reduced to 10^4. In this case the probability of collisions of

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*True, as the atomic number increases χ also increases and in heavy elements such as lead it becomes of order unity;[2] nonetheless, the statement concerning the small probability of electric quadrupole and magnetic dipole transitions (as compared with electric dipole transitions) still applies, as can be seen from Eqs. (1.6)–(1.8).

†It is apparent from what has been said above that there is no contradiction between the "old" and "new" concepts of forbidden lines (see introduction) for the overwhelming majority of atomic spectra. It is only in the x-ray region that contradictions arise.

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* $N = \dfrac{N'}{B + C + A_1 + A_2 + \cdots + A_i + \cdots}$, where N is taken from Eqs. (1.1) and (1.2).

the first and second kind per unit time become much smaller than unity.

We shall consider collisions of the first and second kind in somewhat greater detail because these are frequently used to obtain atoms in metastable states.* It is of interest to note the following features which pertain to these collisions.

Collisions of the first kind between atoms and electrons occur more frequently at values of the electron kinetic energy which are greater than the excitation energy of the atom; on the other hand, collisions of the second kind occur more frequently at low electron velocities. The probability of a collision of the first or second kind between one atom and another atom or an ion is small if the excitation energy and the velocity of the second atom or ion is approximately the same as the energy and velocity of the first atom. It becomes significant, however, when the relative velocities of the colliding particles become comparable with the velocity which a free electron acquires at the excitation potential and increases as these velocities increase.[20]

The probability of a collision of the first or second kind, for which the excitation energy of one atom is transformed into energy of relative motion of the colliding particles, is usually small but may be significant in those cases in which one of the colliding atoms has a small excitation energy as compared with the other or when it can lose (or gain) only a small part of this energy in the collision (the latter situation obtains if the atom has a close-lying excited state). Examples of this type are transitions in the argon atom from the ground state $6\ ^2P_{1/2}$ to the metastable state $6\ ^2P_{3/2}$ in collisions of the first kind with argon atoms (in a discharge at constant temperature),[21] and in transitions of mercury atoms from the excited non-metastable state $6\ ^3P_1$ to the metastable state $6\ ^3P_0$ in collisions of the second kind with argon or nitrogen atoms.[19] Collisions of the second kind between two atoms are very important when one of the atoms excites the other by virtue of loss of its own energy. Such collisions are most probable in the case in which the excitation energies of both atoms are comparable. (A small difference between these energies can be compensated by the relative kinetic energy).† For example, in collisions of mercury atoms, excited to the $6\ ^3P_1$ level with sodium atoms the latter are excited to states, the energy of which are approximately the same as the energy of the $6\ ^3P_1$ state.[19] If there are no such levels the probability of collisions of this kind falls off rapidly. Molecules are especially effective in these collisions since they have available a large number of energy levels so that it is very likely that there will be levels with energies close to the energy of the excited atom. It should also be noted that in collisions of the second kind between two atoms the spin plays an important role. In particular, the most probable collisions of the second kind are those in which the total spin of the colliding atoms is not changed.[22]

A quantitative investigation of collisions of the first and second kind involves great experimental difficulties and work in this field has been started only in recent years. Here we should mention the work of the Soviet physicists Frish, Zapesochnyĭ, Bogdanova et al.[21,23-27]* These papers are concerned chiefly with the role of collisions of the first kind between atoms and electrons in the excitation of allowed lines in electron beams and in gas-discharge plasmas characterized by Maxwellian electron velocity distributions; in the first case an investigation has been made of the effect of cascade transitions on the population of the atomic levels. A quantitative investigation of collisions of the second kind has also been carried out for collisions between excited mercury atoms and sodium atoms; these are responsible for the sensitized fluorescence of sodium vapors. Other important problems have also been treated.

Usually the probability B [cf. Eq. (1.10)] for collisions of the first and second kind is large compared to the radiation probability; hence lines corresponding to the optical transition from a metastable state to a lower state are essentially forbidden. To obtain forbidden lines in atoms of any element the quantity B must be made as small as possible. This objective is usually achieved as follows. Atoms of the element being investigated (for example, O I) are excited to metastable states in a mixture containing an inert gas or some other gas, the atoms of which have high excitation energies compared with the energies of the metastable atoms (for example, the energy of the first metastable state in O I is 5.3 ev while the energy of the first excited state in the atom of the inert gas is of the order of 10 — 20 ev). Thus there are no collisions of the second kind between the metastable atoms of the gas being investigated and the atoms in the mixture. The gas must be kept at a low pressure (approximately 2 mm Hg) for the following

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*The problem of collisions between different particles has been considered in great detail in a recently published monograph by Massey and Burhop.[79]

†These collisions are responsible for the resonance effect in so-called sensitized fluorescence.

*A comprehensive bibliography is given in the review paper by Frish.[23]

TABLE I

Electric dipole			Magnetic dipole			Electric quadrupole			Total 3rd order radiation		
Δm	$\alpha=\frac{\pi}{2}$	$\alpha=0$	Δm	$\alpha=\frac{\pi}{2}$	$\alpha=0$	Δm	$\alpha=\frac{\pi}{2}$	$\alpha=0$	Δm	$\alpha=\frac{\pi}{2}$	$\alpha=0$
+1	σ	right cir.	+1	π	right cir.	+2	σ	—	+3	σ	—
0	π	—	0	σ	—	+1	π	right cir.	+2	π	
−1	σ	left cir.	−1	π	left cir.	0	—	—	+1	σ	right cir.
						−1	π	left cir.	0	π	—
						−2	σ	—	−1	σ	left cir.
									−2	π	—
									−3	σ	—

reasons: at higher pressures there are collisions of the O I atoms with the oxygen molecules, and at lower pressures there are collisions with the walls of the container. The higher the degree of ionization of the gas being investigated the more difficult it is to use the method described above because at increased ionization there is an increase in the number of free electrons with small velocities; this situation results in an increase in the number of collisions of the second kind.

We now consider briefly the problem of removal of an atom from a metastable state via absorption of radiation. The probability of absorption of radiation will be denoted by C. As is well-known, this quantity is proportional to the radiation intensity at the wave length at which the atom in a stable or metastable state is capable of absorption.

The fact that the quantity C is large is one of the important factors which suppresses the production of intense forbidden lines in the atmospheres of many stars. For example, in the atmosphere of the sun the radiation density is so strong that the atom Ca II is removed from its ground $4s^2\ {}^1S$ state by absorption of radiation 2×10^4 times per second.[1] Nevertheless, there are highly ionized atoms (belonging to the first long period in the periodic table) for which C is rather small and for which the conditions for excitation of intense forbidden lines are satisfied. This situation is explained by the fact that the transition of highly ionized atoms from a metastable or ground state to the first non-metastable state occurs at the limit of the ultraviolet, whereas the radiation density of a relatively cold star such as the sun is small in this region of the spectrum. As far as forbidden lines of weakly ionized and neutral atoms are concerned, we may note these can achieve significant intensities only when the radiation density is small in the visible region of the spectrum. This situation (and a reduction in the magnitude of B) are satisfied only in certain non-terrestrial bodies such as gaseous nebulae, young stars in recent stages of development, certain old stars with extensive atmospheres, and so on.

In investigating different orders of magnitude of the sum B + C we may note certain features which pertain to the intensities of forbidden lines. Suppose that a given metastable state of the atom gives some forbidden line. Then, the intensities of these lines [as is apparent from Eq. (1.10)] will be related as the probabilities of the corresponding transitions. The situation is completely different in the case in which the sum B + C is small so that $B + C \ll A_1 + A_2 + \ldots + A_i \ldots$; in this case Eq. (1.9) assumes the form:

$$I = \frac{N' h\nu A_{ik}}{A_1 + A_2 + \ldots + A_i + \ldots}. \qquad (1.11)$$

In the case of a single possible transition or in the case in which $A_{ik} \gg A_1 + A_2 + \ldots + A_{i-1} + (A_i - A_{ik}) + \ldots$, we have

$$I = N' h\nu. \qquad (1.12)$$

It follows from Eq. (1.12) that in the latter case the intensity of the forbidden lines is determined only by the density of metastable atoms; thus the ratio of intensities of the individual lines depends to a considerable degree on the statistical weights and the excitation potentials of the metastable levels.

Of great importance in the study of multipole lines is the theory of the Zeeman effect, which was developed in the 1930's.[1,5,7] The results of theoretical investigations of the simple Zeeman effect are given in Table I. Here Δm denotes the change in the magnetic quantum number, α is the angle between the direction of observation and the magnetic field, π and σ denote the plane of polarization parallel and perpendicular to the magnetic field, right cir. denotes right-circular polarization, left cir. denotes left-circular polarization. It should be noted that the polarization properties of the individual Zeeman components remain the same in the complex Zeeman effect although the components for a given Δm become multiple.

It is apparent from Table I that the longitudinal effect is the same for all order of multipole radiation; this is not true for the transverse effect. The latter is different for each type of multipole radiation so that it can be used to determine the assignment of a line to one or another of the multipole orders. In this way the whole quadrupole series $p^4\,{}^1D - p^4\,{}^1S$, O I; $s\,{}^2S - d\,{}^2D$, Na I and K I was determined, as were the magnetic dipole lines of Pb I, ${}^3P_1 - {}^1S$, λ 4618 A.[28-31]

The theory of the Zeeman effect has been formulated in fairly great quantitative detail. The probability for each of the Zeeman transitions $A^{J,m}_{J',m'}$ has been computed as a function of the quantum number m. As in the case of electric dipole radiation, these probabilities obey the two sum rules:

$$\sum_{m} A^{J,m}_{J',m'} = \sum_{m'} A^{J,m}_{J',m'}, \tag{1.13}$$

where $m' = m \pm 1,\ m \pm 2,\ m \pm 3$ (depending on the order of the multipole) and

$$\sum_{m'} A^{J,m}_{J',m'} = A^{J}_{J'}, \qquad \sum_{m'} A^{J,m'}_{J',m} = \overline{A^{J}_{J'}}, \tag{1.14}$$

where $A^J_{J'}$ and $A^J_{J'}$ are the transition probabilities in the absence of the field; these do not depend on m and satisfy the well-known relation

$$(2J+1)\,A^{J}_{J'} = (2J'+1)\,\overline{A^{J}_{J'}}. \tag{1.15}$$

Different magnetic field cases have been considered: the weak field (anomalous Zeeman effect) and the intermediate and strong field cases (simple Zeeman effect). It has been found in the second case that at some definite field strength the selection rules can be violated. Thus, for example, in the case of quadrupole radiation we find the forbidden components with $\Delta J = \pm 3$.[32]

A theory has also been formulated for multipole radiation of atoms in external fields characterized by various kinds of symmetry. This theory has been of importance in the study of spectra of the rare-earth ions in crystals and solutions. Both the theory and its modifications have been considered in detail by El'yashevich in a monograph devoted to the spectra of rare-earth elements.[33] It is shown in this monograph that multipole radiation is present for any triply-charged rare-earth ion in crystals which have electron configurations of the type $4f^k$ where k varies from 1 in Ce IV to 13 in Yb IV. In transitions between individual levels pertaining to this configuration, it is possible to have both magnetic dipole and electric quadrupole lines and induced electric dipole lines. We shall direct our attention to the first two kinds.

Since the interaction of the ion with the field of the crystal lattice is small, the selection rules for electric quadrupole and magnetic dipole radiation for the quantum numbers J and L remain approximately the same as in the case of free atoms. The situation with regard to the selection rule for the quantum number m is somewhat different. These rules are satisfied rigorously only in cases in which the atomic system possesses an axis of cylindrical symmetry, i.e., an isolated axis of infinite order. In crystals characterized by a rotational axis of n-fold symmetry the selection rules for the quantum number m do not actually lose force but become a particular case of more general selection rules; more precisely, they become the selection rules for the so-called crystallographic quantum number μ, i.e.,

$$\Delta\mu = 0,\ \pm 1 \tag{1.16}$$

for magnetic dipole radiation and

$$\Delta\mu = 0,\ \pm 1,\ \pm 2 \tag{1.17}$$

for electric quadrupole radiation; the quantum number m is related to μ by the expression

$$m = \mu + kn, \tag{1.18}$$

where k is any whole number.[33]

Furthermore, as a consequence of the existence of isolated axes in the crystal the multipole radiation of rare-earth ions must have definite polarization properties; these are characteristic of the various multipoles and of the various axes of symmetry.[33]

An estimate has also been made of the probabilities for both magnetic and quadrupole transitions for rare-earth ions in crystals; this estimate is made using Eqs. (1.1) and (1.2) and taking account of the splitting of the ionic levels and the perturbations of the ionic wave functions due to the lattice fields. According to El'yashevich[33] the maximum theoretical values for these probabilities are

$$A_m = 2\cdot 10^2 \ \text{sec}^{-1}, \qquad (1.19)$$
$$A_q = 0.5 \ \text{sec}^{-1}. \qquad (1.20)$$

The directional properties of the radiation of these ions in crystals has been investigated theoretically by Hellwege,[34,35] whose analysis is based on the assumption that the multipole axes of the ions are oriented in definite directions with respect to the crystal lattice. On this basis theoretical predictions are made of the possible results of experimental investigations of the dependence of intensity on radiation direction for the various multipole lines (taking account of splitting in the ionic field of the crystals). Hellwege has also analyzed in detail the possibility of establishing the fixed direction for the electric and magnetic moments of ions in crystals, using the experimental data for various crystal systems. He has shown that this possibility exists if the following requirements are satisfied:

1. All the multipole moments of a given type must be parallel to one crystallographic direction.
2. This direction must coincide with one of the axes of the indicatrix.

It turns out that the first of these conditions is not satisfied in cubic crystals while the second is not satisfied in triclinal crystals. The fact that the different types of multipole radiation have different directional properties is also used as the basis for the interference method of studying multipole radiation. The interference method was proposed by S. I. Vavilov.[36,37] Vavilov showed that the nature of Fresnel interference for diverging interfering rays (distribution of intensity and polarization) depends strongly on the nature of the elementary radiator, that is to say, on whether the radiator is an electric or magnetic dipole or an electric quadrupole. For example, the dependence of the visibility of the interference bands on the angle φ between the interfering rays is different for dipole and quadrupole electric radiation:[37]

$$\left.\begin{aligned} V_d &= \frac{1+\cos\varphi}{2}\,; \\ V_q &= \frac{1+\cos\varphi}{2} - \sin^2\varphi, \end{aligned}\right\} \qquad (1.21)$$

where $V = \dfrac{I+i}{I-i}$, I and i are the maximum and minimum intensities in the interference field. It is also found that in the case of magnetic dipole radiation there is a displacement of the interference pattern (by one band) as compared with electric dipole radiation.

The theoretical description given above has been verified experimentally. Experimental data obtained by Zaĭdel and Larionov, Hoogschagen, Sevchenko, and others[38-47] in observations of narrow bands in the spectra of triply charged ions of the lanthanides in crystals and in solutions* indicate that the narrow bands are actually due to forbidden transitions between levels in the f-shells. The results of an investigation of polarization and measurements of probabilities for these transitions were found to be in agreement with the theoretical predictions and indicate that some of the transitions are magnetic while the others are due to induced electric dipole radiation. It may be noted that in one of the papers by Freed and Weissman,[47] devoted to an investigation of the spectra of solutions of Eu IV salts, the Vavilov interference method was used successfully to examine the nature of the radiation.

In the early 1940's a theory was developed for hyperfine structure and for the Zeeman effect in second-order, hyperfine multiple radiation. It was established[48] that the intensity ratios for hyperfine components in magnetic dipole radiation obeyed the rules derived in the usual theory for hyperfine structure in electric dipole lines.[49] However the relative intensities of the components of the hyperfine structure of electric quadrupole lines were first obtained by Rubinowicz[50] for the intensities of the components of the fine structure multiplets by substituting the quantum number F, J and I ($\mathbf{F} = \mathbf{J} + \mathbf{I}$, where I is the nuclear spin) for J, L and S. The selection rule for F remains the same as for J, i.e.,

$$\Delta F = 0,\ \pm 1,\ \pm 2,\ \ldots,\ \pm k;\ \ F_1 + F_2 \geqslant k; \qquad (1.22)$$
$$\Delta M = 0,\ \pm 1,\ \pm 2,\ \ldots,$$
$$\pm k\,(M = -F,\ -F+1,\ \ldots,\ F-1,\ F) \qquad (1.23)$$

for 2^k-pole radiation.

These theoretical analyses played an important part in determining the multipole assignments of various forbidden lines. For example: the electric quadrupole assignment of the 5313-A line, the magnetic dipole assignment of the 4618-A line and the mixed nature of the 7330-A line in the spectrum of lead, and the quadrupole assignment of the 2815-A line ($^2D_{5/2} - {}^2S_{1/2}$) in Hg II.[51-54] The theory of hyperfine structure was also used for multipole lines in Bi I, Sb and As I.[55,56]

2. ATOMIC MULTIPOLE RADIATION

We now make a more detailed analysis of different kinds of multipole radiation. We consider first electric quadrupole radiation. We shall not dwell here on the laws of this radiation, developed in the preceding section, which was devoted to the general properties of multipoles, but shall deal chiefly with the multiplet structure of quadrupole lines. As is well known, the intensity ratio for the components of a quadrupole multiplet is equal to the ratio of the corresponding transition probabilities. The relative probabilities $A^{L,J}_{L',J'}$ have been computed for the case of Russell-Saunders coupling and terms of the same multiplicity. These probabilities are functions of the quantum numbers J, L, and S. As an example we present two of the formulas from Condon and Shortley:[4]

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*A comprehensive bibliography is given by El'yashevich.[33]

$$A^{L,\,J}_{L-1,\,J-2}=\frac{Q\,(J-1)\,P\,(J-2)\,P\,(J-1)\,P\,(J)}{(J-1)\,J\,(2J-1)}\,H^2, \qquad (2.1)$$

$$A^{L,\,J}_{L,\,J+2}=\frac{P\,(J+1)\,P\,(J+2)\,Q\,(J)\,Q\,(J+1)}{(J+1)\,(J+2)\,(2J+3)}\,G^2, \qquad (2.2)$$

where

$$H=-e\sum_i\frac{1}{4}\sum_{\alpha''}[(\alpha SL\,|\,x_i\,|\,\alpha''SL)(\alpha''SL\,|\,y_i\,|\,\alpha'SL-1)$$

$$+(\alpha SL\,|\,x_i\,|\,\alpha''SL-1)(\alpha''SL-1\,|\,y_i\,|\,\alpha'SL-1)], \qquad (2.3)$$

$$P\,(J)=(J+L)\,(J+L+1)-S\,(S+1),$$

$$Q\,(J)=S\,(S+1)-(J-L)\,(J-L+1), \qquad (2.4)$$

the expression for G is similar to that for H (cf. reference 4).

All the formulas for the probabilities have twenty-five possible transitions L → L′ = L, L ± 1, L ± 2 and J → J′ = J, J ± 1, J ± 2. It is easy to see that the formulas satisfy the selection rule $J_1 + J_2 \geq 2$. The probabilities for the $A^{L,J}_{L',J'}$ transitions obey the following sum rules:

$$\sum_{J'} A^{L,\,J}_{L',\,J'}=A^{L}_{L'}; \qquad (2.5)$$

$$\sum_{J'} g_{J'}A^{L,\,J}_{L',\,J'}=g_J\overline{A^{L}_{L'}}, \qquad (2.6)$$

where $A^{L}_{L'}$ and A^{L}_{L} are independent of J; J′ = J, J ± 1, ± 2; $g_J = 2J+1$ is the statistical weight for a sublevel with a given J. The rules given in (2.5) and (2.6) are similar to those in (1.13) and (1.15) and the well known rules given by Ornshteĭn, Burger, and Dorgelo for electric dipole radiation. In general, Eqs. (2.1) — (2.5) do not apply for intercombination transitions and for multiplets induced by external fields. Calculations show[57] that in the latter cases the departures from the relations in (2.1) — (2.5) become less important as the multiplet splitting is reduced and as the quantum number L increases. Equations (2.1) — (2.5) have been verified experimentally for the quadrupole multiplets $^2S - {}^2D$ in alkaline metals[58] and

$3d^64s\,^6D - 3d^54s^2\,^6S$, $3d^7\,^4F - 3d^64s\,^4S$, FeII

in the spectrum of the star η-Carinae,[59] although the last multiplet has not been resolved completely. We consider the first multiplet in somewhat greater detail. This multiplet, more precisely the quadrupole doublet $1\,^2S_{1/2} - 3\,^2D_{3/2,5/2}$* (λ 4642.17 A, K I; λ 5165.35 A, Rb I; λ 6894.7 A, Cs I),† was investigated by means of anomalous dispersion in 1929 by Prokof'ev.[58] The Rozhdestvenskiĭ "hook" method was used in this work. Prokof'ev showed that the intensity ratio for the components of the doublet was $\frac{2J+1}{2J'+1} = \frac{2}{3}$ and also for the first time verified experimentally the fact that the ratio of the probabilities for the quadrupole and corresponding (i.e., occupying approximately the same places in the spectral series) electric dipole transitions was of the order of 10^{-6} as predicted by theory. For example, in Rb I the calculated ratio for $A^{3D}_{1S} : A^{2P}_{1S}$ where A^{2P}_{1S} is the probability for electric dipole transition for the first line of the resolved series $1\,^2S_{1/2} - 2\,^2P_{1/2,3/2}$ is 2.9×10^{-6} while the experimental value is 2.7×10^{-6}.[58,5]

Somewhat later Blaton[60–62] developed a theory of anomalous dispersion for electric quadrupole radiation. One result of this theory showed that the quantity N − 1 (N is the refractive index) in the vicinity of a quadrupole absorption line is 4 times larger than in the vicinity of the dipole line with the same transition probability and frequency.

Electric quadrupole lines are also found in x-ray spectra. For example, the line $K - M_{IV}$ in the spectra of elements with atomic number Z = 20 — 50, 56 — 58, 73 — 79, 82, 90 the line $K - N_{IV,V}$ in the spectra of elements with Z = 39 — 50, 56, 57, 73 — 74, 77 — 79, and the line $L_{III} - N_{VI,VII}$ in the spectra of elements with Z = 72 — 82, 90, 92 and so on.[63]

Theoretical and experimental investigations[5,58] have also been made of the dependence of the intensity of electric quadrupole lines formed in a given spectral series as a function of the quantum number n. The behavior of these lines is completely different from the behavior of lines associated with induced radiation, frequently even those which obey the same selection rules. Whereas the intensity of the quadrupole lines falls off rapidly with increasing n, the intensity of the induced lines increases up to a certain value of n and then falls off. These features of the behavior of quadrupole and induced lines are frequently used experimentally as a means of distinguishing between the two effects.

Of the remaining features of electric quadrupole radiation, we note the small width of quadrupole lines as compared with dipole lines (a consequence of the long life time of metastable atoms Δt and the relation $\Delta E \cdot \Delta t \geq h/2\pi$, where where ΔE is the width of the level) and the directional dependence of the radiation intensity. For example a maximum radiation intensity is found at an angle $\alpha = 45°$ rather than $\alpha = 90°$ (α is the angle between the radiation direction and the multipole axis) as is the case in dipole radiation.

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*Numbers 1, 3, and 2 are introduced purely arbitrarily (for listing the terms).

†Here we give the first lines of the doublets being considered.

We now consider magnetic dipole radiation. The analyses of magnetic and electric dipole radiation are very similar both in the classical and quantum mechanical formulations. For example, the formulas for the intensities of the electric and magnetic fields of a magnetic dipole are obtained from the corresponding formulas for electric dipole radiation by making the substitutions $\mathbf{E} \rightarrow \mathbf{H}$, $\mathbf{H} \rightarrow -\mathbf{E}$, and $\mathbf{D} \rightarrow \mathbf{M}$, where $\mathbf{D}$ and $\mathbf{M}$ are the electric and magnetic dipole moments respectively. The formulas for the relative intensities of the components of ordinary multiplets and hyperfine structure multiplets are the same for both types of radiation. There is also a similarity between the Zeeman effects. As is apparent from Table I, the data on polarization properties of magnetic dipole lines are obtained from the corresponding data for electric dipole lines by interchanging π and σ.

An important distinguishing feature of magnetic dipole radiation is the fact that its production is intimately related with the existence of spin in the electron and the fact that the electron has different gyromagnetic ratios for the orbital and inherent magnetic moments. If there were no spin, or if the gyromagnetic ratios were the same, the magnetic moment of the atom would be proportional to the total angular momentum and would become an integral of the equations of motion. Its components would be expressed by diagonal matrices in the quantum theory formulation and would vanish for transitions.

All magnetic dipole lines can be divided into three groups:

(1) "Pure lines," i.e., lines that obey the selection rule for Russell-Saunders coupling (with the necessary exclusion of the rule $J_1 + J_2 = 1$) since it is only in this case that they cannot also be in an electric quadrupole radiation state,

(2) intercombination lines, and

(3) "practical" magnetic dipole lines, i.e., lines due to magnetic dipole and electric quadrupole radiation, with the former predominating (these lines will be discussed below).

Of the first two groups, the second has been investigated in greater detail. This group includes magnetic dipole lines, some of which have been mentioned above, and also a series of other lines which are found chiefly in the spectra of celestial bodies. As a consequence of the fact that they arise chiefly by virtue of perturbation, these lines are characterized by small transition probabilities, even as compared with the probabilities of "pure" electric quadrupole transitions. For example,[2] the probability for the intercombination magnetic dipole transition ${}^1S_0 \rightarrow {}^3P_1$ in OI is 9×10^{-2} sec^{-1} while the probability for the electric quadrupole transition ${}^1S_0 \rightarrow {}^1D_2$ in OI is $2\ \text{sec}^{-1}$. As the atomic number increases the probability for intercombination transitions also increases (in particular, in the case of lead the transition ${}^1S_0 \rightarrow {}^3P_1$ has a probability of $50\ \text{sec}^{-1}$).

According to Mrozowski[2] the lines in the first group cannot be resolved by ordinary spectroscopic methods.* They arise predominantly in transitions between components of both fine structure and hyperfine structure of the same term. A more careful investigation has been made of the transitions between levels of the hyperfine structure of the ground states of the alkali atoms. These levels are due to the magnetic interaction between nuclei having nonvanishing spin and the outer electrons. Since the ground state is characterized by $J = \frac{1}{2}$, the levels are determined by the quantum numbers $F = I - \frac{1}{2}$ and $F = I + \frac{1}{2}$, where I is the nuclear spin. These transitions are defined by the selection rule $\Delta F = \pm 1$. These transitions are investigated most effectively by the magnetic resonance methods. The effectiveness of this method results from the following.[64-66]

As is well known, hyperfine levels are very close to each other and for this reason the Bohr frequency corresponding to the level difference is extremely low — say 1.5×10^8 to $1.2 \times 10^{10}\ \text{sec}^{-1}$, that is to say, these frequencies fall in the radiofrequency region. Because of the low frequencies, the lifetime of the excited level of the hyperfine structure is very long and the intensity of the corresponding spontaneous magnetic dipole lines is so weak that direct observation is essentially impossible. To strengthen these lines, it is necessary to reduce the lifetime, say to 10^{-4} sec.[66] This possibility is realized inside a magnetic resonance apparatus which is traversed by a beam of excited atoms. As is well known (cf. Fig. 1), four magnetic fields are used in a magnetic resonance system: two deflection fields H_A and H_B, a fixed field H_C, and an oscillating field H_ω. In general the roles played by these fields are as follows.

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*With the exclusion of lines corresponding to the transition ${}^3P_1 - {}^3P_0$, which are observed in the spectrum of the solar corona.

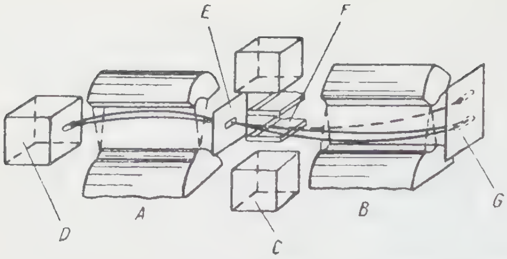

FIG. 1. Motion of an atomic beam in a magnetic resonance apparatus: D) heater, E) slit, G) detector, A, B, C) magnets. (H_A and H_B are the deflection fields, H_C is the fixed field.) F) region of high-frequency field, H_ω.

The oscillating magnetic field H_ω acting on the excited atoms at the appropriate frequency, produces rather intense induced magnetic dipole lines which can be observed either in absorption or emission. The presence of the fixed field results in a Zeeman effect for these lines. The emission or absorption of any of the Zeeman components corresponds to a change in the effective magnetic moment of the atom and this change is detected by means of the deflection fields H_A and H_B. It should be noted that the magnitude of this change depends to a considerable degree on the field H_C. It has been shown[66] that at small values of H_C the above mentioned change is much larger than is the case in strong fields,* although the Zeeman splitting goes in the opposite way. Hence in resolving these lines one usually uses a weak magnetic field for H_C (several hundred gausses). The Zeeman splitting is then used to find the frequency $\Delta\nu$ corresponding to magnetic dipole transition between hyperfine structure levels in the ground state of the atom.

As an example we may present the data obtained in an experiment carried out with this method in lithium. The field H_C ranged from 0.25 to 1.5 gauss for Li^6 and 0.15 to 3500 gausses for Li^7. In the latter case only those Zeeman components were investigated which corresponded to the selection rules $\Delta F = 0$, $\Delta M = \pm 1$, where M is the magnetic quantum number. The values of $\Delta\nu$ are found to be $(228.28 \pm 0.01) \times 10^6\ \mathrm{sec}^{-1}$ and $(803.54 \pm 0.04) \times 10^6\ \mathrm{sec}^{-1}$ for Li^6 and Li^7 respectively. Using the measured values of $\Delta\nu$ it is possible to determine the ratio of the magnetic moments of the two isotopes; this ratio agrees with the results obtained by nuclear resonance experiments.[65–68]

The results of measurements of $\Delta\nu$ for the other alkali metals (and indium) are shown in Table II. The third column contains $\Delta\nu$ in wave numbers. As is apparent from Table II, the uncertainty in the measurements is rather small, approximately 0.01%.[66]

The quantity $\Delta\nu$ has been measured with extremely high precision in hydrogen and deuterium, as is indicated by the following data:[69]

$$\Delta\nu(HI) = (1420.40573 \pm 0.00005)\,10^6\ \mathrm{sec}^{-1}, \quad (2.7)$$

$$\Delta\nu(DI) = (327.384302 \pm 0.000030)\,10^6\ \mathrm{sec}^{-1}. \quad (2.8)$$

Lines at frequencies given by (2.7) and (2.8) have been observed in the radio spectra of interstellar gas.

Recently the magnetic resonance method has been used to study hyperfine structure in excited atomic states of hydrogen and the alkali metals. The results were used to determine the nuclear electric quadrupole moments.[70–74]

A large number of forbidden lines (in particular the above mentioned "practical" magnetic dipole lines) are due to electric quadrupole and magnetic dipole radiation simultaneously, that is to say, so-called mixed radiation. As an example we have lines corresponding to the transitions ${}^1D \rightarrow {}^3P$, ${}^3P_2 \rightarrow {}^3P_1$ in the p^2 configuration; ${}^2P \rightarrow {}^2D$, ${}^2P \rightarrow {}^4S$ and ${}^2D \rightarrow {}^4S$ in the p^3 configuration, and so on. The mixed nature and the relative contribution due to each type of second-order radiation in the intensity of any forbidden line assigned to the p^2, p^3, and p^4 configurations are given in reference 8. The probabilities for these second-order multipole transitions are expressed as functions of a known (tabulated for various elements) parameter χ (cf. above) and the unknown radial integral (1.3). For example, the corresponding strengths for the lines in the ${}^1D_2 \rightarrow {}^3P_1$ transition for small deviations from Russell-Saunders coupling are:

$$S_m({}^1D_2', {}^3P_1) = \frac{125}{144}\chi^2\left(1 + \frac{5}{6}\chi - \frac{25}{48}\chi^2 + \ldots\right), \quad (2.9)$$

$$S_q({}^1D_2', {}^3P_1) = \frac{125}{16}s_q^2\chi^2\left(1 + \frac{5}{6}\chi + \ldots\right), \quad (2.10)$$

where s_q is the radial integral. It is apparent from Eqs. (2.9) — (2.10) that to find the relative

TABLE II

Atom	$\Delta\nu \cdot 10^{-6}\mathrm{sec}^{-1}$	$\Delta\nu$ (in cm^{-1})
Na^{23}	1771.75 ± 0.01	0.059102
K^{39}	461.75 ± 0.02	0.015403
K^{40}	1285.7 ± 0.01	0.042887
K^{41}	254.02 ± 0.02	0.008474
Rb^{85}	3035.7 ± 0.2	0.10127
Rb^{87}	6834.1 ± 1.0	0.22797
Jn^{113}	11387.0 ± 4	0.3799
Jn^{115}	114139.0 ± 3	0.3807
Cs^{113}	9192.6 ± 0.5	0.30665

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*So long as the effective magnetic moment does not change sign in the transition.

probabilities for the multipole transitions it is necessary to know the integral s_q. It is extremely difficult to compute this integral because the electron density distribution in the outer part of the atom is not known with the required accuracy. Pasternack[16] has estimated one of these integrals for light atoms, using an approximation method (the Hartree-Fock fields and hydrogen-like wave functions with a constant shielding term). More exact values of this integral, especially for the heavy atoms, have been obtained by semi-empirical methods. From the intensity ratios of two lines which derive from the same excited level it is possible to determine the ratio of the probabilities for the magnetic dipole and electric quadrupole transitions at the outset; then, comparing this ratio with the theoretical ratios obtained by expressions such as (2.9) and (2.10) (for different lines) one can determine s_q. The calculation (and the considerations given above) can be verified by applying the same method to other lines which derive from the same level.

The method described above for determining s_q and the use of this determination for finding the percentage composition of the radiation has been applied to Pb I. On the basis of the measurements the ratio of the line intensities for λ 4618 A, $^1S_0 - {}^3P_1$ and λ 5313 A, $^1S_0 - {}^3P_2$ in lead and the formulas in (2.7) and (2.8), a value $s_q^2 = 240$ is obtained.[2] Then, from the known value of s_q and (2.9) and (2.10) a theoretical determination is made of the electric quadrupole percentage in the mixed line λ 7330 A, $^1D_2 - {}^3P_1$, in Pb I (about 6%). However a direct measurement of the intensities of the electric quadrupole and magnetic dipole Zeeman components of the λ 7330 A line gives the electric quadrupole percentage as 2.2%. According to Mrozowski the second result is more reliable. In reference 2 examples are given of forbidden multiplets in Te I, O I and Bi I which are especially convenient for carrying out experiments to determine s_q.

The Zeeman effect offers a powerful method for investigating mixed multipole radiation. The theory of the Zeeman effect for mixed multipole lines has been given in references 8 and 48. The most important results of this work is the conclusion that there is an interference effect between the electric quadrupole and magnetic dipole radiation. The calculated theoretical intensities of the Zeeman components corresponding to the selection rule $\Delta m = \pm 1$ consists of three parts: two of these correspond to the probabilities for pure magnetic dipole and pure electric quadrupole transitions, while the latter results form the interference of the two radiations. It has been shown by calculation that the presence of these interference terms does not affect the total probability for a transition between two levels; the only effect is a change in the angular distribution of the radiation. The dependence of the interference terms on the percentage composition of the mixed radiation has also been established. For example, if the ratio of intensities of the Zeeman π components for "pure" dipole magnetic triplet are (in accordance with the intensity rule) 6 : 3 : 1, in the presence of electric quadrupole radiation the indicated ratio is (transverse) $(6-2x):(3+x):(1-x)$, while the longitudinal factor is $(6+2x):(3-x):(1-x)$, where x is the interference correction, which is proportional to the square root of the percentage of electric quadrupole radiation. In particular,[2] for the 7330-A line x is 0.6.

So far the interference effect has been observed only in the observation of the transverse Zeeman effect of the mixed line λ 7330 A, $^1D_2 - {}^3P_1$ in Pb I.[53] The intensities of the individual π components have been measured and the results of the measurements have been compared with the values of these intensities computed on the basis of the work in reference 48. The interference corrections and the hyperfine structure lines produced as a result of the presence of the odd isotope Pb^{207} are found to be very important for obtaining good agreement between the experimental and theoretical results.

Of the other work on mixed multipole radiation we may mention the pure electric quadrupole nature of the $^2D \rightarrow {}^4S$ transition in N I,[75-76] and the almost pure magnetic dipole nature of the $^2P \rightarrow {}^4S$ transition in N I;[2] estimates have also been made of the component percentages in the radiation of the well-known nebular lines λ 4959 A, $^1D_2 \rightarrow {}^3P_1$ and λ 5007 A, $^1D_2 \rightarrow {}^3P_2$ in O III.[77] Because of the small percentage of electric quadrupole radiation (0.1%) the latter may be assumed essentially magnetic dipole lines. However the radiation composition has not been determined for very many of these lines and further work will be required for solution of this problem.

In concluding the present section, we consider the problem of observing third-order multipole radiation in atomic spectra. Lines of this type have not been observed in the optical region and their observation in the x-ray region is open to doubt.[78] The difficulty in observing magnetic quadrupole and electric octupole lines is a result of the following. As has already been mentioned, the probability of third-order multipole transitions is very small, even as compared with second-order multipole transitions. For λ4580 A these are respectively:[78]

$$A'_m = 10^{-4}\ \text{sec}^{-1}, \tag{2.11}$$
$$A'_q = 4 \cdot 10^{-7}\ \text{sec}^{-1}. \tag{2.12}$$

Consequently the time interval between two successive excitations of the atom (for example between two collisions with other atoms) must be several hours (A'_m) or even more than a month (A'_q) in order for the atom to lose its excitation energy by emission of a photon. Furthermore it should be noted that these lines obey the Laporte selection rule, as do electric dipole lines, and are produced in transitions between states with different electron configurations. When $|J - J'| \le 1$ it is completely possible for both types of lines to be excited simultaneously; the magnetic quadrupole and electric octupole are very much weaker and are rather completely masked by the electric dipole lines. It is true that there can be those lines which satisfy the selection rule $2 \le |L - L'| \le 3$, which are not subject to the masking effect indicated above; however, these are extremely weak.[78] From what has been indicated above it follows that these lines can be observed only under conditions which obtain in gaseous nebulae and only in cases in which the atom has metastable levels and other levels between which only third-order multipole transitions can take place. As has been shown in reference 78, cases of this kind are extremely rare.

Obviously everything that has been said with respect to the observation of third-order multipole lines applies for higher order multipoles.

3. FORBIDDEN LINES IN THE SPECTRA OF CELESTIAL BODIES

Forbidden lines have been observed in the spectra of planetary nebulae, the solar corona, new stars, and other celestial bodies. The wave length region available to astronomical observation lies between 3,000 and 10,000 A.* Although forbidden lines were discovered a long time ago (as far back as the 1880's), because of the extreme difficulty of producing them in the laboratory (cf. reference 2) the identification of these lines has been very slow. Even at the present time there are unidentified lines in the spectra of the solar corona and other celestial bodies.

The method of identification is simple in concept but complicated in execution. The wave number of the investigated line is compared with the difference of any two terms of the tentatively proposed atom; these terms are taken from a preliminary careful analysis of the spectrum of this element. If it is found that these differences coincide the investigated line is assigned to a definite element and transition. If there is no complete spectral analysis of the atoms or ions of the element in question the terms are determined by extrapolation or interpolation from atomic spectra of other elements which have the same spectral structure. The validity of the identification, as can be seen from the foregoing, depends to a considerable degree on the accurate determination of the wave lengths of the lines being investigated and the terms in the various atoms or ions. It has been found that almost all forbidden lines in the spectra of celestial bodies are due to second-order spontaneous multipole radiation (i.e., magnetic dipole or electric quadrupole radiation) for neutral or ionized atoms of the various elements.

The most intense forbidden lines are found in the spectra of planetary nebulae. This situation arises because of the physical conditions which prevail. The density of matter in these nebulae is extremely small (of the order of 10^{-18} g/cm^3) and the temperature is relatively low (approximately 10,000°K); thus there is a long mean time between collisions (the order of a minute) and the resulting radiation density is small ($10^{-6} - 10^{-8}$ of the radiation density at the surface of the sun). Consequently neither collisions nor the radiation field are able to keep metastable atoms from decaying spontaneously to lower levels and, as is apparent from Eqs. (1.11) – (1.12), the intensity of the forbidden lines is determined chiefly by the number of such atoms. Since this number is large in these nebulae the intensity of the forbidden lines is appreciable. However, the accumulation of metastable atoms depends on the excitation mechanism.

Astrophysicists have considered three excitation mechanisms for these spectral lines: (1) photoionization with subsequent recombination, (2) fluorescence, and (3) excitation by electron impact. In all three cases the emission of the nebulae derives from the radiant energy of the central star.

It is difficult to believe that the forbidden lines are excited by the first of these mechanisms. In the case of ionized atoms such as O III, N II, etc., this mechanism cannot be reconciled with the observed data. This can be shown by using the example of the well-known nebular lines λ5007 A and λ4959 A from O III. Let us assume that these lines are excited by a recombination mechanism, i.e., the O III atoms are first ionized and then capture free electrons; this capture is followed by cascade allowed transitions to the metastable state, whence the system decays to a lower level via the emission of forbidden lines. However, because the ionization potentials in O III and He II (54.5 ev and 54.2 ev) are almost the same,[80] both of these atoms should absorb energy from the star in the same region of the spectrum and the O III and He II lines should be of approximately of the same intensity. However, in actual fact, the O III line is much stronger than the He II line.

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*If we neglect the radio-frequency region, which has become important in recent years (cf. below).

The accumulation of metastable atoms (resulting in the excitation of forbidden lines) by means of the other two mechanisms was first considered theoretically by Ambartsumyan.[81] He showed that if the excitation of atoms to metastable levels is due exclusively to radiation, in order for metastable atoms to accumulate the following condition must be satisfied:

$$w \ll \frac{A_{21}}{A_{23}}, \tag{3.1}$$

where w is the ratio of the spectral radiation densities of the particular nebula and the central star. A_{21} is the probability for emission of a forbidden line, A_{23} is the probability for a transition to a higher level, the subscript 2 denotes the metastable level and the subscripts 1 and 3 denote levels below and above the metastable level. Because w is so small ($w = 10^{-13}$) the condition in (3.1) is usually satisfied in planetary nebulae (with the exception of those cases in which the lifetime of the metastable state is extremely long; for example, the $2\,^3S$ state in helium). If, however, excitation of the atoms obtains by virtue of collisions with free electrons (the electrons usually arise in photo ionization of atomic hydrogen) the accumulation of metastable states requires that the following inequality be satisfied:

$$a_{21} \ll A_{21}, \tag{3.2}$$

where a_{21} is the probability of a collision of the second kind between the metastable atom and an electron while A_{21} is the probability for a spontaneous transition. This situation also obtains in these nebulae, as is apparent from a consideration of the physical conditions which exist.

The latest calculations[82] for O III show that the efficiency of collision excitation of forbidden lines requires that still another requirement must be satisfied:

$$N_e > 4 \cdot 10^2, \tag{3.3}$$

where N_e is the density of free electrons. This inequality is also satisfied in these nebulae; according to an estimate of the electron density carried out by various methods,[11] N_e is approximately $10^3 - 10^4$.

It should be noted that the inequalities in (3.1) and (3.3) also are satisfied in other celestial bodies whose spectra exhibit forbidden lines in spite of the fact that the physical conditions are somewhat different from those in planetary nebulae.

The prevailing opinion is that collision excitation of forbidden lines is more effective than fluorescence excitation. This situation is indicated by the lower excitation energies (order of 1 — 10 ev) of the majority of metastable levels as compared with the kinetic energies of the free electrons. The electron temperatures in various planetary nebulae generally lie within the limits 6,000 — 10,000°K,[10] corresponding to a mean kinetic energy of 1 — 1.5 ev for the electrons.

According to collision theory the number of electron collisions F_{12} which transfer atoms from the ground state 1 to the excited state 2 is given by the following expression when the electron velocity distribution is Maxwellian:[82]

$$F_{12} = 8.54 \cdot 10^{-6} \frac{N_1 N_e}{T_e^{\frac{1}{2}}} \frac{\Omega(1,2)}{2J_1 + 1} e^{\frac{-\hbar\omega_{21}}{kT_e}}, \tag{3.4}$$

where N_1 is the number of atoms in state 1, T_e and N_e are respectively the temperature and density of the free electrons, $2J_1 + 1$ is the statistical weight for state 1, $\hbar\omega_{21}$ is the excitation energy, $\Omega(1, 2)$ is the so-called collision strength (analogous to the line strength), which is related to the effective cross section $\sigma(1, 2)$ for collisions between an atom and electron with mass m and velocity v by the expression

$$\sigma(1,2) = \frac{1}{2J_1 + 1} \frac{h^2}{4\pi m^2 v^2} \Omega(1,2). \tag{3.5}$$

The number of collisions of the second kind F_{21}, which transfer excited atoms back to the ground state, is computed from the expression

$$F_{21} = 8.54 \cdot 10^{-6} \frac{N_2 N_e}{T_e^{\frac{1}{2}}} \frac{\Omega(1,2)}{2J_2 + 1} \tag{3.6}$$

and represents only several percent of the number of optical transitions, under the conditions which obtain in these nebulae.[10]

If the atom has several metastable levels, through the use of statistical equilibrium it is possible to determine the relative populations from statistical equilibrium considerations; from this knowledge it is then possible to determine the intensity ratios for the corresponding forbidden lines. For example, if there are two metastable levels, 2 and 3, where $E_3 > E_2$ from statistical equilibrium considerations we have:[10]

$$F_{13} + F_{23} = N_3(A_{31} + A_{32}) + F_{31} + F_{32}, \tag{3.7}$$

$$F_{12} + F_{32} + N_3 A_{32} = N_2 A_{21} + F_{21} + F_{23}, \tag{3.8}$$

TABLE III

$q=2$	$\Omega(1,2)$	$\Omega(1,3)$	$q=3$	$\Omega(1,2)$	$\Omega(1,3)$	$q=4$	$\Omega(1,2)$	$\Omega(1,3)$
NII	2.39	0.223	OII	1.44	0.218	FII	**0.95**	**0.057**
OIII	1.73	0.195	FIII	**1.00**	**0.221**	NeIII	0.76	0.077
FIV	**1.21**	**0.172**	NeIV	**0.68**	**0.234**	NaIV	**0.61**	**0.092**
NeV	**0.84**	**0.157**	NaV	**0.43**	**0.255**	MgV	**0.54**	**0.112**

where A_{21}, A_{31}, and A_{32} are the probabilities for the corresponding optical transitions. The left sides of Eqs. (3.7) and (3.8) represent the transitions which increase the populations of levels 3 and 2 respectively while the right sides indicate the transitions which reduce these populations. By substituting the values of F and A in Eqs. (3.7) and (3.8) and eliminating N_1 it is possible to find the ratio N_3/N_2, the relative populations in levels 3 and 2, and the quantity $N_2A_{21}h\omega_{21}/N_3A_{32}h\omega_{32}$, the relative intensity of the 1 — 2 and 2 — 3 lines.

So far the theory indicated above for collision excitation has been verified only for forbidden lines in O III, which are the most intense lines in these nebular spectra. According to Ambartsumyan[80] the ratio of the total intensity of well-known nebular lines N_1 (λ4958.93 A, $^3P_1 - {}^1D_2$, O III) and N_2 (λ5006.86 A, $^3P_2 - {}^1D_2$, O III) to the ratio of the aurora line λ4363.19 A, $^1D_2 - {}^1S_0$, O III is:

$$\frac{I_{N_1}+I_{N_2}}{I_{\lambda\,4363}} = 4.5e^{\frac{33000}{T_e}}, \tag{3.9}$$

where T_e is the electron temperature.

Equation (3.9) is found to be in satisfactory agreement with the observed data. The theory developed for forbidden lines in O III should make it possible in the future to use the absolute intensities to determine the temperature of the central star and the abundance of O III in these nebulae.[12]

Equations (3.4) — (3.9) indicate the important role played in this theory by the accuracy of the determination of the numerical values of the parameters Ω and the transition probability A. Because it is extremely difficult to determine these quantities experimentally, for the most part they have been found purely theoretically, using quantum-mechanical methods. The calculation of the parameter Ω (the effective cross section for electron excitation of forbidden lines in various atoms and ions) has been undertaken by Hebb and Menzel (O III), Aller (O II) and others.[82,83-85] The calculations are based on the quantum theory of collisions.[79,86] The partial wave method is used and an analytic approximation of the wave function is obtained by the Slater method,[82] the Hartree-Fock method, and so on. The most accurate values of σ and Ω for forbidden transitions in various ions (N II, O II, O III, and Ne III) have been obtained by Seaton, who used the Hartree-Fock method.[85] However even in the most accurate calculation errors of ±40% are possible (in the absolute values of σ and Ω). Seaton also has made estimates of the corresponding parameters Ω for the ions F IV, Ne V, F III, Ne IV, Na V, F II, Na IV, and Mg V (with accuracy up to a factor of 2). Table III lists the values of Ω from Seaton for transitions in various ions from the normal level ^3P (level 1) to the metastable ^1D and ^1S levels (levels 2 and 3) in the np^q configuration ($q = 2$ and $q = 4$) and also from the normal ^4S (level 1) to metastable ^2D and ^2P levels (levels 2 and 3) of the np^q ($q = 3$) configuration. The numerical values of Ω obtained by these estimates are shown in heavy type.

If Ω is known, the effective cross sections σ are computed from Eq. (3.5). In those cases in which the kinetic energy of the colliding electron is comparable with the excitation energy of the atom or ion these cross sections turn out to be sizeable — approximately 10 — 100 times greater than the gas-kinetic cross sections.

We have already indicated the great value of a knowledge of both the relative and absolute probabilities for forbidden transitions for lines which are of interest in astrophysics. Probability calculations have been given chiefly by Shortley, Aller, Baker, and Menzel.[8] The accuracy of the results of these calculations depends to a large extent on the accuracy with which the wave functions for complex atoms are known, i.e., to the extent to which it is possible to compute various types of atomic interactions (spin-spin, spin-orbit, configuration). As an example, Table IV lists the results of the most accurate (at the present time) calculation of the transitions for $1s^2\,2s^2\,2p^2$ configurations for the iso-electronic sequence C I, N II, O III, and F IV.[87] The calculation has been carried out with refined wave functions which take into account the correlation between electrons; these functions have been found by a variational method which is a development of the method described in reference 88. As is apparent from

TABLE IV

	CI	NII	OIII	FIV
$A_m\ (^1D_2, ^3P_2)$	$0.0^{3}18$	0.0027	0.0195	0.0937
$A_q\ (^1D_2, ^3P_2)$	$0.0^{5}15$	$0.0^{4}114$	$0.0^{4}51$	$0.0^{3}161$
$A_m\ (^1D_2, ^3P_1)$	$0.0^{4}60$	$0.0^{2}92$	0.0066	0.034
$A_q\ (^1D_2, ^3P_1)$	$0.0^{6}21$	$0.0^{5}176$	$0.0^{5}78$	$0.0^{4}26$
$A_q\ (^1D_2, ^3P_0)$	$0.0^{7}32$	$0.0^{6}30$	$0.0^{5}14$	$0.0^{5}47$
$A_q\ (^1S_0, ^1D_2)$	0.62	1.350	2.00	2.60
$A_m\ (^1S_0, ^3P_1)$	0.002	0.030	0.214	0.94
$A_q\ (^1S_0, ^3P_2)$	$0.0^{4}23$	$0.0^{3}16$	$0.0^{3}89$	0.0028

Table IV, in the probabilities for mixed multiple transitions $^1D_2 \rightarrow {}^3P_1$ and $^1D_2 \rightarrow {}^3P_2$ the magnetic dipole part is considerably greater than the electric-quadrupole part.

By taking account of the various interactions between electrons it is possible to get better values for the probabilities for forbidden transitions and for atomic energy levels;[89-90] these more refined values provide improved means of identification for lines. In particular, it has been shown in references 91 and 92 that taking account of configuration interactions makes it possible to avoid certain uncertainties in the identification of the Ca XV lines in the corona. In 1954 Layzer[91] established the fact that in highly ionized atoms the strongest interaction takes place between configurations which have the same sequence of principle quantum numbers and the same parity. In work carried out by Garstang[92] a more general expression for the strength of lines which correspond to all possible forbidden transitions between levels assigned to the $1s^2\ 2s^2\ 2p^2$ configuration was found taking account of the perturbing effect of the $1s^2\ 2p^4$ configuration. On the basis of an analysis of these expressions and the appropriate expressions from reference 8, Garstang has shown that if in these formulas one takes the line strengths derived by neglecting the configuration interaction, using the experimental values rather than the theoretical values for the terms, the largest part of the configuration interaction is still taken into account. This result is illustrated by a detailed calculation for Ca XV. In reference 92 formulas have also been obtained for the transition probability between terms of the $1s^2\ 2s^2\ 2p^4$ configuration which take account of the interaction with $1s^2\ 2p^6$ configuration. This interaction makes possible the following two-electron configuration transitions:

$$2p^6\,{}^1S_0 - 2s^2 2p^4\,{}^3P_1,\ \ 2p^6\,{}^1S_0 - 2s^2 2p^4\,{}^3P_2$$
$$\text{and } 2p^6\,{}^1S_0 - 2s^2 2p^4\,{}^1D_2{}^* .$$

We now present a brief analysis of individual forbidden lines, starting from the nature of the metastable states of atoms with different electronic systems. Knowing the exact values of the differences between metastable states and the ground levels for different atoms it is possible to predict all possible forbidden lines.

One-electron systems (H I, He II, Li III, etc.). These systems are characterized by one metastable state — $2s\ ^2S_{1/2}$.* For a long time, in accordance with the Dirac theory, it was assumed that the $2s\ ^2S_{1/2}$ level coincides with the $2p\ ^2P_{1/2}$ level. In 1947, it was shown by means of the magnetic-resonance method that these levels are shifted with respect to each other by 1062 ± 5 Mcs.[94] This shift was interpreted in terms of quantum electrodynamics.[95] Calculations showed[96] that the $2s\ ^2S_{1/2} \rightarrow 1s\ ^2S_{1/2}$ transition could be accompanied by the emission of one or two photons, with the two-photon transition being almost 10^8 times more probable than the single-photon transition. The lifetime of the atom in the metastable $2s\ ^2S_{1/2}$ state is 0.14 seconds. Neither one-photon nor two-photon transitions have been observed in the spectra of celestial bodies.

Two-electron systems (He I, Li II, Be III, etc.). The metastable states are the $1s\ 2s\ ^1S$ and $1s\ 2s\ ^3S$ states. The forbidden transitions are: $1s\ 2s\ ^1S \rightarrow 1s^2\ ^1S$ and $1s\ 2s\ ^3S \rightarrow 1s^2\ ^1S$. Because strictly normal coupling obtains in the helium atom the spontaneous transition $1s\ 2s\ ^3S \rightarrow 1s^2\ ^1S$ is not very likely in either the one-photon or two-photon case. For this reason transitions from the ^3S level to the ^1S level take place chiefly by means of collisions. However the $1s\ 2s\ ^1S \rightarrow 1s^2\ ^1S$ transitions are more probable if accompanied by the emission of two photons. It is believed that a two-photon transition of this kind takes place in nebulae.[13]

Three-electron systems (Li I, Be II, B III, etc.) do not have metastable levels.

Four-electron systems (Be I, B II, C III, N IV, etc.). The basic metastable states are $2s\ 2p\ ^3P_0$

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*Two-electron transitions are discussed in reference 93.

*Metastable states which arise as a consequence of level splitting into hyperfine sub-levels are not taken into account. Transitions between the hyperfine sub-levels are considered above.

TABLE V

		$^3P_1\ ^1D_2$	$^3P_2\ ^1D_2$	$^1D_2\ -^1S_0$	$^3P_1\ -^1S_0$	$^1D_2$	$^1S_0$	First non-metastable
CI	Calculated	9822.9	9849.5±0.3	8727.4±0.1	4621.4±0.1	1.3	2.7	9
NII	Calculated	6548.4	6583.9±0.1	5755.0±0.1	3063.0±0.1	1.9	4.0	20
	Observed	6548.06	6583.37	5754.57				
OIII	Calculated	4959.5	5007.6±0.2	4363.2±0.1				
	Observed	4958.93±0.03	5006.86±0.02	4363.19±0.01		2.5	5.3	36
FIV	Calculated	3996.3	4059.3±0.3	3532.2±0.3		3.1	6.6	58
	Observed	3997.40±0.04	4060.23±0.03					
NeV	Calculated	3344	3424	2972		3.8		86
	Observed	3345.84±0.01	3425.87±0.03					

and 2s 2p $^3P_2$. As an approximation it may also be assumed that the 2s 2p $^3P_1$ level is metastable, especially in the light elements, since the probability for the 2s 2p $^3P_1 \rightarrow 2s^2\ ^1S$ transition is very small in these elements because of the almost pure normal coupling; the order of magnitude of this transition is the same as that for ordinary forbidden transitions. In Be I the $^1S \rightarrow {}^3P$ line lies in a region available to observation and corresponds to wavelengths lying in the limits λ 4548 ± 5 A.[1]

Five-electron systems (B I, C II, N III, O IV, etc.). There are two metastable states: $2s^2\ 2p\ ^2P_{1/2}$ and $2s\ 2p^2\ ^4P$. The metastability of the latter is explained by the small probability for intercombination transitions. The $2s\ 2p^2\ ^4P \rightarrow 2s^2\ 2p\ ^2P_{1/2}$ transitions have been observed astronomically only for B I, at wavelengths of 3300 ± 300 A). For the atoms indicated above the $2s^2\ 2p\ ^2P_{3/2} \rightarrow 2s^2\ 2p\ ^2P_{1/2}$ transition lies in the far infrared; however as the atomic number of the element is increased it slowly shifts towards wavelengths which are amenable to astronomical observation. It has been observed in the spectrum of the solar corona only for argon XIV (λ 4412 A).[97]*

Six-electron systems (C I, N II, O III, F IV, Ne V, etc.). The ground state is the $^3P_0$ state; the metastable states are the $^3P_1$, $^3P_2$, $^1D_2$ and $^1S_0$ states. All these belong to the $2s^2\ 2p^2$ configuration. To a considerable degree it may also be assumed that the $2s\ 2p^3\ ^5S$ level is metastable since the intercombination transitions $^3P - {}^5S$, $^1D - {}^5S$ and $^1S - {}^5S$ are characterized by low probabilities. The $^3P - {}^5S$ lines may be significant under atmospheric conditions and are found in the observed region of the spectrum only for C I. With regard to the transitions between the $^3P_0$, $^3P_1$ and $^3P_2$ terms we may note that two of these have been observed in the highly ionized atom Ca XV. These are the corona lines λ 5694.42 A, $^3P_1 \rightarrow {}^3P_0$ and λ 5446 A, $^3P_2 \rightarrow {}^3P_1$.*

The calculations indicate that the probability of the forbidden transition $^1S \rightarrow {}^3P_0$ is zero while the probabilities for the transitions $^1D_2 \rightarrow {}^3P_0$ and $^1S \rightarrow {}^3P_2$ are small (1000 times smaller than the probability for the $^1S \rightarrow {}^3P_1$ transition).[1] Consequently the main forbidden lines are: $^3P_1 - {}^1D_2$, $^3P_2 - {}^1D_2$, $^1D_2 - {}^1S_0$ and $^3P_1 - {}^1S_0$. There is some historical background with regard to the terminology of these forbidden lines. The first two lines are called nebular lines since lines of this type were most intense in nebulae, while the third line is called an aurora line since the most intense line in the spectra of the polar aurora and night sky arises from the $^1S_0 \rightarrow {}^1D_2$ transition. Lines arising from $S^1 \rightarrow {}^3P$ transitions are called trans-aurora lines.

In Table V are given the results of the identification of these lines for various elements according to the data of Bowen.[1,99] Columns 4 — 6 list data on the excitation potentials of the metastable levels ^1D and ^1S as compared with the lowest potentials for the non-metastable levels. All of the former are considerably below the latter.

Only one of the forbidden lines in C I has been observed: λ 4621.5 A, $^3P_1 - {}^1S$.[99] Lines of the other elements have been found in spectra of various celestial bodies and the data of the observations are in excellent agreement with the calculated results. It may be noted that the lines in Ne V were predicted by extrapolation.

The lines investigated in greatest detail are those of O III (cf. Fig. 2). Only three of these (given in Table V) are available to astronomical observation. Bowen first proposed that these were due to electron impact (these lines were used to verify the theory of collision excitation).

The relative intensities of the $^3P - {}^1D_2$ and

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*It had been assumed earlier that the 4359-A corona line was due to the $^2P_{3/2} \rightarrow {}^2P_{1/2}$ transition. The correction was made by Edlen in 1954.[97]

*This identification now seems to be open to question[97-98] (cf. below).

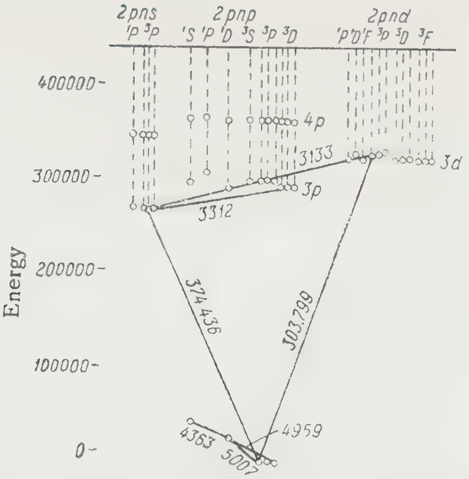

FIG. 2. Diagram for the terms in the O III ion. The forbidden transitions are indicated in the lower part of the diagram.

$^1D_2 - {}^1S_0$ lines are different in the spectra of different celestial bodies. Whereas in nebulae the nebular lines are more intense than the aurora lines, in the polar aurora (and also in early stages of new stars) the opposite hold true. This is explained by the difference in the prevailing physical conditions. In the nebulae the intensity of the lines is determined chiefly by the number of metastable states and not by the transition probabilities, because almost all of the metastable atoms make spontaneous transitions to lower states. Since the $^1D_2$ state has a lower excitation potential and a higher statistical weight than the $^1S_0$ state, the number of atoms in the $^1D_2$ state is greater than in the $^1S_0$; consequently the $3P - {}^1D_2$ line is more intense than the $^1D_2 - {}^1S_0$ line.

In the polar aurora, where the density of matter and radiation is not as small as in nebulae, the intensity of the lines depends to a considerable degree on transition probabilities. Since the probability for the $^1S_0 \rightarrow {}^1D_2$ transition is much greater than that of the $^1D_2 \rightarrow 3P$ transition (cf. Table IV),* the aurora line is more intense than the nebular line. These conclusions may be generalized to all forbidden lines of the $P - D$ and $D - S$ type for the p^2 and p^4 configurations and lines of the $S - D$ and $D - P$ type for the p^3 configuration since the metastable D states are characterized by much lower excitation potentials and longer lifetimes than the S and P states.

Seven-electron systems (NI, OII, FIII, NeIV, NaV, MgVI, etc.). The metastable states, which are chiefly $s^2 p^3$ configuration are the $^2D_{5/2}$, $^2D_{3/2}$, $^2P_{1/2}$, and $^2P_{3/2}$. The stable level is the ^4S level. The observed forbidden multiplets $^4S - {}^2D$ and $^2D - {}^2P$ are given in Table VI, taken from Bowen.[1,99]

The first of the multiplets in OII (λ 3726.16 A, λ 3728.91 A) are the brightest lines in the spectra of the nebulae and interstellar space. The lifetime of the ^2D state is approximately 5 hours and is almost 15,000 times greater than that of the ^2P state, while the excitation potential is 1½ times smaller. Thus, the features indicated above with respect to the ^2D and ^2P levels are verified in OII. The NI lines $^4S - {}^2D$ (λ 5199 A) and $^4S - {}^2P$ (λ 3466 A) have been observed in the spectra of polar aurora; however the observation of the NI lines $^2D - {}^2P$ (λ 10400 A) in the infrared region of the radiation of the night sky are open to question[100] (cf. below).

TABLE VI

		$^4S - {}^2D_{3/2}$	$^4S - D_{5/2}$	$^2D_{5/2} - {}^2P_{3/2}$	$^2D_{5/2} - {}^2P_{1/2}$	$^2D_{3/2} - {}^2P_{3/2}$	$^2D_{3/2} - {}^2P_{1/2}$
NI	Calculated	5198.5	5200.7	10397.8		10407.3	
	Observed	5197.94	5200.41			7318.6	7319.4
OII	Calculated	3726.2	3729.1±1				
	Observed	3726.04 ±0.02	3728.80 ±0.03	7319.92±0.08		7330.19±0.06	
NeIV	Observed			4714.25 ±0.03	4715.61 ±0.06	4724.15 ±0.02	4725.62 ±0.02

Eight-electron systems (OI, FII, NeIII, NaIV, MgV, AlVI, etc.). A feature of these systems is the fact that all the metastable states of the $2p^4$ and $2p^2$ configurations are ^3P, ^1D, and ^1S. The first configuration differs from the second only in the inversion of the 3P term (the stable state is the $^3P_2$ state). The main transitions are $^1D_2 \rightarrow {}^3P_1$, $^1D_2 \rightarrow {}^3P_2$, $^1S_0 \rightarrow {}^1D_2$ and $^1S_0 \rightarrow {}^3P_1$ (the transitions $^1D_2 \rightarrow {}^3P_0$, $^1S_0 \rightarrow {}^3P_0$ and $^1S_0 \rightarrow {}^3P_2$ are characterized by much smaller probabilities.

The following lines of OI are observed in the nebulae: λ 6363.82 ± 0.02 A, $^3P_1 - {}^1D_2$; λ 6300.31 ± 0.004 A, $^3P_2 - {}^1D_2$; and the following lines from NeIII: λ 3967.47 ± 0.002 A, $^3P_1 - {}^1D_2$ and λ 3868.76 ± 0.02 A, $^3P_2 - {}^1D_2$.[99] The solar corona exhibits the following line from Ca XIII: λ 4086.3 A, $^3P_2 - {}^3P_1$.[98] Some of the forbidden lines from A XI have been observed in the spectra of new stars.[101]

Forbidden lines of OI corresponding to the

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*This is true both for O III and O I.

TABLE VII

		$^3P_2-{}^1D_2$	$^3P_2-{}^1D_2$	$^1D_2-{}^1S_0$	$^3P_1-{}^1S_0$
SIII	Observed			6312.06±0.05	3721.8
ClIV	,,	7530.54±0.01	8045.63±0.05	5323.29±0.10	
AV	,,	6435.10±0.05	7005.67±0.09	4625.54±0.04	

TABLE VIII

		$^4S-{}^2D_{5/2}$	$^4S-{}^2D_{3/2}$	$^4S-{}^2P_{3/2}$	$^4S-{}^2P_{1/2}$	$^2D_{5/2}-{}^2P_{3/2}$	$^2D_{3/2}-{}^2P_{3/2}$	$^2D_{3/2}-{}^2P_{1/2}$
SII	Observed	6716.42	6730.78	4068.6 ±0.03	4076.35 ±0.03			
ClIII	,,	5577.66 ±0.08	5537.60 ±0.24					
AIV	,,	4711.33 ±0.02	4740.2 ±0.02			7237.26 ±0.22	7170.62 ±0.08	7262.76 ±0.24
KV	,,	4122.63 ±0.05	4163.3 ±0.05					

TABLE IX

		$^3P_2-{}^1D_2$	$^3P_1-{}^1D_2$	$^1D_2-{}^1S_0$	$^3P_1-{}^1S_0$
SI	Observed			7726.5	6526.9
AIII	,,	7135.80±0.05	7751.06±0.15	5191.82±0.03	
KIV	,,	6101.83±0.08		4510.9±0.08	
CaV	,,	5309.18±0.04	6086.92±0.11		

transitions $^1S_0 \rightarrow {}^1D_2$ and $^1D_2 \rightarrow {}^3P$ have been observed in the spectra of the polar aurora and the radiation of the night sky (these are considered in greater detail below).

Nine-electron systems (F I, Ne II, Na III, Mg IV, etc.). Only two forbidden lines have been observed (in the solar corona): λ 3328 A, Ca XII and λ 5536 A, Ar X;[98] both of these correspond to the transition $^2P_{1/2} \rightarrow {}^2P_{3/2}$. The other transitions between metastable states do not lie in the region in which astronomical observations are possible.

Ten-electron systems (Ne I, Na II, Mg III, Al IV, etc.). All the forbidden transitions lie in the far ultraviolet and have not been observed.

Eleven-electron systems (Na I, Mg II, Al III, etc.) have no metastable states.

Twelve-electron systems (Mg I, Al II, Si III, etc.). The metastable states are: 3s 3p $^3P_0$ and 3s 3p $^3P_2$; the stable state is $3s^2\ {}^1S_0$. Forbidden transitions have not been observed for these systems.

Thirteen-electron systems (Al I, Si II, P III, S IV, etc.). Only two forbidden lines have been observed: λ 5302.9 A from Fe XIV and λ 3001 A from Ni XVI, corresponding to the transition $^2P_{3/2} \rightarrow {}^2P_{1/2}$. The first of these is the brightest line in the solar corona. It has also been observed in the spectra of new stars.[101]

Fourteen-electron systems (Si I, P II, S III, Cl IV, A V, K VI, etc.). The metastable states which are of astrophysical interest are ^1S, ^1D, and ^3P. All of these are in the $3s^2\ 3p^2$ configuration. The forbidden lines observed in the spectra of nebulae are given in Table VII.[99] In addition, identification has been made of three corona lines which are assigned to transitions between components of the ^3P term: namely, λ 6701.8 A, Ni XV ($^3P_2 \rightarrow {}^3P_1$), λ 10746.8 A F XIII ($^3P_1 \rightarrow {}^3P_0$) and λ 10798 A Fe XIII ($^3P_2 \rightarrow {}^3P_1$), and one line from Fe XIII λ 3388 A assigned to the transition $^1D_2 \rightarrow {}^3P_2$.[98]

Fifteen-electron systems (P I, S II, Cl III, A IV, K V, Ca VI, etc.). The basic configuration is $3s^2\ 3p^3$. The stable level is ^4S; the metastable levels are ^2D and ^2P. In Table VIII are given the results of observations in nebulae of the forbidden lines $^4S - {}^2P$, $^4S - {}^2D$ and $^2D - {}^2P$.[99] The $^4S - {}^2P$ lines in Cl III are masked by one of the lines from Ne III. Transitions between components of the metastable levels have not been observed.

Sixteen-electron systems (S I, Cl II, A III, K IV, Ca V, etc.). The metastable states are assigned to the ground configurations $3s^2\ 3p^4$ are ^3P, ^1D, and ^1S. The results of observations of forbidden lines assigned to this system are given in Table IX.[99] These lines are observed chiefly in the spectra of nebulae. Forbidden lines have not been observed in Cl II. Both components of $^3P_0 - {}^1D$ in Ca V have also been observed in the spectra of new stars, where λ 6087 A of Ca V is one of the brightest lines in the spectra. The following corona lines are assigned to sixteen-electron systems: λ 7891.9 A Fe XI, $^3P_1 \rightarrow {}^3P_2$ and λ 5116 A Ni XIII, $^3P_1 \rightarrow {}^3P_2$.[98]

17, 18 and 19-electron systems, with a few exceptions, have no metastable states characterized by transitions which lie in regions available to astronomical observations. The spectrum of the solar corona exhibits two forbidden lines: λ 6374.5 A Fe X, $^2P_{1/2} \rightarrow {}^2P_{3/2}$ and λ 4232.4 A Ni XII, $^2P_{3/2} \rightarrow {}^2P_{1/2}$; the first of these lines is assigned to the seventeen-electron system while the second is assigned to a nineteen-electron system. The line from Fe X has also been observed in the spectra of new stars.[102]

The terms of other electron systems have not been classified with sufficient completeness and the identification of the forbidden lines for these configurations in the spectra of celestial bodies is far from complete. The most reliable investigations have been of lines of various iron ions — Fe II, Fe III, Fe V, Fe VI, Fe VII, and Fe VIII[103] (the Fe lines in the corona have been already mentioned above). Twenty-six lines of this kind are observed in nebulae. Also observed are eight forbidden lines from Mn V and Mn VI and ions of titanium, chromium and other elements in the first long period of the periodic table (K, Ca, Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, and Zn). The absence of lines for heavier elements is apparently a result of the fact that these elements do not occur in large amounts in the nebulae.

Usually the atoms that are more highly ionized emit more intense forbidden lines; this can be explained as follows. As is well known, the difference between the low excitation potential of metastable levels and the relatively high excitation potential of the first non-metastable level increases with the degree of ionization. For example, whereas the excitation potential of the first non-metastable level for Mn II and Fe II is 5 volts, in Mn VI and Fe VI these potentials are 65 — 70 volts; at the same time the excitation potential of the metastable levels remains approximately the same in both cases: 1.9 — 2.5 volts.[1] Consequently transitions of a highly ionized atom from a metastable level to the first non-metastable level and back occur in the far ultraviolet and the forbidden lines of highly ionized atoms may arise even when the total intensity of the radiation is large if the spectral density of the radiation in the far ultraviolet is small. An example of this kind is the presence or absence of forbidden lines of highly ionized atoms in slightly ionized atoms in the solar corona and in the spectra of other celestial bodies, under suitable physical conditions.

The majority of the forbidden lines are due to mixed radiation, i.e., electric quadrupole and magnetic dipole radiation. Only a relatively few of these lines are pure electric quadrupole, intercombination or pure magnetic dipole lines. For example in the p^2 and p^4 configurations the pure electric quadrupole transitions are $^1S - {}^1D$, $^3P_2 - {}^3P_0$ and $^1D - {}^3P_0$ while the pure magnetic dipole transitions are $^1S - {}^3P_1$ and $^3P_1 - {}^3P_0$. In the case of the p^3 configuration there is only one pure electric quadrupole transition $^2D_{5/2} - {}^2P_{1/2}$.

The two radio lines at 21 cm and 91.5 cm discovered recently (in 1951 and 1955 respectively) in spectra from interstellar gas are pure magnetic dipole lines. These lines arise in transitions between sublevels of the hyperfine structure of the ground states of atomic hydrogen (21 cm) and deuterium (91.5 cm). The lifetime of the metastable sublevel of the hyperfine structure of atomic hydrogen is extremely long — on the order of 10^7 years. In deuterium this lifetime is a hundred times larger. The possibility of a spontaneous radiated transition to the normal level is due to the extremely rarified nature of interstellar gas.

Investigations of radio waves from hydrogen are of extreme importance in astronomy. At the present time, on the basis of observations of this radiation it has been possible to draw some very important conclusions as to the density and temperature of interstellar gas, the structure and motion of the galaxy, the hydrogen mass which surrounds the Magellanic clouds and so on.[104]

We shall consider in somewhat greater detail the forbidden lines observed in the spectra of the polar aurora, the radiation of the night sky, and the solar corona, since an investigation of these lines is of great importance in the study of the physical processes which are involved.

The green line from O I ($^1D_2 - {}^1S_0$), the wavelength of which is 5577.3445 ± 0.0027 A according to the data of Wegard and Harang (1937) along with the so-called "red triplet" $^3P - {}^1D$ (Fig. 3)

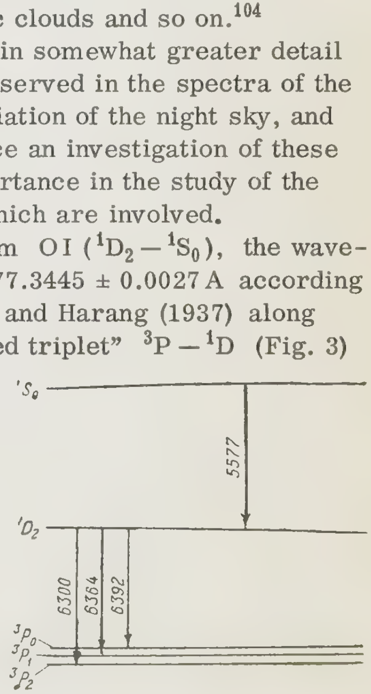

FIG. 3. Forbidden transitions in atomic oxygen.

TABLE X

OI	Transition	$^1S_0 \to {}^1D_2$	$^1S_0 \to {}^3P$	$^1D_2 \to {}^3P_0$	$^1D_2 \to {}^3P_1$	$^1D_2 \to {}^3P_2$
	Probability	2.2	0.0908	0.0000022	0.0026	0.0078

are the brightest lines in the spectra of the polar aurora and the radiation of the night sky. The identification of the forbidden lines from OI has been carried out by laboratory methods,[106] as mentioned in the introduction. The most precise values for the transition probabilities for these lines have been obtained by Pasternak[16] (cf. Table X).

Starting from the data in Table X it is easy to find that the mean lifetime for the $^1S_0$ level is 0.43 sec; the mean lifetime for the $^1D_2$ level is 95 sec. This great difference in lifetimes for the $^1D_2$ and $^1S_0$ levels is responsible for the change in color of the polar aurora as a function of height. In the upper part of the polar aurora the pressure is lower than in the bottom part. Hence the number of collisions which metastable atoms experience there [OI ($^1D_2$)] is much smaller and almost all of these atoms can radiate; as a consequence the red triplet is found to be brightest. With a reduction in height the number of collisions increases and the total number of metastable atoms [OI($^1D_2$)] does not radiate, and some make transitions to the metastable $^1S_0$ state. However, as a consequence of the shorter lifetime atoms in the $^1S_0$ state these radiate to a greater degree than the OI ($^1D_2$) atoms. As a result, the intensity of the red lines falls off with height while that of the green line increases. Obviously, to obtain a complete quantitative picture of the behavior of these lines the excitation mechanism must be known.

Three different mechanisms have been proposed for the excitation of forbidden lines in the spectra of the polar aurora and the radiation of the night sky: collision effects, photo-chemical effects, and fluorescence.

The first of these is somewhat similar to the mechanism for excitation of forbidden transitions in nebulae. The basic role is played by electrons with low energies (approximately 7 — 10 ev). In collisions between such electrons and hydrogen molecules the latter dissociate into two atoms (dissociation energy 5.09 ev); one of these remains in the ground state while the second is excited to the metastable $^1D_2$ state (excitation potential 2 volts) or to the $^1S_0$ state (excitation potential 4.2 volts). This is followed by spontaneous transitions $^1S_0 \to {}^1D_2$ and $^1D_2 \to {}^3P$ of the OI atoms with the emission of the green and red lines. These optical transitions are possible because of the low density of the upper layers of the atmosphere.

The next question concerns the origin of these low-energy electrons. According to Dauvillier[107-108] these electrons are a result of ionization of atoms by fast particles emitted by the sun. In the composition of the corpuscular radiation of the sun* there are light ions and a large number of electrons, some with energies ranging up to 10^6 ev; ionization of the atoms and molecules of the atmosphere takes place chiefly as a result of collisions with these electrons.

Even if the collision excitation mechanism is able to explain the production of forbidden radiation in the polar aurora it is certainly not capable of explaining the radiation of the night sky. In contrast with the polar aurora, the radiation of the night sky exhibits a very weak latitude effect, if any.[113,100] Dauvillier, in order to establish the validity of electron excitation for forbidden lines assumes in this case that the sun, in addition to emitting powerful periodic fluxes, also emits a constant low intensity flux of charged particles.[108] However the marked differences in the spectra of the polar aurora and the radiation of the night sky, and other effects, in particular the fact that the green line in the spectrum of the night sky exhibits an intensity maximum soon after midnight, indicates some other mechanism for the excitation of the forbidden radiation, probably a photo-chemical mechanism.

According to Chapman the photo-chemical excitation mechanism may be described as follows.[100] During the day oxygen molecules dissociate into atoms as a result of the absorption of ultraviolet radiation from the sun. During the night these oxygen atoms recombine when triple collisions take place; the result is the excitation of one of the atoms to the metastable ${}^1S_0$ level in accordance with the relation

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*At the present time the corpuscular theory for the production of the polar aurora is generally accepted since no other theory can explain the relation between the polar aurora and the periodic processes that take place on the sun (for example the relation to sun spots), the dependence of these effects on the magnetic field of the earth (the so-called latitude effect), or other characteristic features (the shape and time variation). Although this theory, in the form in which it has been formulated by Birkelaud, Störmer, and Boguslavskiĭ[109-111, 105] has a number of shortcomings, it would seem to be the only available means of explaining these effects. Obviously some modifications of the theory will be required. An important attempt along these lines is that suggested by Chapman and Ferraro.[112]

$$O + O + O \rightarrow O_2 + O\,({}^1S_0). \qquad (3.10)$$

This is followed by the transitions

$$OI - {}^1S_0 \rightarrow {}^1D_2 \text{ and } {}^1D_2 \rightarrow {}^3P \qquad (3.11)$$

which are accompanied by the emission of the green line and the red triplet.

According to a calculation carried out by Elvey,[115] the recombination process described by (3.10) takes place with greatest intensity at a height of 160 km; on the other hand the total height of the atmospheric layer in which radiation of the forbidden lines from O I is observed is approximately 50 km (from 120 km to 170 km).

The intensity of the green line does not reach a maximum immediately at twilight but considerably later, around midnight. Khvostikov explains this effect by the fact that the height at which maximum photo-ionization of O_2 molescules takes place does not coincide with the height at which maximum recombination, described in Eq. (3.10), occurs. The first height level is associated with the height at which the highest concentration of oxygen atoms is found and is much higher than the second; the delay is due to the time required for the O I atoms to descend by diffusion to the height at which the recombination is a maximum.

However, there are some difficulties in the photo-chemical excitation hypothesis. The dissociation energy of the oxygen molecule, which is liberated in recombination, is 5.1 ev while the excitation energy of the ${}^1S_0$ level is 4.2 ev. Hence part of the energy goes into relative kinetic energy of the particles which are produced and should lead to Doppler broadening of the emitted forbidden lines. This broadening has not yet been observed. The chief shortcoming of the photo-chemical theory is the fact that it cannot explain the polarization of the forbidden lines from O I; this polarization effect has been established by Khvostikov.[116]

It is possible that both excitation mechanisms (collision and photo-chemical) operate simultaneously, with the first predominating in the polar aurora and the second in the radiation of the night sky.

A third mechanism—fluorescence—can also explain certain of the features of the radiation of the twilight sky.

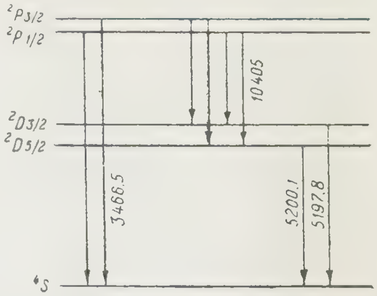

FIG. 4. Forbidden transitions in atomic nitrogen.

The increase in intensity of the red line from O I (${}^3P_2 - {}^1D_2$, λ 6300A) at twilight with the absence of a twilight effect in the green line from O I (${}^1D_2 - {}^1S_0$, λ 5577A) indicates that the excitation mechanisms are different for the red and green lines. On the basis of a careful analysis of the excitation conditions of these lines, taking account of the dependence of the density of oxygen atoms on height on the diffusion of O I atoms and the fact that the brightness of the red line increases strongly with the appearance of the first slash even at the uppermost layers of the atmosphere, Khvostikov[100] has proposed a resonance fluorescence mechanism for the production of the λ 6300A line at twilight. The O I atoms (${}^3P_2$) absorb radiation from that part of the continuous spectrum of the sun which corresponds to wavelengths near λ 6300A and make transition to the ${}^1D_2$ level; then, in returning, they radiate the red line.* The high intensity of the O I line at λ 6300A when the sun has set below the horizon is due to the abnormally high selective reflection of the sun's rays at 6300A in the upper layers of the atmosphere, which contain atomic oxygen.[100]

Great interest attaches to the presence of forbidden lines of atomic nitrogen in the radiation from the high levels of the atmosphere. We have already noted that only two of the forbidden lines of N I (cf. Fig. 4), namely ${}^4S - {}^2P$ λ 3466A and ${}^4S - {}^2D$ λ 5199A have been observed in the spectrum of the polar aurora; these are found to be much weaker than the forbidden lines of O I. Searches for forbidden lines of N I in the radiation spectrum of the night sky have not as yet been successful. In order to facilitate identification a good deal of preliminary work has been carried out on these lines. The transition probabilities,[16] precise wavelengths,[118] and excitation potentials of the metastable level have been determined; also, a self-consistent field analysis has been used to compute the relative intensity of the components of the doublet ${}^4S - {}^2D : I({}^4S_{3/2} - {}^2D_{5/2}) / I({}^4S_{3/2} - {}^2D_{3/2}) = 0.87.$[119]

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*The absorption of solar radiation also occurs in the polar aurora, which is illuminated by the sun, but leads to a different effect—a considerable weakening of the green line. It is probable that this effect is due to absorption of ultraviolet photons by metastable atoms [O I (${}^1S_0$)].[117]

The analysis of the excitation of forbidden lines in N I is as yet not complete. It is assumed that these lines arise chiefly as a result of inelastic collisions between nitrogen molecules and atoms and metastable oxygen atoms. However, an estimate of the intensity ratio for $^4S-^2D$ and $^4S-^2P$ lines made on this basis does not agree with the observed data.[100]

As we have seen, the forbidden lines observed in the spectrum of the solar corona may be assigned to a few elements, the atoms of which are highly ionized (Fe X — XI, XIII — XIV; Ca XII — XIII, XV; Ni XIII, XV — XVI; A X, XIV). The absence of forbidden lines from other elements may be explained either by the small quantities of these elements in the solar corona, the small concentration of metastable ions, or the fact that the lines from the metastable ions lies in a region of the spectrum which cannot be observed.

Almost all of the corona lines have been identified by Edlen (the first two lines were identified in 1939). Four lines remain unidentified: λ 3454, λ 4567, λ 3801 and λ 4359 A. The identification was attended by great difficulties. Only four lines were identified on the basis of the location of the corresponding term differences as obtained from resolved lines in the spectra of Fe X — XI, Ca XII — XIII.[120-121] The remaining lines were identified by a complicated extrapolation procedure in which intermediate electron coupling and various features of atomic spectra were taken into account. For example, in finding the ^2P terms assigned to the $3s^2\,3p$ and $3s^2\,3p^5$ configurations, the so-called "proper" doublet rule was used: $\nu(^2P_{3/2}-{}^2P_{1/2}) \sim (Z-\sigma)^4$, where Z is the atomic number and σ is a constant which is essentially independent of Z.[98]

When lines are associated with a given ion the identification of the corona lines is verified by a comparison of the relative intensities obtained from the observed data with those obtained theoretically. The theoretical calculation is based on the assumption that the corona lines are excited by electron impact* and is carried out by the method described below. The comparison of the theoretical and observed values of the relative intensities of these lines has been found satisfactory.[98] The only discrepancy occurs for lines from Ca XV. Edlen has identified the λ 5694 A line with the $^3P_1 \rightarrow {}^3P_0$ transition in Ca XV while Waldmeyer assigns the line λ′5446 A to the $^3P_2 \rightarrow {}^3P_1$ transition. The observed intensity ratio for these lines (the ratio of the first to second) is approximately 6. The theoretical calculations carried out by Shklovskiĭ[98] give a value lying in the range 1.05 — 1.58 (for all possible values of the concentration of free electrons and electron temperature in the corona). If we assume that the λ 5694 A line corresponds to the $^3P_2 \rightarrow {}^3P_1$ transition and the λ 5446 A line corresponds to the $^3P_1 \rightarrow {}^3P_0$ transition, according to Shklovskiĭ the results of the theory (0.38 — 0.97)[98] show a still greater contradiction with the observations. In this connection Shklovskiĭ has maintained that the identification of the λ 5694 A and λ 5446 A lines is incorrect. Later, however, it was found according to the data of reference 122 that the λ 5445 A line was considerably weakened by certain Fraunhoffer lines. If account is taken of this weakening, the intensity ratio for the λ 5694 A and λ 5446 A lines becomes 1.6, in good agreement with the first of the Shklovskiĭ results indicated above.

The rather sizeable discrepancy between the differences of the extrapolated terms $^3P_1-{}^3P_0$ and $^3P_2-{}^3P_1$ and the wave numbers of the corona lines λ 5694 A and λ 5446 A which was first found was later removed by taking account of the configuration interaction in the extrapolation procedure, as we have already indicated.[91,92,97]

The corona lines are characterized by sizeable widths (approximately 1 A); this value far exceeds the natural width for spontaneous forbidden lines and indicates the existence of high velocities for the radiating ions. It is impossible to explain this Doppler broadening by the motion of matter in the corona since the velocity associated with this motion is small (5 — 10 km/sec). It is assumed at the present time that the Doppler broadening of the corona lines is a consequence of the high temperature of the matter in the corona; this temperature reaches millions of degrees. Other indications of this high temperature are the high degree of ionization of F, Ca, Ni and A, the absence of recombination lines for hydrogen in the spectrum of the corona, and a number of other effects.[80] The mechanism by which the solar corona is heated has been analyzed in detail by Shklovskiĭ.[98] According to this author the heating of the corona results from the existence of electric fields inside the corona;* although these fields are weak they continuously transfer kinetic energy to charged particles, chiefly electrons. Shklovskiĭ has shown that even very weak fields (of the order of 10^{-8} v/cm) are sufficient for achieving a high electron temperature in the corona.

Artificial earth satellites have been of great importance in investigations of radiation in the solar corona and in the upper levels of the earth's atmosphere. On the basis of results obtained with rocket experiments Krasovskiĭ[123] concludes that the mechanism described in Eq. (3.11) cannot be used to explain the emission of the line λ 5577 A and the red triplet. A careful analysis has been made of the possibility of other reactions for the excitation of metastable levels in oxygen and Krasovskiĭ has shown that the reactions responsible for the excitation of the $^1S_0$ and $^1D_0$ levels must be different; he has proposed the following as more probable reactions:

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*We are not neglecting photo-excitation, but this mechanism is important only in certain cases.[98]

*The variable magnetic fields on the sun are postulated as the source of the electric fields in the corona.

(1) for the $^1S_0$ level

$$O_2^* + CO \to CO_2 + O\,(^1S_0),$$
$$OH^* + OH \to H_2O + O\,(^1S_0), \qquad (3.12)$$

(2) for the $^1D_2$ level

$$O_2^* + O \to O_2^* + O(^1D_2),$$
$$O_2^* + H \to OH^* + O\,(^1D_2), \qquad (3.13)$$
$$OH^* + O \to OH^* + O\,(^1D_2),$$
$$OH^* + H \to H_2^* + O\,(^1D_2), \qquad (3.14)$$

where O_2^* and OH_2^* are used to indicate vibration excitation of the oxygen and hydroxyl molecules.

Rocket experiments have also served to furnish an interesting correlation between the ultraviolet radiation from the sun (in the region 0 – 20 A) and the intensity of certain corona lines (λ 5303 A Fe XIV, λ 6702 A Ni XV).[124]

4. FORBIDDEN LINES INDUCED BY VARIOUS FIELDS

It is possible to obtain forbidden radiation of two kinds in discharge tubes. The first kind is characterized by lines whose intensity reaches a maximum at low current densities and falls off as the current is increased. The lines of the second kind are characterized by different behavior: the intensity of these lines is proportional to the square of the current density. The first group of lines is due to spontaneous radiation, such as has been considered in the earlier sections; the second kind is radiation which is induced by ionic fields. Although the field due to neighboring ions which acts on an atom varies in magnitude and direction, because of the relatively slow variation of the Coulomb field with distance it may be assumed as a first approximation that the ionic field is constant over the entire atom.[2] Hence it is of importance to study forbidden lines which are induced by external constant electric or magnetic fields.

The analysis of electric dipole radiation induced by a fixed electric field is relatively easy. The effect of the fixed electric field on the atom may be considered as a limiting case; i.e., the effect of a variable periodic electric field of zero frequency.* The quantum theory of Raman scattering of light gives the following expression for the dipole moment induced by a field $\mathbf{E}$ of frequency ω:[125]

$$\mathbf{d}_{mk}^{(i)} = -e^{i(\omega_{mk}+\omega)t}\frac{1}{2h}\sum_i\left\{\frac{(\mathbf{E}_0\cdot\mathbf{d}_{ik})\,\mathbf{d}_{mi}}{\omega_{ki}-\omega}+\frac{(\mathbf{E}_0\cdot\mathbf{d}_{im}^*)\,\mathbf{d}_{ik}}{\omega_{mi}+\omega}\right\}$$
$$-e^{i(\omega_{mk}-\omega)t}\frac{1}{2h}\sum_i\left\{\frac{(\mathbf{E}_0\cdot\mathbf{d}_{ik})\,\mathbf{d}_{mi}}{\omega_{ki}+\omega}+\frac{(\mathbf{E}_0\cdot\mathbf{d}_{im}^*)\,\mathbf{d}_{ik}}{\omega_{mi}-\omega}\right\}, \qquad (4.1)$$

where $\mathbf{d}_{ik}$ and $\mathbf{d}_{mi}$ are the dipole moments of the system in the absence of the field, ω_{mk} and ω_{mi} are the Bohr frequencies, $\mathbf{E}_0$ is the field amplitude. If we set $\omega = 0$ in Eq. (4.1) we obtain the dipole moment induced by the fixed electric field:

$$\mathbf{d}_{mk}^{(i)} = -e^{i\omega_{mk}t}\frac{1}{h}\sum_i\left\{\frac{(\mathbf{E}_0\cdot\mathbf{d}_{ik})\,\mathbf{d}_{mi}}{\omega_{ki}}+\frac{(\mathbf{E}_0\cdot\mathbf{d}_{im}^*)\,\mathbf{d}_{ik}}{\omega_{mi}}\right\}. \qquad (4.2)$$

It is apparent from Eq. (4.2) that the induced dipole moment $\mathbf{d}_{mk}^{(i)}$ differs from zero only in the case in which there is a level i and levels m and k such that electric dipole transitions are possible in the absence of a field. Assuming this, we obtain for the induced electric dipole lines the selection rules $\Delta J = 0, \pm 1, \pm 2$, even states ⇄ even states, odd states ⇄ odd states and, in the case of Russell-Saunders coupling, $\Delta L = 0, \pm 1, \pm 2$, $\Delta S = 0$, i.e., the same selection rules (with the exception of $J_1 + J_2 \geq 2$) as for electric quadrupole radiation. It also follows from Eq. (4.2) that the probability for induced electric dipole transitions is proportional to the square of the electric field intensity of current density, in complete agreement with experimental results. Furthermore, this probability depends to a considerable extent on the location of level i with respect to the k and m levels. If level i is close to either k or m, one of the denominators in Eq. (4.2) is markedly reduced and the probability for an induced transition is increased considerably. Since the energy levels tend to approach each other as the principle quantum number n is increased the intensity of the induced electric dipole lines must first increase (with the number of the line index) to some maximum and then fall off (as a consequence of the fact that $\mathbf{d}_{mi}$ is reduced with increasing n). The existence of this maximum serves to explain the experimental fact that with a gradual increase in the field first one observes only those lines of the forbidden series which correspond to these maxima; thereafter the other lines are gradually observed. The unique dependence of the intensity of induced lines on the quantum number n sharply distinguishes these lines from spontaneous electric quadrupole lines. The induced dipole lines are also distinguished from the spontaneous quadrupole lines by the considerable width, a result of the Stark effect. In the general case the sum rules do not apply for induced forbidden lines (as contrasted with the quadrupole lines); it is only in the case of large L and small multiplet splitting that they have any meaning.[126,57]

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*The radiation of atoms induced by periodic fields exhibits, as is well known, resonance effects.

All these features of electric dipole radiation induced by a fixed electric field were first reported experimentally in the lines of the forbidden series in He I and K I, observed in absorption.[127–129]

It should be noted that forbidden lines induced by a rather strong (not molecular) electric field are polarized.[157] The degree of polarization of the different components has been studied by Miliyanchuk, who has established, for example, that the π-component in the $n^2\ S_{1/2} - m^2\ S_{1/2}$ lines of single-electron atoms observed in a direction perpendicular to the direction of the electric field is weaker than that of the σ component.[158]

If the density of charged particles in the discharge is high enough or if the atoms are in high excited states, the molecular field over the entire atom cannot be assumed to be uniform. The electric dipole radiation induced by a weak inhomogeneous electric field has been investigated theoretically in great detail by Miliyanchuk.[158–162] The selection rules for this radiation are found to be the same as those for octupole electric radiation, with the exception of the requirement $J_1 + J_2 \geq 3$ which here takes the form $J_1 + J_2 \geq 1$. The intensity variation of the lines is almost the same in series induced by non-uniform and uniform electric fields, but in the first case the intensity of the lines increases much more rapidly with the quantum number n (as n^8) than in the second case (as n^4).[160]

If uniform and non-uniform electric fields act simultaneously (as is usually the case in a discharge), in the first approximation the non-uniform field has no effect on the intensity of the forbidden lines induced by the uniform field.[161] This theoretical conclusion has been verified in the work carried out by Prilezhaeva and her colleagues,[163] who measured the intensity of the forbidden lines in the P — P series of lithium in the spectrum of a carbon arc.

Forbidden transitions induced by uniform and non-uniform electric fields are easily distinguished from each other since they obey different Laporte rules. However, it is more difficult to distinguish the electric dipole transitions induced by non-uniform electric fields from the spontaneous dipole transitions (electric) since most of the selection rules are the same in both cases. However, one can make use of the Zeeman effect to distinguish between these lines.[159]

In a non-uniform electric field (or in a magnetic field) one sometimes encounters a so-called "complementary multiplet." The essence of this effect is that additional components with $\Delta J = \pm 2, \pm 3$ appear in an ordinary electric dipole multiplet. The complementary multiplet $4\ ^3P - 4\ ^3D$ Zn I observed in the arc is apparently an effect of this kind.[164]

Examples of forbidden lines induced by a non-uniform electric field are the lines of the $2\ ^2S - m^2\ F$ in Tl I, observed in the positive column of a glow discharge[165] and the weak lines $1\ ^1S_0 - 5\ ^1F_3$, $1\ ^1S_0 - 6\ ^1F_3$, in Ba I and $1\ ^2S - 6\ ^2F$ in K I observed in a spark discharge.[166]

A rather detailed investigation of electric dipole radiation of rare-earth ions, induced by electric fields in crystals and solutions, has been carried out in reference 33. The production of induced electric dipole transitions between the levels of the f^k-shell of the ion and the crystal, i.e., transitions between levels of the same parity, is due to the absence of a center of symmetry in the field, as is the case in many point symmetry groups in crystals. The central symmetry of the field can also be disturbed by ionic vibrations. The absence of a center of symmetry in the field leads to the appearance of an odd part U'_{odd} in the lattice potential where U'_{odd} consists of two terms — a static term and a dynamic term:[33]

$$U'_{\text{odd}} = U_{\text{odd}} + U_{\text{vib}}. \qquad (4.3)$$

Here U_{odd} represents the interaction of the lattice field with the ion in an equilibrium position while U_{vib} represents the same interaction when the ion is not at the equilibrium position.

The order of magnitude of U_{vib} is determined from the expression

$$U_{\text{vib}} \sim U_{\text{odd}} \cdot \frac{R}{r_0}, \qquad (4.4)$$

where U_{even} is the even part of the lattice potential, R is the amplitude of the ionic vibrations, and r_0 is of the order of the dimensions of the electron shell. The existence of an odd part in the lattice potential leads to a violation of the selection rules. For the quantities J and L we obtain the selection rules $|\Delta J| \leq 4$ and $|\Delta L| \leq 4$ in the case of a field without a center of symmetry and $|\Delta J| \leq 2$ and $|\Delta L| \leq 2$ in the case of a field with a center of symmetry, but with account being taken of the odd dynamic part of the lattice potential U_{vib}. It is also possible to have transitions which obey the selection rules $|\Delta J| \leq 6$, $|\Delta L| \leq 6$ and $|\Delta J| \leq 4$, and $|\Delta L| \leq 4$ respectively, but these are characterized by extremely small probabilities.[33] The selection rules for the quantum number m are the same as those in the case of magnetic and quadrupole radiation which become in the corresponding rules for the crystallographic number $\mu : \mu = 0, \pm 1$ if we do not take account of the dynamic part of the lattice potential and $|\Delta\mu| \leq 2$ or $|\Delta\mu| \leq 4$ if we consider the linear or cubic terms of this part respectively. In a particular case, when the crystal is characterized by inversion axes of order n, these selection rules assume the form

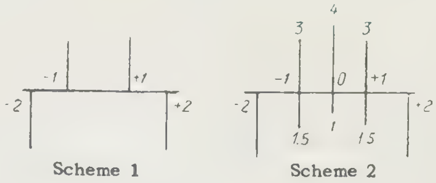
$$\Delta\mu = \pm \frac{n}{2}, \quad \pm\left(1 \pm \frac{n}{2}\right). \qquad (4.5)$$

The maximum probability for induced electric dipole transitions has been computed by El'yashevich[33] and found to be 10^4 sec^{-1}; as is apparent from Eqs. (1.19) — (1.20) this is much larger than the maximum probabilities for quadrupole and magnetic transitions.

The induced electric dipole radiation of rare-earth ions in crystals and solutions has been observed experimentally by a number of the investigators who have already been mentioned.

The Zeeman effect is very characteristic of induced electric dipole lines, and is frequently used as a means of identification. Both the longitudinal and transverse effects are different for the dipole and spontaneous quadrupole lines. Thus, for longitudinal observation in the case of quadrupole radiation, components with $\Delta m = \pm 2$ are absent; in the case of dipole radiation induced by a constant electric field, these components appear. The same applies to the component with $\Delta m = 0$ under transverse observation. In addition it has been found that for this induced dipole radiation the sum of the Zeeman σ and π components are not equal.[2]

The Zeeman effect has also been studied in detail for electric dipole lines induced by isotropic electric fields.[130–132] These fields are usually produced by a random distribution of ions; under these conditions the mean values of the field components are the same in any direction. In this case the Zeeman pattern is again different for spontaneous quadrupole and induced electric dipole lines. In Fig. 5, as an example, we show the Zeeman components for these two types of lines produced in P — S transitions in He I as obtained under transverse observation.[133] The schemes refer to the quadrupole (1) and dipole (2) lines where the upper and lower vertical lines denote the σ and π components respectively; the numbers on the horizontal lines denote the values of the respective Δm components while the lengths of these lines denote the relative intensities for the σ and π components are equal (in contrast with the behavior of lines induced by a fixed electric field).



Scheme 1 Scheme 2

FIG. 5. Zeeman effect for electric dipole lines induced by isotropic electric fields (detailed explanation given in text).

An investigation has also been made of the Zeeman effect in lines induced by molecular electric fields in which there are no components parallel to the magnetic field. In this case it is found that for transverse observation the Zeeman splitting of the induced dipole lines arising from P — F and P — P transitions is almost identical with the Zeeman splitting of spontaneous electric quadrupole and magnetic dipole lines respectively. These splittings are different, however, for longitudinal observation. By means of this effect it has been possible to establish the induced nature of the lines $2\,{}^1P - 4\,{}^3F$ (λ 4921 A), $2\,{}^3P_{1,2} - 4\,{}^3F$ (λ 4471 A) and $2\,{}^1P - 5\,{}^1P$ (λ 4383 A) in the spectrum of helium. One of these lines, $2\,{}^3P_{1,2} - 4\,{}^3F$, has been observed in the spectra of certain class B stars, indicating the existence of ionic fields. The P — G lines in the spectrum of helium have been assigned to induced radiation (rather than spontaneous octupole radiation as had been believed earlier).[133–136]

Miliyanchuk[159] has investigated the Zeeman effect in lines induced by a weakly inhomogeneous electric field for the general case. The selection rules for the magnetic quantum numbers are found to be $\Delta m = 0, \pm 1, \pm 2, \pm 3$. The polarization of the component characterized by $\Delta m = \pm 3$ is the same as that of the component with $\Delta m = \pm 1$ in the resolved line. The remaining components are elliptically polarized for all directions of observation. The shifts and intensities of the components $m \rightarrow m + \delta$ and $-m \rightarrow -m - \delta$ $(\delta = 0, 1, 2, 3)$ are different.

Forbidden atomic transitions can be induced in a strong magnetic field alone (without an electric field). For this effect to take place it is necessary that the field strength be sufficient to break the coupling between the L and S moments of the atom; under these conditions, the perturbed wave functions cannot be found in the first approximation alone. Thus the moment $J = L + S$ is no longer constant in direction and the selection rule $\Delta J = 0, \pm 1$ no longer applies. A calculation of the corrections which arise in the higher approximations leads to the additional selection rule $\Delta J = \pm 2$; in very strong fields we find the selection rule $\Delta J = \pm 3$. It should be noted that forbidden lines can appear in relatively weak fields if these fields are strong enough to produce a "partial" Paschen-Back effect. Forbidden lines induced by a magnetic field have been observed in the $(P - D)$ triplets of Ca I, Zn I, Cd I and in the $(P - D)$ doublets of Al I and Ca II.[37]

All the above considerations relating to the production of forbidden fine-structure lines under the effect of a magnetic field also apply to hyperfine structure. Because the hyperfine splitting of terms is small, even relatively small fields break the coupling between **J** and **I**, leading to the appearance of the forbidden transitions $\Delta F = \pm 2, \pm 3$.

The production of forbidden lines of the type described above is also possible in the spectra of atoms or molecules which have magnetic moments if these atoms or molecules approach each other very closely; under these conditions the magnetic fields which are set up become rather large. Since the Zeeman splitting is proportional to the first power of the magnetic field these forbidden lines should be considerably more diffuse than the forbidden lines induced by an electric field (in the latter case the relative sharpness of the forbidden lines arises by virtue of the quadratic Stark effect[2]).

We may also consider one other type of forbidden line which appears in an external magnetic field. As is well known, the Zeeman components of lines for which there is no fine structure obey the following selection rules: $\Delta m_J = 0$, $\Delta m_I = 0$ (π components) and $\Delta m_J = \pm 1$, $\Delta m_I = 0$, (σ components), where m_J and m_I are quantum numbers which determine the projection of the moments **J** and **I** respectively. If the lines exhibit hyperfine structure the first of the indicated selection rules refers to the number m_F, i.e., the π components obey the rule $\Delta m_F = 0$ while the σ components obey the rule $\Delta m_F = \pm 1$ where m_F is the magnetic quantum number, corresponding to the total angular momentum $\mathbf{F} = \mathbf{J} + \mathbf{I}$. Since $m_F = m_J + m_I$, the selection rule for m_F will be satisfied for various changes of the quantum numbers m_J and m_I which lead to the appearance of "extra" Zeeman components. For example, in addition to the resolved π-components which satisfy the selection rules $\Delta m_I = 0$, $\Delta m_F = 0$, and $\Delta m_J = 0$, there must be π components that obey the selection rules $\Delta m_F = 0$, $\Delta m_J = \pm 1$ and $\Delta m_I = \pm 1$. The intensity of these components is proportional to $(Ahc/2g(J)\mu_0 H)^2$ (A is the hyperfine structure constant, g is the Landé g factor, H is the magnetic field) and consequently decreases with increasing field and with diminishing A.[138-139] Forbidden π and σ components have been observed experimentally by Gerasimov and Frish,[140] who observed the Zeeman effect for the absorption lines in cesium (λ 4593.2 A, λ 4555.3 A, and λ 8521.1 A), rubidium (λ 7800.3 A), and sodium (λ 5889.56 A). It should be noted that the hyperfine structure of the lines listed above was not resolved; the existence of the effect was indicated indirectly — in the appearance of the forbidden components.

TABLE XI

Polarization	Component shift		Fractional parts of normal splitting
	$\Delta\lambda$ (Å)	$\Delta\nu$ (cm^{-1})	
π	*0.232	*−1.090	$-\frac{4}{3}\Delta\nu_0 = -1.109$
	0.117	−0.552	$-\frac{2}{3}\Delta\nu_0 = -0.554$
	−0.117	0.552	$\frac{2}{3}\Delta\nu_0 = 0.554$
	*−0.232	*1.090	$\frac{4}{3}\Delta\nu_0 = 1.109$
σ	0.237	−1.114	$-\frac{4}{3}\Delta\nu_0 = -1.109$
	*0.107	*−0.503	$-\frac{2}{3}\Delta\nu_0 = -0.554$
	−0.107	0.597	$\frac{2}{3}\Delta\nu_0 = 0.554$
	−0.237	1.114	$\frac{4}{3}\Delta\nu_0 = 1.109$

Table XI shows the results of these observations on one of the cesium lines (λ 4593.2 A, $6\,{}^2S_{1/2} - 7\,{}^2P_{1/2}$) through the use of a magnetic field of 17,800 gausses; also shown are the corresponding displacements of the components of the fractional parts of the normal Lorentz splitting (the forbidden components are denoted by the asterisks). As we have seen the forbidden π components are found close to $\pm \frac{4}{3}\Delta\nu_0$, while the σ components are close to $\pm \frac{2}{3}\Delta\nu_0$ where the location of the latter exhibits some asymmetry. This asymmetry is explained by the non-uniform intensity of the forbidden components; this effect means that their common center of gravity is somewhat displaced.[140]*

Forbidden lines arise in the magnetic field of the nucleus (without external fields). The existence of nonvanishing spin and magnetic moment in a nucleus leads not only to hyperfine structure but also to the appearance of forbidden transitions. As is well known, the hyperfine terms are found by means of perturbation theory.[141] The perturbation operator used is the energy of the magnetic interaction between the electron shell and the nuclear spin

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*The existence of an asymmetry in the displacements and intensities of the forbidden components is explained by Miliyanchuk[158] by the fact that under the experimental conditions of Frish and Gerasimov these components arise under the effect of the intermolecular inhomogeneous field.

$$V = -\mu_I \cdot \mathbf{H}_J, \qquad (4.6)$$

where μ_I is the magnetic moment of a nucleus with spin I, while H_J is the magnetic field produced by the electron shell with a total angular momentum J (neglecting the spin of the nucleus). The corrections for the energy in the first approximation are computed from the unperturbed functions in the representation γ, J^2, I^2, F^2, F_z where $F = J + I$ is the total angular momentum (including the spin of the nucleus), γ represents all the remaining mechanical variables, which are not written explicitly. In this case the selection rules for the various multipole transitions remain the same as in the case in which there is no interaction between the magnetic moment of the nucleus and the atomic shell (if we do not take account of the additional analogous rules for the quantum numbers F and m_F). The result of a calculation will be more precise, the larger the spin-orbital interaction as compared with the interaction V. If this second quantity is not small compared with the first, in order to obtain the energy correction and the selection rules it is necessary to use the first-approximation wave functions. These are of the form

$$\varphi(\gamma, J, F) = \psi^{(0)}(\gamma, J, F) + \sum_{\gamma', J' \neq J} \frac{(\gamma', J', F' \,|\, V \,|\, \gamma, J, F)}{E(\gamma', J') - E(\gamma, J)} \psi^{(0)}(\gamma', J', F'), \qquad (4.7)$$

where $(\gamma', J', F' | V | \gamma, J, F)$ is the nondiagonal perturbation matrix element and $E(\gamma', J') - E(\gamma, J)$ is the difference between the fine-structure levels. The nondiagonal elements are comparable in magnitude with the difference between the hyperfine structure sublevels and vanish when $\Delta F \neq 0$; $\Delta J \neq 0, \pm 1$.[142]

It is apparent from Eq. (4.7) that the hyperfine structure terms for given J and F are, in the first approximation, linear combinations of the other terms of the zeroth approximation with the same F but with the other quantum numbers different; all terms will be larger for a value of J′ for which the difference in levels of the fine structure $E(\gamma', J') - E(\gamma, J)$ is smaller. The existence of these linear combinations or, to put it another way, this mixing of fine structure states, leads to violations of the selection rule and thus explains the production of forbidden lines.

Forbidden lines induced by the magnetic field of the nucleus have already been observed in atomic spectra of bivalent elements — mercury and cadmium. These lines are Hg I: λ 2269.80 A, $6\,s^2\;{}^1S_0 - 6\,sp\;{}^3P_2$ (first investigated in 1927 by Rayleigh),[143] λ 2655.58 A, $6\,s^2\;{}^1S_0 - 6\,sp\;{}^3P_0$ (first obtained in fluorescence by Wood) and λ 2967.5 A, $6\,sp\;{}^3P_0 - 6\,sd\;{}^1D_2$,[144] and also the lines in Cd I:

λ 3141 Å, $5s^2\,{}^1S_0 - 5sp\,{}^3P_2$ and λ 3320 Å, $5s^2\,{}^1S_0 - 5sp\,{}^3P_0$,

first observed in 1931.[145] The nature of these lines was not established immediately. For some time they were something of a puzzle. On the one hand they could not be assigned to one of the spontaneous multipole radiation lines since they did not obey any of the appropriate selection rules; on the other hand, from an analysis of the excitation conditions it was impossible to draw any conclusion as to the possible production of these lines by virtue of external fields.* Bowen was the first to propose that these lines were induced by the magnetic field of the nucleus. This theoretical proposal of Bowen was first investigated by Goudsmit and Bacher in 1932 who used the line λ 2967.5 A.[146] Taking account of the interaction of the nuclear spin with only one of the outer s electrons in the mercury atom (the interaction with d electrons is a hundred times weaker) and considering only the first term in the summation in (4.7), (using the ${}^3D_1$ term (because of the fact that it was closest to the ${}^1D_2$ term), these authors were able to obtain corrections for the hyperfine structure sublevels of the ${}^1D_2$ and ${}^3D_2$ terms for both odd isotopes of mercury: Hg^{199} ($I = 1/2$) and Hg^{201} ($I = 3/2$). It was found that the difference between the hyperfine structure levels with the same F for the terms ${}^1D_2$ and ${}^3D_1$ increases as compared with the ideal scheme (i.e., as compared with the results of calculations in the zeroth approximation). Goudsmit and Bacher also verified the appearance of the forbidden line ${}^3P_0 - {}^1D_2$, also in the case in which the interacting terms have only one sublevel with the same F; the ratio of intensities for the forbidden and corresponding displaced line is given by the simple expression

$$\frac{I}{I_0} = \frac{\delta}{\Delta + \delta}, \qquad (4.8)$$

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*Later Miliyanchuk[160] showed theoretically that at high pressures the probability for forbidden transitions of the type ${}^1S_0 - {}^3P_2$ to a considerable degree was affected by the inhomogeneity of the molecular field.

where Δ is the distance between the indicated sublevels when the term interaction is neglected, and δ is the change in this distance due to the interaction of the terms.

Three papers have been devoted to the theory of other forbidden lines. In all this work the same hypothesis has been used concerning the production of these lines under the effect of the magnetic moment of the nucleus. In the first of these papers[147] calculations have been made of the relative intensities of the hyperfine structure components of the Hg I line $^1S_0 - {}^3P_2$. In the second[142] investigation has been made of the forbidden line Hg I $^1S_0 - {}^3P_0$. The intensity ratio was obtained for the two odd isotopes Hg^{199} and Hg^{201} which have spins $I_1 = 1/2$ and $I_2 = 3/2$ respectively (the spin of the other isotopes is zero); in particular,

$$\frac{I}{I_0} = \frac{g^2(I_1)\, I_1 (I_1 + 1)\, p_1}{g^2(I_2)\, I_2 (I_2 + 1)\, p_2}, \tag{4.9}$$

where $g(I_1)$ and $g(I_2)$ are the gyromagnetic ratios and p_1 and p_2 are the percentage composition of these isotopes. It has been established that the term causing the strongest perturbation of the $^3P_0$ terms is $^3P_1$; on the assumption that the nondiagonal and diagonal elements of the perturbation matrix are the same $(\gamma', J', F' | V | \gamma, J, F)$ an estimate has been made of the intensity ratio for the transitions $A(^3P_0 \rightarrow {}^1S_0)$ and $A(^3P_1 \rightarrow {}^1S_0)$. In the last paper[148] an investigation has been made of the forbidden transitions $^3P_0 \rightarrow {}^1S_0$ and $^3P_2 \rightarrow {}^1S_0$ for mercury, cadmium and zinc. The $^3P_2$ term is also most highly perturbed by the $^3P_1$ term. It has been found that the ratio in (4.9) also applies for the transition $^3P_2 \rightarrow {}^1S_0$.

In addition, a semi-empirical method (using the experimental data on hyperfine structure and the calculated results of the nondiagonal elements of the perturbation matrix) has been used to obtain numerical values for the ratios of the probabilities for the $^3P_0 \rightarrow {}^1S_0$ and $^3P_2 \rightarrow {}^1S_0$ transitions as compared with the resonance transition $^3P_1 \rightarrow {}^1S_0$ for mercury, cadmium, and zinc. These numerical values are found to be of the same order of magnitude. However, lines corresponding to these transitions have been found only in mercury and cadmium. The absence of such lines in zinc may be explained by the fact that the multiplet levels $^3P_0$, $^3P_1$, and $^3P_2$ for the zinc atom are very close to each other so that transitions of the atom from the metastable levels $^3P_0$ and $^3P_2$ to the non-metastable level $^3P_1$ are enhanced, thereby implying that it is difficult to achieve the concentration of metastable atoms required to obtain an observable intensity for the $^1S_0 - {}^3P_0$ and $^1S_0 - {}^3P_2$ lines.

To supplement the cited results of the theoretical work, we should also note that the Bowen hypothesis concerning the origin of the lines $^1S_0 - {}^3P_0$, $^1S_0 - {}^3P_2$ and $^3P_0 - {}^1D_2$, leads in itself to the following features of the lines: First, only the odd isotopes Hg I, Cd I, and Zn I should radiate (the magnetic moments of nuclei of the even isotopes of these elements are zero); second, the hyperfine structure of these elements should be different for different values of the spin; third, when several odd isotopes are present an isotopic shift should be observed.

All these features and the results of the theoretical work cited above have been verified in the experimental work of Gaviola, Mrozowski and other authors.[149–153] Gaviola[149] has found the ratio of the probabilities for Hg I $A(^3P_0 \rightarrow {}^1S_0)$ and Hg I $A(^3P_1 \rightarrow {}^1S_0)$. Mrozowski[150–151] has investigated all three forbidden lines of mercury. These lines were obtained by optical excitation in mercury vapor containing an admixture of nitrogen. Mrozowski was able to establish the presence of an isotopic shift in all of these lines and has measured the relative intensities of the hyperfine structure components for the $^1S_0 - {}^3P_2$ and $^3P_0 - {}^1D_2$ lines; he has also observed other features which are found to be in good agreement with theory. In references 152 and 153 it has been shown that the intensity of these lines for both mercury and cadmium are proportional to the number of odd isotopes (the lines were excited in mercury vapor and cadmium vapor containing different isotopic composition).

On the basis of all that has been said concerning the effect of the magnetic moment of the nucleus on forbidden transitions, it can be concluded that the interaction between the electric quadrupole moments of the nucleus with the electron shells should also lead to the production of forbidden lines. As yet only the effect of the electric quadrupole moment of the nucleus on the hyperfine structure levels of various atoms has been investigated (in particular, europium, potassium, cesium, and so on[154–155]). The electric quadrupole moments of nuclei are determined mainly from the experimental data on the hyperfine structure.

CONCLUSION

Of all the possible forbidden lines, only a relatively few have been obtained under laboratory conditions.* These lines have been observed in both absorption and emission spectra. In absorption† it is especially convenient to investigate the lines that arise in transitions between high metastable states and ground states, because it is very difficult to obtain a high density of atoms in high metastable states. The absorption method of studying forbidden lines has advantages and disadvantages. On the one hand this method allows a considerable simplification in the measurements of the transition probabilities since there is no concern with the populations of metastable levels; on the other hand it is extremely difficult to measure the intensities of weak absorption lines and this makes it necessary to study very thick layers of gases or vapors. It should also be noted that the absorption method can be used only for studying transitions in which one level must correspond to the ground state.

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*This is discussed in greater detail in Mrozowski.[2]

†In absorption spectra both spontaneous and induced forbidden lines have been observed; for example, the $^2S - m\,^2D$ series in the alkali metals,[93] the $^1S_0 - {}^3P_2$ line mercury[143], etc.

Emission forbidden lines are obtained most frequently in discharge tubes. These tubes can have internal or external electrodes; the external electrodes are especially convenient in cases in which the required vapor pressure can be achieved only at high temperatures. As has already been indicated, to obtain sufficiently intense spontaneous forbidden lines, the investigated gas must be mixed with small amounts of intert gases or other gases whose atoms or molecules have high excitation potentials. The presence of an inert gas leads to an increase in the probability of excitation of the investigated atoms, since elastic collisions of atoms of this gas with free electrons reduce the mean kinetic energy of the latter to values comparable with the energy of the metastable levels. The actual role played by the inert gas in the production of forbidden lines has as yet not been clarified.

Another important method of obtaining forbidden lines is the fluorescence method. In this case the atoms of the material investigated first absorb radiation and make a transition to high excited states; then, by radiation or collision with other atoms, they make transitions to metastable levels. So far this method has been applied only in mercury (Wood and Gaviola[149]). Mercury vapors have been investigated at room temperature; in this work helium or nitrogen was present in the fluorescence tube. The mercury atoms are excited to the $^3P_1$ level by irradiation with resonance radiation at λ 2537 A and then, via collisions, make transitions to the metastable $^3P_0$ state. It would appear to be desirable to apply this method in other cases.

A third important method for the production and investigation of forbidden lines is the radio-spectroscopy method. We have considered the method and its application above.

All optical methods involve great difficulties in the measurements of intensities of weak forbidden lines, all the more so since the latter are frequently close to smeared-out lines. In order to overcome these difficulties, generally speaking, one prefers to take steps which result in an improvement in the excitation conditions for the forbidden lines rather than improvement of the detection apparatus, since small changes in the excitation conditions can result in intensity increases of several hundred times.[2]

Under laboratory conditions one usually investigates forbidden lines in neutral atoms. Forbidden lines of singly ionized atoms can be obtained with reasonable intensity only at very low pressures and only in the absence of an inert gas.[2] Research in the laboratory production of forbidden lines in highly ionized gases is just beginning.* This problem is one of great importance.

Up to this time the overwhelming majority of forbidden lines observed in the spectra of celestial bodies have not been obtained in the laboratory and a second very important problem is a complete and accurate calculation of the transition probabilities associated with these lines. A calculation of this kind for the basic configurations of atoms in the first two short periods of the periodic table has been carried out rather carefully, as we have already indicated. However, the calculation of the probabilities for forbidden transitions pertaining to the configurations d^n, $d^{n-1}s$, etc., which are characteristic of atoms in the first long period, has only just begun (with the exception of the work by Pasternack and Garstang[16-156]). It should be noted that in order to obtain accurate numerical values for these probabilities one must have highly accurate values for the radial integral s_q.

Although the general theory of forbidden lines is of interest in astrophysics and has been considered in a rather large number of papers, cases in which this theory has been applied to actual lines are rare. At the present time only the theory of "nebular" lines may be considered well-developed.

A number of problems relating to the theoretical and experimental aspects of forbidden atomic radiation are still not solved. These include the following: (1) further development of the theory of inter-configuration and other interactions, (2) the identification of the remaining unidentified four corona lines and other lines in the spectra of celestial bodies, (3) the discovery of spontaneous multipole lines higher than the second order, (4) further investigations of the interference effect in mixed radiation, (5) the observation of pure magnetic dipole lines in the optical region, and so on.

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*If we neglect ions in crystals and solutions.

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New Apparatus and Methods of Measurement

REAL SPECTRAL APPARATUS

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1. IDEAL AND REAL APPARATUS

1. Real spectral apparatus are not, as is well known, ideal harmonic analyzers of radiation i.e., the distribution of energy over the spectrum obtained with the help of real spectral apparatus differs from the "ideal" or "true" distribution, which gives the Fourier expansion of the radiation being investigated. In any real apparatus, the differences of the observed distribution from the true (or as we shall say, the distortion) result from many very different reasons. The following classification of the distorting factors by the nature of the distortions produced by them is useful. In the first group we list distortions produced in the investigation of a spectrum of even comparatively crude structure, i.e., with slowly changing brightness, for example in the investigation of a continuous spectrum. Among such distortions are the change with the wavelength (a) of the solid angle of the exit pupil of the filter, (b) of its dispersion, (c) of the angular and linear magnification, (d) of the sensitivity of the radiation detector, etc. In the majority of cases, these factors lead to a slow change in the scale along the spectrogram or to a slow change of the coefficient of proportionality between the illumination (or current) measured and the true spectral density of the radiation. We can also include among these factors parasitic exposure of the spectrogram, brought about by scattering of radiation on parts of the apparatus and leading as a rule to an addition of some slowly changing quantity to the true distribution. It is usually comparitively easy to take these distortions into consideration and at the present time there are workable methods for such consideration (see, for example, references 1, 2). In what follows we shall not consider these factors.

In the study of the fine structure of a spectrum (singlet, doublet, etc, spectral lines, narrow bands, and so forth), over the range of which one cannot take into account distortions of the first group, there are important distortions of another type, which we shall place in a second group. These distortions are brought about by the fact that even in monochromatic radiation a real spectral apparatus gives some distribution of energy over a spectrogram of finite width. The form of this distribution and its width are determined by various factors: diffraction on the diaphragms of the optical system of the spectral apparatus, its aberrations, the finite aperture widths, the time lag of the recording instrument, scattering in the light-sensitive photographic emulsion, etc. It is important to emphasize that, independent of its nature, the finite width of the distribution in a monochromatic radiation is common to all real spectral apparatus and determines to a significant degree the possibilities of investigation of fine details or structure of a spectrum.

In addition to distortions of a systematic character, brought about by factors of the first and second groups, a difference between the "true" and "observed" distributions is also caused by factors that lead to random errors of measurement. We shall put these factors in a third group. The appearance of random errors is connected with the change of parameters of the optical system during the time of the experiment, by fluctuation phenomena in the light detector, and for similar reasons. In a number of cases, changes in the brightness of the light source and fluctuations of the liminous flux connected with the corpuscular character of light also lead to random measurement errors.

The demarcation for random and systematic distortions is expedient because they are produced by physically different causes and consequently require for their reduction different (and sometimes even contradictory) changes in the apparatus. However, the possibilities of the apparatus as a whole are limited by factors of the second and the third groups taken together; in consideration of such questions as the resolution, accuracy of measurement of the real energy distribution, etc., it is necessary to take their joint action into account.

2. Let us now consider a quantitative description of the distortions brought about by factors of the second group. Let the true energy distribution over the spectrum be described by the function $\varphi(x)$, while the distribution obtained by real apparatus for monochromatic radiation be represented

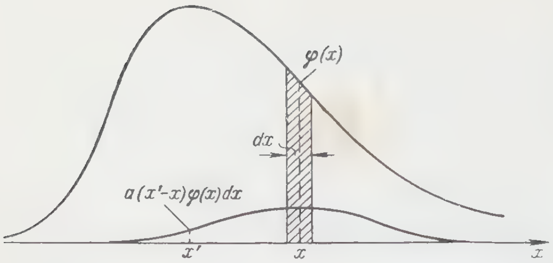

FIG. 1. For the derivation of Eq. (1).

by the function $a(x)$, which we shall call the apparatus function of the spectral equipment. The observed distribution can be plotted in the following way. Each monochromatic component $\varphi(x)dx$ of the true radiation is replaced by the apparatus function, as a result of which, at some arbitrary point x', there is created an illumination (or current) $a(x'-x)\varphi(x)dx$ (see Fig. 1). Other monochromatic components of the true distribution also make a corresponding contribution to the illumination at the point x', and as a result the observed distribution $f(x')$ will be expressed by the following integral

$$f(x') = \int_{-\infty}^{\infty} a(x'-x)\varphi(x)\,dx. \tag{1}$$

The function $a(x)$, and consequently the formula (1), take into account distortions both in the optical and recording parts of the apparatus. It is easy to see that each of these parts can be characterized by its own apparatus function $a_1(x)$ and $a_2(x)$. For example, the distribution of the illumination $f_1(x')$ in the focal plane of the objective of the recorder is

$$f_1(x') = \int_{-\infty}^{\infty} a_1(x'-x)\varphi(x)\,dx. \tag{2}$$

It is easy to show that the apparatus function of the spectral equipment $a(x)$ is expressed in terms of $a_1(x)$ and $a_2(x)$ in the following way:

$$a(x) = \int_{-\infty}^{\infty} a_1(x-y)\,a_2(y)\,dy. \tag{3}$$

Applicability of Eqs. (1) – (3) can be limited by the following circumstances. In the derivation of (1) it was assumed that the apparatus function was the same at different wavelengths. It is easy to see that the condition for invariance ought to be satisfied essentially only in a comparatively small region around the point x', where $a(x'-x)$ is appreciably different from 0.

This condition is satisfied in the overwhelming number of cases, although in regions of the spectrum at great distances from one another, the apparatus function will change appreciably. For example, if the apparatus function is almost completely determined by diffraction on the aperture diaphragm, then its width is proportional to the wavelength and it is doubled in the transition, say, from 5000 A to 10,000 A. However, the apparatus function is essentially different from zero only in a region with dimensions on the order of $2\lambda/R$, where R is the Rayleigh resolving power, and where it is quite clear that over the range of such a region we can neglect the change in the apparatus function. Besides, these conditions are sometimes not observed, and it is necessary at the same time to consider factors of the first and second groups (for example, see reference 3). In what follows such cases will not be considered.

It is appropriate to make some remarks of a terminological character. Several names are given in the literature for the function $a(x)$: instrumental contour, apparatus function, distortion function, etc. In English and American literature on infrared spectroscopy the term "slit function" is used. Everywhere in what follows we shall use the term "apparatus function" both for all apparatus as a whole and for the description of particular distorting factors: the apparatus function of the slit, the apparatus function of the diffraction, the apparatus function of the photo layer, etc., where by apparatus function of any factor is meant the observed distribution of illumination (current) for monochromatic radiation and for vanishingly small effects of the remaining distorting factors.

Up to now we have spoken about the true and observed energy distribution over the wavelengths, consequently identifying the arguments x, x', y with the wavelength. However, by x, y, etc., we can also understand the frequencies either of the coordinate in the density spectrum or the coordinate in the recorded spectrum, obtained by a microphotometer or by scanning of the spectrum by some recording device.

This situation is evidently connected with the fact that we are limiting ourselves to a consideration only of distortion factors of the second group, and of comparatively narrow spectral intervals, where the conversion coefficients (type of linear dispersion) from the wavelength to the coordinates or to the frequencies we can regard as constant.

Finally, we note that the relations of type (1) have been given quite a definite name in mathematical literature, that is, we say that the function $f(x)$ expressed by Eq. (1) is the convolution of $a(x)$ and $\varphi(x)$. In the physical literature, particular names are sometimes used for the different special cases. For example, in the investigation of the distorting action of a recording instrument alone, Eq. (1) is called the "general time equation of the spectrometer" in reference 4. For convolutions of two functions there exists a set of mathematical theorems; in particular, convolution possesses the properties of commutation, association, and distribution,[5] i.e.:

$$\int_{-\infty}^{\infty} a(x-y)\varphi(y)\,dy = \int_{-\infty}^{\infty} \varphi(x-y)a(y)\,dy;$$

$$\int_{-\infty}^{\infty} g(x-y)\,dy \int_{-\infty}^{\infty} a(y-z)\varphi(z)\,dz$$

$$= \int_{-\infty}^{\infty} \varphi(x-y)\,dy \int_{-\infty}^{\infty} a(y-z)g(z)\,dz;$$

$$\int_{-\infty}^{\infty} a(x-y)[g(y)+\varphi(y)]\,dy$$

$$= \int_{-\infty}^{\infty} a(x-y)g(y)\,dy + \int_{-\infty}^{\infty} a(x-y)\varphi(y)\,dy. \qquad (4)$$

An important property of convolution consists in the fact that its integral is equal to the product of the integrals of the functions $a(x)$ and $\varphi(x)$:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a(x-y)\varphi(y)\,dy\,dx = \int_{-\infty}^{\infty} a(z)\,dz \int_{-\infty}^{\infty} \varphi(y)\,dy. \qquad (5)$$

Some other properties of the convolution will be considered below.

It should be borne in mind that for the appropriate use of the concept of the true and the energy distribution over the spectrum we need a definite normalization of the apparatus function $a(x)$. This is connected with the dual role of the slit (or slits) in the spectral apparatus. The finite width of the slit leads to a distortion of the spectral distribution and consequently the slit ought to be infinitely narrow in the ideal apparatus. On the other hand, for an infinitely narrow slit, there is an infinitely small current in the apparatus and it is not possible to operate with real quantities. This difficulty can be avoided if it is contrived that as we reduce the width of the slit we increase the illumination of the spectrum, so that the total energy coming into the apparatus remains unchanged. Mathematically, this means that the apparatus function must be normalized over the area, for example, in the following way:*

$$\int_{-\infty}^{\infty} a(x)\,dx = 1. \qquad (6)$$

For such a normalization, the total energy of the true distribution is equal to the total energy of the observed distribution. Actually, by integrating (1) over x', and taking (5) and (6) into account, we obtain

$$\int_{-\infty}^{\infty} f(x')\,dx' = \int_{-\infty}^{\infty} a(z)\,dz \int_{-\infty}^{\infty} \varphi(y)\,dy = \int_{-\infty}^{\infty} \varphi(y)\,dy. \qquad (7)$$

Thus, if we assume (6), then the presence of the apparatus function leads only to a redistribution of the energy over the spectrum, i.e., to a change in the shape of the line, the band, etc. This makes it possible to isolate the problem of the distortion of the shape of the distribution from problems connected with the absolute value of the illumination, light intensity, etc. For this purpose it suffices to normalize the functions $f(x)$ and $\varphi(x)$ in a manner similar to (6):

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \varphi(x)\,dx = 1. \qquad (8)$$

We emphasize that the relation (7) is the direct consequence of the definition of an apparatus function which does not depend either on the form of $\varphi(x)$ or on the nature of $a(x)$. Therefore, the experimental verification of (7), obtained in reference 6, shows only that the region of integration in (7), always finite in the experiment, was taken sufficiently large.

Normalization of (6) and (8) makes it possible to use the theory of δ-functions* for the description of limiting cases — monochromatic radiation and ideal spectral apparatus. If $a(x) = \delta(x)$ then:[7]

$$f(x') = \int_{-\infty}^{\infty} \delta(x'-x)\varphi(x)\,dx = \varphi(x'),$$

i.e., an instrument whose apparatus function is a δ-function is an ideal non-distorting device. In the other limiting case of monochromatic radiation, we have

$$\varphi(x) = \delta(x); \quad f(x') = \int_{-\infty}^{\infty} a(x'-x)\delta(x)\,dx = a(x'),$$

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*For periodic apparatus functions of interference spectroscopes, the integration is carried out over a single period.

*The δ function is formally defined in the following way (see, for example, reference 7):

$$\delta(x) = \begin{cases} \infty, & x=0; \\ 0, & x\neq 0; \end{cases} \quad \int_{-\infty}^{\infty} \delta(x)\,dx = 1.$$

In what follows we shall take for granted the following property of the function:

$$\int_{-\infty}^{\infty} g(x)\delta(x)\,dx = g(0); \quad \delta(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{ix\omega}\,d\omega.$$

in correspondence with the definition of the apparatus function.

3. A relation of the type (1) connects the true and observed quantities not only in the case of spectroscopic measurements but also in many other regions of physical measurement. The distortion of signal presented to the input of an electrical linear system is described by Eq. (1) (see, for example, reference 8) while, by analogy, the apparatus function is the response of the system to an instantaneous pulse. The theory of the phosphoroscope and also of other apparatus with time scanning of rapidly alternating processes can also be reduced to Eq. (1). The methods of Michelson for the determination of the structure of spectral lines and the structure of astronomical objects[9] are quantitatively described by Eq. (1). The methods of x-ray determination of the dimensions of particles are also described in terms of a relation of the type (1) (see, for example, reference 10). Equation (1) arises in a number of problems of astrophysics,[11] the theory of broadening of spectral lines,[11,12] and in other regions. The distortions introduced by real optical apparatus and radio telescopes are described by the two-dimensional analogue of Eq. (1):

$$f(x', y') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a(x'-y,\ y'-y)\,\varphi(x, y)\,dx\,dy,$$

where (x, y) physically means the distribution of the measured quantity (intensity, current, brightness temperature) for a point source of radiation.[13]

In all these and in many other cases we are dealing with a linear integral relation between the true distribution of some physical quantity $\varphi(x)$ with an arbitrary argument x and the experimentally determined distribution $f(x)$, where the kernel of the equation is characteristic for the given equipment or apparatus function $a(x)$. In spite of the physical diversity of these quantities and the manifold concrete forms of the function $a(x)$, the statement of the problems arising in connection with Eq. (1), and the methods of mathematical solution of these problems have a very general and sometimes almost identical physical measurement in completely different regions. Therefore, many results obtained in other regions can be useful, and are sometimes used directly for the spectral apparatus. This applies particularly to the introduction into the theory of real apparatus the so-called spectral approach or the method of Fourier analysis, which has, as is well-known, a very general significance in the theory of linear electrical systems and in many other cases.

The method of quantitative description of the distorting action of a real spectral instrument set forth above is not the only one possible. Actually, we can construct the observed distribution by the following, more general method. An arbitrary true distribution $\varphi(x)$ is expanded into a sum of certain standard distributions; the distortion of the standard distribution by the real apparatus is taken into account and then the distorted standard distributions are combined forming the observed distribution $f(x)$. In the mathematical technique which was explained above, we used monochromatic radiation as a standard distribution; the real apparatus corresponding to this was described by the apparatus function. Such a form of the standard distribution has a very clear physical meaning and historically was first used for the analysis of the properties of real optical and spectral equipment. However, this by no means signifies that it is impossible to make effective use of other standard distributions. For example, in the theory of linear electrical systems, the step and harmonic distributions are also employed.

The possibility and usefulness of the spectral approach for real optical apparatus, i.e., the use in this case of the harmonic standard distribution was first explicitly shown by Abbe in his theory of the formation of the image of non-luminous objects (see, for example, reference 14). Later, L. I. Mandel'shtam[15,16] and Rayleigh[17] extended the concept of Abbe to the case of partially coherent and luminous objects. Another early application of the spectral approach in optical measurements was associated with the investigation of the hyperfine structure of spectral lines, carried out by Michelson on his double-beam interferometer. Evidently, he was also the first to construct the harmonic analyzer for the analysis of spectroscopic data.[9] However, widespread recognition of the spectral approach in the region of interest to us has come about only during the last 15 – 20 years in connection with the development of information theory and the application of its results and of the mathematical methods for spectral apparatus (see, for example, references 18 – 22).

The fruitfulness of the spectral approach results from the following significant property of the harmonic distribution of the intensity, a property established by L. I. Mandel'shtam.[16] If the true distribution has the form

$$\varphi(x) = \cos(\omega x + \beta), \tag{9}$$

then the observed distribution will also be harmonic with the same frequency ω but, generally speaking, with a different amplitude and phase.[16] Actually, substitution of (9) in (1) quickly demonstrates that

$$f(x) = A\cos(\omega x + \beta'), \tag{10}$$

where the quantities A and β' depend on ω and on the form of the apparatus function. It will be shown below that this is correct in the general case of an arbitrary real apparatus. The inverse is also substantiated, that is, only a harmonic distribution preserves its form under the distorting action of real apparatus.

In connection with the general method of the construction of the observed distribution, let us represent the true distribution in the form of a superposition of harmonic distributions, i.e., in the form of a Fourier integral:[23]

$$\varphi(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi(\omega)\, e^{ix\omega}\, d\omega. \tag{11}$$

Introducing this in (1), and changing the order of integration, we obtain:

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} A(\omega)\,\Phi(\omega)\, e^{ix\omega}\, d\omega, \tag{12}$$

where

$$A(\omega) = \int_{-\infty}^{\infty} a(x)\, e^{-i\omega x}\, dx \tag{13}$$

is the Fourier transform of the apparatus function. It is easily seen from (12) that the Fourier transform of the observed distribution is

$$F(\omega) = \int_{-\infty}^{\infty} f(x)\, e^{-i\omega x}\, dx, \tag{14}$$

i.e., the amplitude of the harmonic component of the observed distribution is equal to

$$F(\omega) = A(\omega)\,\Phi(\omega). \tag{15}$$

The meaning of the transformation of Eq. (1) to Eqs. (12) and (15) is evidently as follows. Each harmonic component of the true distribution with frequency ω and amplitude $\Phi(\omega)$ is changed over by the real apparatus into the same harmonic component of the observed distribution with the same frequency but, in general, a different amplitude $F(\omega)$, which is given by Eq. (15). The sum of all the harmonics distorted by the apparatus gives the observed distribution (12).

In the case of electrical systems, a relation is valid that is analogous to (15). This relation exists between the amplitude of the harmonic signal at the input of the system $\Phi(\omega)$, and at its output $F(\omega)$. The complex function $A(\omega)$ is known as the phase-frequency characteristic of the system. In optical apparatus generally (in particular in spectral apparatus), this function does not usually have a simple physical meaning, except for the case when the apparatus function is determined only by diffraction, aberrations and slits. In this case, as can be shown, $A(\omega)$ is determined by the amplitude of the electromagnetic field in the plane of the aperture diaphragm of the system, i.e., by the first (according to Abbe) image, and ω is proportional to the coordinate in this plane.[24] In other cases it is not possible to find such a physical interpretation.

From the mathematical point of view, Eqs. (1) and (15) express a general theorem on convolution and its Fourier transform. Thus, the Fourier transform of the convolution of two functions is the product of the Fourier transforms of these functions. On the basis of the equation for the inversion of a Fourier integral, we can also substantiate the fact that the Fourier transform of the product of two functions is the convolution of the Fourier transform of these functions (see, for example, reference 23).

Besides the methods of analysis of real instruments considered above, another method exists and is rapidly being advanced. It is based on representations and the mathematical techniques of information theory (see, for example, references 19 — 22, 25 — 27). In the present review, however, this approach will not be considered.

4. As has been pointed out, the apparatus function of a real spectral instrument is determined by many and varied factors. Depending on which of these factors plays the decisive role, the apparatus function will have one or another form and will have a varying width. In the most refined instruments, the apparatus function is almost completely determined by diffraction at the aperture.[28,29] If, as is usually the case with spectral instruments, the aperture is rectangular then, as is well-known:[30]

$$a(x) = \frac{1}{s_0}\left[\frac{\sin \pi x/s_0}{\pi x/s_0}\right]^2, \tag{16}$$

where $s_0 = \lambda f/D$, λ is the wavelength, D is the width of the cross-section of the beam at the exist of the dispersing system and f is the focal length of the objective. The Fourier transform of such an apparatus function has the form

$$A(\omega) = \begin{cases} 1 - \dfrac{s_0}{2\pi}|\omega|, & |\omega| \leqslant \dfrac{2\pi}{s_0}; \\ 0 & |\omega| \geqslant \dfrac{2\pi}{s_0}. \end{cases} \tag{16a}$$

In a number of instruments there is another limiting case, namely, the apparatus function in them is determined only by the widths of the slits, and the diffraction, aberrations, etc., can be neglected. The apparatus function of such a spectrograph has the form of a rectangle

$$a(x)=\begin{cases}\dfrac{1}{s}, & |x|<\dfrac{s}{2};\\[2mm] 0 & |x|>\dfrac{s}{2},\end{cases}\tag{17}$$

and its Fourier transform is

$$A(\omega)=\frac{\sin s\omega/2}{s\omega/2}.\tag{17a}$$

The apparatus function of a monochromator, taking into account only slit distortions, is the faltung of two rectangular functions:[31]

$$a(x)=\begin{cases}\dfrac{1}{s_1}, & s_1>s_2,\ |x|\leqslant\dfrac{s_1-s_2}{2};\\[2mm] \dfrac{1}{s_1s_2}\left[\dfrac{s_1+s_2}{2}-|x|\right], & \dfrac{s_1-s_2}{2}\leqslant|x|\leqslant\dfrac{s_1+s_2}{2};\\[2mm] 0 & \dfrac{s_1+s_2}{2}\leqslant|x|,\end{cases}\tag{18}$$

where s_2 is the exit width and s_1 is the width of the geometric image of the initial slit. In particular, for the most important case in practice, $s_1 = s_2 = s$, Eq. (18) takes the form

$$a(x)=\begin{cases}\dfrac{1}{s}\left[1-\dfrac{|x|}{s}\right], & |x|\leqslant s;\\[2mm] 0, & |x|\geqslant s,\end{cases}\tag{19}$$

that is, $a(x)$ has a triangular shape. The Fourier transform of the functions determined by Eqs. (18) and (19) can be written without carryout out direct computations but on the basis of the theorem of the Fourier transform of the convolution of two functions mentioned above. Inasmuch as (18) is the convolution of two rectangular apparatus functions, then its Fourier transform is the product of two functions of the form of (17a):

$$A(\omega)=\frac{\sin s_1\omega/2}{s_1\omega/2}\cdot\frac{\sin s_2\omega/2}{s_2\omega/2}.\tag{18a}$$

For the triangular apparatus function (19) we have

$$A(\omega)=\left[\frac{\sin s\omega/2}{s\omega/2}\right]^2.\tag{19a}$$

Equations (16) — (19) rarely correspond to reality with a sufficient degree of accuracy and, beginning with the researches of Schuster,[32] many attempts were made at a simultaneous account of slit and diffraction distortions. If we consider the slit to be self-luminous and assume, consequently, that the light waves which form the diffraction image of an infinitely narrow element of the slit do not interfere with the waves reaching the image from neighboring elements, then the apparatus function is the convolution of the diffraction and slit apparatus functions. For a spectrograph, for example, we have

$$\left.\begin{aligned}a(x)&=\frac{1}{ss_0}\int_{-s/2}^{s/2}\left\{\frac{\sin[\pi(x-y)/s_0]}{\pi(x-y)/s_0}\right\}^2dy;\\ A(\omega)&=\begin{cases}\dfrac{\sin s\omega/2}{s\omega/2}\left[1-\dfrac{s_0}{2\pi}|\omega|\right], & |\omega|\leqslant\dfrac{2\pi}{s_0},\\[2mm] 0 & |\omega|\geqslant\dfrac{2\pi}{s}.\end{cases}\end{aligned}\right\}\tag{20}$$

The complicated character of these equations makes difficult their practical use and forces us to look for an approximate representation of the apparatus function with the help of elementary functions. For the case $s \gg s_0$, such approximate expressions have been found both for the spectrograph[33,34] and for the monochromator.[35] Account of the partial coherence of illumination of the slit in cases of practical interest leads only to a comparatively small correction to the width of the apparatus function and scarcely changes its form at all.[24,36,37]

The situation is complicated somewhat by the presence of any appreciable aberrations. The few calculations made in recent times[24,38,39] do not permit us to draw any universal conclusions which are valid for a wide class of instruments. Therefore, one usually makes use of an experimentally measured apparatus function which is given either graphically or in tabular form, or of approximate analytic expressions of suitably simple form. In many cases, when the diffraction and aberration distortions are not very large, the Gaussian curve is a suitable approximation for the monochromator:[24,40]

$$\begin{gathered}a(x)=\frac{2\sqrt{\ln 2}}{\sqrt{\pi}\,\alpha}\exp\left\{-\frac{4\ln 2}{\alpha^2}x^2\right\};\\ A(\omega)=\exp\left\{-\frac{\omega^2\alpha^2}{16\ln 2}\right\}.\end{gathered}\tag{21}$$

For spectrographs of small, medium, and sometimes also of large dispersion, one uses the following approximate representations:[41,42]

$$a(x)=\frac{\alpha/2\pi}{x^2+(\alpha/2)^2};\quad A(\omega)=\exp\left\{-\frac{\alpha}{2}|\omega|\right\}.\tag{22}$$

The possibility of such an approximation is connected with the fact that in the spectrographs mentioned the apparatus function is determined to a significant degree by the scattering of light in the emulsion. The apparatus function of the emulsion is, according to references 43 — 45, well described by the experimental curve

$$\begin{gathered}A(\omega)=\frac{1}{1+\left[\dfrac{\alpha\omega}{2\ln 2}\right]^2},\\ a(x)=\frac{\ln 2}{\alpha}\exp\left\{-\frac{2\ln 2}{\alpha}|x|\right\},\end{gathered}\tag{23}$$

which in its "wings" is comparatively close to the curve of the dispersion form with the same width α.

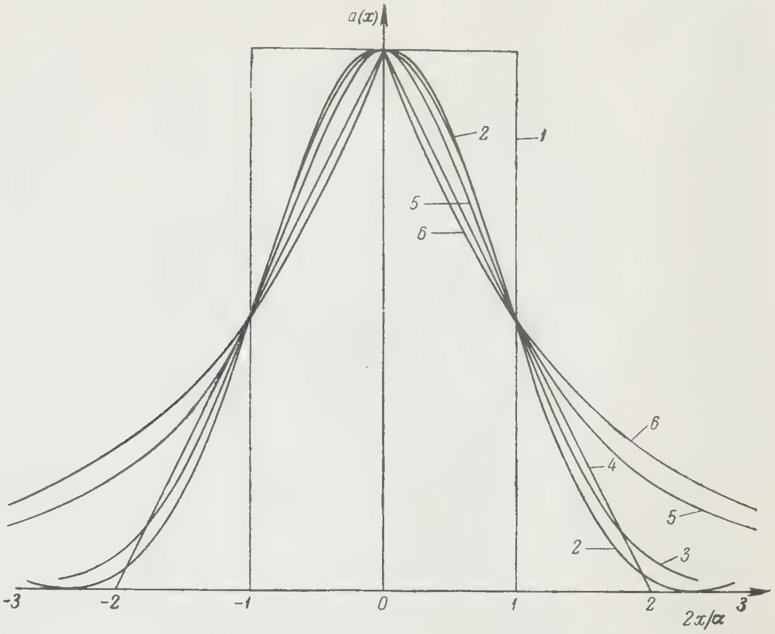

FIG. 2. Apparatus functions for various forms of width α: 1) slit shaped (17); 2) diffraction (16); 3) Gaussian (21); 4) triangular (19); 5) dispersion (22); 6) exponential (23).

Recently, a more complicated approximate apparatus function has been employed in a number of works.[46–49] This function is obtained with the help of the Voigt function, which is the convolution of the dispersion and Gaussian functions

$$\left.\begin{aligned} a(x) &= \frac{\sqrt{\ln 2}}{\pi^{3/2}}\frac{\alpha_1}{\alpha_2}\int\limits_{-\infty}^{\infty}\frac{\exp\left\{-\frac{4\ln 2}{\alpha_2^2}y^2\right\}}{(x-y)^2+(\alpha_1/2)^2}\,dy;\\ A(\omega) &= \exp\left\{-\frac{\alpha_1}{2}|\omega|-\frac{\alpha_2^2\omega^2}{16\ln 2}\right\}.\end{aligned}\right\}\qquad(24)$$

Graphs are shown in Fig. 2 for the functions under consideration. Their Fourier transforms are shown in Fig. 3. For ease in comparison, all the apparatus functions are given for a single width and for the same value at the maximum.

In the investigation of spectra of Rayleigh and Raman scattering, additional distortions are connected with the non-monochromatic character of the resultant radiation. The observed shape of the line is represented in the form of the convolution of the true line shape and of the observed line shape of the scattering, which is obtained for monochromatic excitation.[50,33] In a number of cases the total apparatus function, which takes into account the distortions just mentioned, virtually coincides with the observed shape of the excitation line.[51,41]

Apparatus functions of interference spectroscopes (the Fabry-Perot etalon, the Lummer-Gehreke plate, the Michelson echelon) are widely known (see, for example, references 2, 52, 53), and we shall not give their expressions here. We

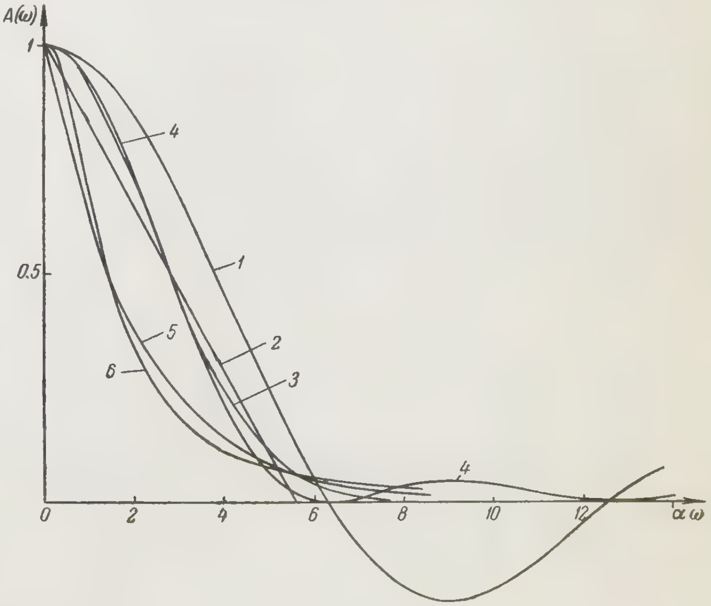

FIG. 3. Fourier transforms of apparatus functions. The number of each curve corresponds to that of Fig. 2.

note only that, in view of the relatively small number of interfering rays in these instruments, we are sometimes obliged to take into consideration the periodic character of their apparatus functions, i.e., to consider the superposition of the interference bands of adjacent orders.

The apparatus function of a recording instrument which is governed by its intertia and the distortions connected with it were considered in references 6, 54 — 58. We shall not dwell on these papers, inasmuch as the fundamental results contained in them were set forth in the review of reference 4. Moreover, as will be seen from what follows (see Sec. 3), the recording conditions must be so chosen that these distortions are small in comparison with the distortions brought about by the apparatus function of the optical part of the instrument, and in comparison with the random errors of measurement. Therefòre, we shall only give a few results obtained in reference 58 after reference 4 was published.

A recording instrument equivalent to an RC circuit is described, as is well-known, by the following apparatus function and its Fourier transform:

$$\left.\begin{aligned} a(x) &= \begin{cases} 0, & x<0; \\ \frac{1}{v\tau} e^{-x/v\tau}, & x>0; \end{cases} \\ A(\omega) &= \frac{1}{1+iv\tau\omega} = \frac{e^{-i\tan^{-1}v\tau\omega}}{\sqrt{1+(v\tau\omega)^2}}, \end{aligned}\right\} \qquad (25)$$

where τ is a time constant and v is the speed of scanning. For sufficiently slow scanning such an apparatus function reduces to the displacement of a spectrum of arbitrary shape by a quantity equal to

$$\Delta x = v\tau. \qquad (26)$$

The condition of sufficient slowness of scanning for the case, for example, of a single line means that $v\tau \ll \delta$, where δ is the observed line width. If we take into account the shift of the spectrum just noted, then in slow scanning the relative error of recording the illumination at an arbitrary point in the curve is given by the expression

$$p = k(v\tau)^2, \qquad (26a)$$

where the coefficient k is determined by the second derivative of the observed distribution at the point in which we are interested. For example, for the maximum of a single line of Gaussian shape, $k = 4 \ln 2/\delta^2$. On the basis of Eqs. (26) and (26a), we can choose the proper conditions for scanning the spectrum (see Sec. 3).

5. We now turn to a consideration of the distorting factors of the third kind, i.e., factors which lead to the appearance of random errors of measurement. A formal generalization of the basic relation (1) of the theory of real instruments, which takes into account the presence of random errors of measurement, evidently consists in the addition to the right side of Eq. (1) of some random function which we shall designate by $\xi(x)$:[24]

$$f(x) = \int_{-\infty}^{\infty} a(x-x')\varphi(x')\,dx' + \xi(x). \qquad (27)$$

Just as the value of the apparatus function a(x) was necessary for the description of the total effect of the distorting factors of the second group, so must we, for the description of random errors, give statistical mean characteristics of the function $\xi(x)$, which naturally depend on the origin of $\xi(x)$, that is, on those conditions that led to the random errors.

We shall not undertake to set forth the well-known results of the theory of errors nor give any exhaustive analysis of errors for spectroscopic measurements. This is partially connected with the fact that these questions have been exhaustively treated in the literature (see, for example, references 1, 8, 59 — 66). Another basic reason is the circumstance that at the present time analysis of Eq. (27) i.e., the simultaneous consideration of both systematic and random errors, has been carried out in the literature only for a single type of function $\xi(x)$, namely, for the case in which the mean characteristics of $\xi(x)$ do not depend on f(x). In particular, such a situation is realized in contemporary infrared spectrophotometers, where the random errors are almost entirely determined by fluctuation processes in the radiation detector (Johnson noise of the voltmeter, photo-resistance, etc.). In photographic recording, the random errors are obviously proportional to the intensity on the photographic plate,[1] and the theory outlined below is not applicable to this case.

Under the limitations outlined above, the mean values of the errors is equal to zero, so that (6) and (8) are left unchanged by normalization. The random function $\xi(x)$ can be given, with completeness that is sufficient for our purposes, by the correlation function:[8]

$$\psi(|x-x'|) = \overline{\xi(x)\xi(x')}, \qquad (28)$$

which characterizes the interdependence of random errors in different parts of the spectrogram. For $x = x'$, the correlation function is clearly equal to the mean square error

$$\psi(0)=\overline{\xi^2}=\overline{\Delta f^2}. \quad (29)$$

If the electrical inertia of the radiation detector is much less than the inertia of the recording instrument, which is usually the case, then, as can be shown,[8] the correlation function $\psi(x)$ is equal to

$$\psi(z)=c\int_{-\infty}^{\infty} a_2(x)\, a_2(z+x)\, dx, \quad (30)$$

That is, it is expressed by the apparatus function of the recording instrument. The coefficient c is proportional to the mean-square error of recording and can be determined from the condition (29). We recall that the function $f(x)$ satisfies the normalization condition (8) and, consequently, for the transition from $\xi(x)$ to fluctuations in the recording itself, it is necessary to multiply $\xi(x)$ by the integral of the directly recorded observed distribution, and to multiply the mean square combination of the type (28) by the square of this integral.

We can see from Eq. (30) that the correlation function is essentially different from zero in an interval of the order of the time constant of the recording equipment. This means that the random errors are essentially dependent only at points which are distant from each other by intervals of the order of the time constant, which is physically evident. At points at greater distances from each other, the errors are virtually statistically independent.

In spectral language, the description of random errors is given by the Fourier transform of the correlation function

$$\Psi(\omega)=\int_{-\infty}^{\infty} \psi(z)\, e^{-i\omega z}\, dz, \quad (31)$$

usually called the mean spectral density of the intensity of the fluctuations.[8] From the formula for the inversion of the Fourier integral, we have

$$\psi(z)=\frac{1}{2\pi}\int_{-\infty}^{\infty} \Psi(\omega)\, e^{iz\omega}\, d\omega. \quad (32)$$

Setting z equal to 0 in this equation, and taking (29) into account, we obtain:

$$\psi(0)=\overline{\xi^2}=\frac{1}{2\pi}\int_{-\infty}^{\infty} \Psi(\omega)\, d\omega. \quad (33)$$

This equation has the following interpretation. In a recording of the energy distribution over the spectrum, in addition to the "useful signal" [the first term in (27)] with the spectrum $A(\omega)\Phi(\omega)$, there are present fluctuations, the random errors $\xi(x)$, which can be represented in the form of a set of random harmonics, the mean power of which referred to a unit interval of frequency is $\psi(\omega)$. Consequently, we can consider $\psi(\omega)\,d\omega$ as the mean fluctuation power which takes place in an infinitely narrow band of frequencies $d\omega$. The mean square error of the recording is the sum of all these harmonic fluctuations.

We shall now give the expression $\psi(\omega)$ in terms of the Fourier transform $A_2(\omega)$ of the apparatus function of the recording instrument, i.e., in terms of its frequency characteristic. Carrying out the Fourier transformation on the left and right hand sides of Eq. (30), we obtain the result that

$$\Psi(\omega)=c\,|A_2(\omega)|^2. \quad (34)$$

This equation, as also Eq. (30), is valid only when the inertia of the recording instrument far exceeds the electrical inertia of the radiation detector. Such is usually the case for spectroscopic measurements.

In the opposite case, it is necessary to multiply the right hand side of (34) by the spectral density of the fluctuation power in the radiation detector. By virtue of the normalization (8), we have $A_2(0) = 1$. Consequently, c is nothing else than the spectral density of the intensity of the fluctuations at $\omega = 0$:

$$c=\Psi(0). \quad (35)$$

2. REDUCTION TO AN IDEAL INSTRUMENT

1. In the analysis of the general equation (27) it is appropriate to consider the limiting case of absolutely accurate measurements, which are described by Eq. (1). The present section is devoted to precisely this case, while Sec. 3 will contain the consequences which follow from the presence of random errors.

Depending on circumstances, various problems connected with Eq. (1) will be of interest, and in correspondence with the physical arrangement of the problem there will be different mathematical problems. Historically, the first statement of the problem was the following: if the true distribution consists of two monochromatic lines which are separated by a distance d, i.e.,

$$\varphi(x)=\frac{1}{2}\left[\delta\left(x-x_0-\frac{d}{2}\right)+\delta\left(x-x_0+\frac{d}{2}\right)\right],$$

then for what value of d at the point $x = x_0$ will there be a minimum of intensity with a definite amount of contrast? This problem is directly connected with the resolution problem or the problem of the determination of the minimum resolved interval of wavelengths, which will be considered in Sec. 4, and is a special case of the more general

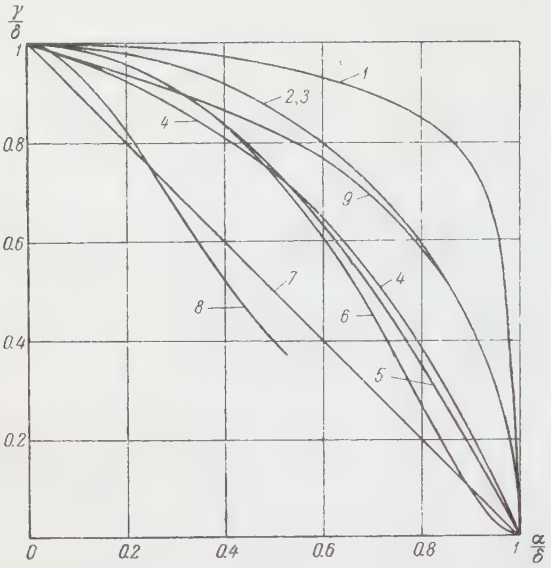

FIG. 4. Relation between the width δ of the convolution f(x) of the functions $\varphi(x)$ and a(x) with the widths γ and α for various forms of and a(x): 1) rectangular (α) and diffraction (γ); 2) slit shaped (α) and dispersion (γ); 3) Gaussian (α) and Gaussian (γ); 4) diffraction (α) and dispersion (γ); 5) Gaussian (α) and dispersion (γ); 6) triangular (α) and dispersion (γ); 7) dispersion (α) and dispersion (γ); 8) exponential (α) and dispersion (γ); 9) Gaussian (α) and triangular (γ).

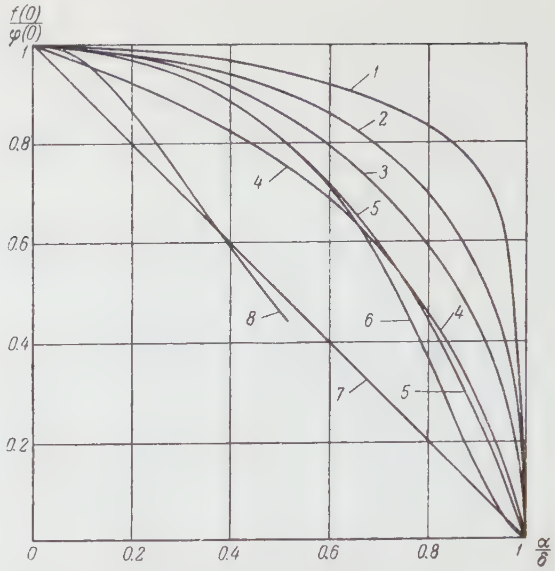

FIG. 5. Maximum value of the convolution f(0) of the functions $\varphi(x)$ and a(x) in dependence on the ratio of the widths $\alpha(\delta)$ of the functions a(x) and f(x). The identification of the curves follows that of Fig. 4.

problem of the calculation of the observed distribution f(x) from the given true distribution $\varphi(x)$ and the apparatus function a(x). In spite of the essential simplicity of the mathematical solution of the problem, which reduces to the calculation of the integral in (1), such calculations rarely lead to simple expressions f(x) in terms of elementary functions. In the majority of cases, one has to be satisfied with graphical or numerical results. Nonetheless, such material is useful in practical research. Therefore, we have plotted in Fig. 4 graphs giving the relation between the widths of the true and observed distributions for various forms of $\varphi(x)$ and a(x). In Fig. 5 there are given the dependences of the intensity at the maximum of the line. These parameters are the most used for all sorts of estimates.

The problem of establishing such distributions $\varphi(x)$, which are transmitted with a real instrument without distortion or in the form of an identical distribution, i.e., distributions for which the following relation holds, is of the utmost importance:

$$f(x)=\lambda\varphi(x). \tag{36}$$

The solution of this problem, which was considered by L. I. Mandel'shtam,[16] consists in the fact that only a sinusoidal distribution satisfies the condition (36) in the general case. Actually the true distribution will not be distorted by a real instrument only in that case in which all its harmonic components will be diminished in amplitude by the same amount, that is, if

$$A(\omega)=\text{const.} \tag{37}$$

The analysis carried in Sec. 1 showed that the Fourier transform of the apparatus functions of real spectral instruments do not satisfy this condition and therefore we can only talk about a sufficiently small difference of f(x) and $\varphi(x)$. However, we can establish a criterion of smallness only upon consideration of random errors of measurement.*

The most important practical problem of the theory is the problem of the reduction to an ideal instrument. By reduction to an ideal instrument we mean the finding of the true distribution for a given observed distribution f(x) and apparatus function a(x).† The particular problem of reduction will be treated in Secs. 2 and 3.

2. As is seen from (1), for absolutely accurate measurements, the reduction leads to a solution of the linear integral equation of first order with a different kernel a(x − y) and the observed distribution f(x) on the left hand side of the equa-

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*For certain special forms of $\varphi(x)$ and a(x), the condition (36) will be satisfied (see reference 16 and reference 23, pages 389 and 441, respectively). However, such apparatus functions are not possible in spectral instruments.

†Sometimes other terminology is used, for example, "correction of experimental results for apparatus distortion," "elimination of the apparatus function," etc.

tion. Many different methods of solution in this equation are described in the literature; these will be described below. Here, for a clarification of the connection between the various methods, their advantages and disadvantages, it is appropriate to adopt the point of view of the spectral approach.

A general solution of the problem of reduction from the point of view of the spectral approach consists in the following: each harmonic component of the observed distribution is reduced, that is, $F(\omega)$ is multiplied by $1/A(\omega)$, and all the reduced harmonics are summed. Formally, this reduces to a solution of Eq. (15) relative to $\Phi(\omega)$, and the substitution of the resultant expression in (11):[23]

$$\varphi(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{F(\omega)}{A(\omega)}e^{ix\omega}\,d\omega. \tag{38}$$

Equation (38) gives the general solution of Eq. (16) and, at the same time, the general solution of the problem of reduction for absolutely accurate measurements. However, in recent times, Eq. (38) has rarely been used for direct treatment of experimental data. This is explained by the fact that the spectral instrument does not measure $F(\omega)$ but rather the energy distribution $f(x)$,* and direct use of (38) requires rather difficult preliminary conversion from $f(x)$ to $F(\omega)$. However, with the development of machine-computation technology, this process will not be difficult and perhaps the reduction will be carried out immediately from (38). At the present time in most cases we attempt to select for $F(\omega)$ and $A(\omega)$ simple analytic expressions for which the integral in (38) is computed in explicit form, or so to transform (38) that it would be possible to carry out direct operations on $f(x)$ leading to $\varphi(x)$. In connection with this, the different methods of reduction found in the literature lead to the following classification: (1) analytical methods that start out from a given or an approximate $F(\omega)$ and $A(\omega)$ in terms of simple functions which make possible the integration in (38) in explicit form; (2) methods that reduce the calculation of (38) to a succession of such operations on $f(x)$ (differentiation, integration, etc.), independent of the form $f(x)$; usually these methods are graphical or numerical; (3) finally, there are mixed methods; thus there is, for example, a variant of the method of successive approximations, in which the zeroth approximation is the solution obtained by an analytic method and the subsequent approximations are graphical or numerical.

First we shall consider analytic methods. Approximations of $F(\omega)$ and $A(\omega)$ by expressions of the form

$$\exp\{-\beta_1|\omega|-\beta_2\omega^2\}. \tag{39}$$

have received widespread use. For example, if

$$F(\omega)=\exp\{-\delta|\omega|\};\quad A(\omega)=\exp\{-\alpha|\omega|\}, \tag{40}$$

which corresponds to a description of the observed distribution and the apparatus function in terms of the dispersion curves of the form

$$f(x)=\frac{\delta/\pi}{x^2+\delta^2};\quad a(x)=\frac{\alpha/\pi}{x^2+\alpha^2}, \tag{41}$$

then

$$\left.\begin{aligned}&\Phi(\omega)=\exp\{-(\delta-\alpha)|\omega|\};\\&\varphi(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{-(\delta-\alpha)|\omega|+ix\omega}\,d\omega=\frac{(\delta-\alpha)\pi}{x^2+(\delta-\alpha)^2},\end{aligned}\right\} \tag{42}$$

that is, the true distribution also has a dispersion form in which its width 2γ is equal to the difference in width of the observed distribution and the apparatus function $2\gamma=2(\delta-\alpha)$. The approximations of (40) or (41) are often applied to the investigation of lines of emission spectra of atoms (see, for example, reference 70), Rayleigh lines and Raman scattering of light,[33,41,51,71,72] in x-rays,[10,73,74] astrophysics,[11,75] and in other cases. In these regions, and also in infrared spectroscopy there is application of another special case of Eq. (39):[4,11,12,33,40,75,76]

$$F(\omega)=\exp\{-\delta^2\omega^2/4\};\quad A(\omega)=\exp\{-\alpha^2\omega^2/4\}, \tag{42'}$$

which corresponds to the approximation of the observed distribution and apparatus function by Gaussian curves

$$f(x)=\frac{1}{\sqrt{\pi}\,\delta}\exp\{-x^2/\delta^2\};\ d(x)=\frac{1}{\sqrt{\pi}\,\alpha}\exp\{-x^2/\alpha^2\}. \tag{43}$$

In this case the true distribution also has a Gaussian form

$$\Phi(\omega)=\exp\{-(\delta^2-\alpha^2)\omega^2/4\};$$

$$\varphi(x)=\frac{1}{\sqrt{\pi(\delta^2-\alpha^2)}}\exp\{-x^2/(\delta^2-\alpha^2)\}, \tag{44}$$

and its width is equal to

$$2\gamma=2\sqrt{\delta^2-\alpha^2}. \tag{45}$$

Use of the complete expression (39), that is, the approximation

$$F(\omega)=\exp\{-\delta_1|\omega|-\delta_2^2\omega^2/4\};$$

$$A(\omega)=\exp\{-\alpha_1|\omega|-\alpha_2^2\omega^2/4\}, \tag{46}$$

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*An exception in this relation is the determination of the structure of a line by means of a double beam interferometer,[9] where $|F(\omega)|$ coincides with the "visibility curve," and the treatment of the results consists of the harmonic analysis of the "visibility curve." It should be noted that in references 67, 68 there are applied several improvements of the method, which permit us to avoid the well-known indeterminacy which takes place in the experiments of Michelson.[69]

leads to a similar form for $\Phi(\omega)$ and a representation of the true distribution $\varphi(x)$ by means of the Voigt function

$$\left.\begin{aligned}\Phi(\omega)&=\exp\{-(\delta_1-\alpha_1)|\omega|-(\delta_2^2-\alpha_2^2)\omega^2/4\};\\ \varphi(x)&=\frac{\delta_1-\alpha_1}{\pi\sqrt{\pi(\delta_2^2-\alpha_2^2)}}\int_{-\infty}^{\infty}\frac{\exp\{-y^2/(\delta_2^2-\alpha_2^2)\}}{(x-y)^2+(\delta_1-\alpha_1)^2}\,dy.\end{aligned}\right\}\tag{47}$$

In spite of the rather rough expression for $\varphi(x)$, treatment of the experimental data by this method is comparatively simple, thanks to the excellent tables of the Voigt function in reference 77. In references 12, 47 – 49, and 75 methods are worked out in detail for the determination of the parameters α_1, α_2, δ_1, δ_2 from observed quantities; these parameters are necessary for the calculation of $\varphi(x)$ by (47).

A characteristic feature of the foregoing methods and of other methods (see references 51, 74, 78, 79) is the description of the initial curves by relatively simple expressions that depend on one or two parameters, wherein, in the introduction of the second parameter, the calculations as a rule become significantly more complicated. It must be assumed that introduction of still another parameter would not be appropriate. Moreover, in many cases the approximations considered are inadequate. In connection with this, more flexible methods have been developed, directly connected with the specific form of the observed distribution.

3. Consideration of approximate universal methods of reduction is properly begun with such a transformation of Eq. (38) that we isolate that part which is connected with a non-ideal instrument. The apparatus function of an ideal instrument is a δ-function, while its Fourier transform is equal to unity:

$$\int_{-\infty}^{\infty}\delta(x)\,e^{-i\omega x}\,dx=1.\tag{48}$$

Therefore, it is natural to add and subtract unity to $A^{-1}(\omega)$ under the integral sign in (38); we then have

$$\begin{aligned}\varphi(x)&=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)[A^{-1}(\omega)-1]\,e^{ix\omega}\,d\omega+\frac{1}{2\pi}\int_{\infty}^{\infty}F(\omega)\,e^{ix\omega}\,d\omega\\ &=f(x)+\frac{1}{2\pi}\int_{-\infty}^{\infty}[A^{-1}(\omega)-1]\,F(\omega)\,e^{ix\omega}\,d\omega.\end{aligned}\tag{49}$$

The second term in this equation is often a relatively small correction which vanishes in an ideal instrument, and the major part of approximating methods reduced to some particular variation of calculation of this term.

It should be emphasized that $A(\omega)$ is characteristic for the function of the given instrument, while both the energy distribution over the spectrum and, consequently, the function $F(\omega)$ can be extremely varied. In connection with these methods, it is natural to base the reduction only on assumptions concerning the properties of $A(\omega)$, but not of $F(\omega)$. Therefore, a majority of the approximate universal methods of solution of Eq. (1) obtained with the help of the expansion of $A^{-1}(\omega)-1$ in functional series.

Let us first consider an expansion in a power series

$$A^{-1}(\omega)-1=\sum_n b_n\omega^n.\tag{50}$$

We substitute this expression in (38) and change the order of integration and summation:

$$\varphi(x)=f(x)+\sum_n b_n\frac{1}{2\pi}\int_{-\infty}^{\infty}\omega^n F(\omega)\,e^{ix\omega}\,d\omega.$$

We can compute the resultant integrals by differentiating $f(x)$ n times:

$$\frac{d^nf}{dx^n}=\frac{d^n}{dx^n}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)\,e^{ix\omega}\,d\omega\right\}=\frac{i^n}{2\pi}\int_{-\infty}^{\infty}\omega^n F(\omega)\,e^{ix\omega}\,d\omega.$$

Consequently,

$$\varphi(x)=f(x)+\sum_n\frac{b_n}{i^n}\frac{d^nf}{dx^n}.\tag{51}$$

Thus, expansion of $A^{-1}(\omega)-1$ in a power series corresponds to the expression of $\varphi(x)$ in terms of $f(x)$ and its derivatives, while the coefficients are determined only by the apparatus function. Solution in the form (51) was obtained by Eddington,[80] Hardy and Young,[81] and other authors[49,82,83] for certain specific forms of the apparatus functions of spectral instruments. In reference 84 an attempt was made to correct the distortions introduced by the inertia of the thermal light detector with the aid of a differentiating network, which is a customary method for high-frequency correction in radio-electronics. It is easy to see that this corresponds to the calculation of the first term of Eq. (51).

Equation (51) takes an especially simple form upon elimination of the apparatus function of a photographic emulsion. According to (23), only b_2 differs from zero in this case, that is,[49]

$$\varphi(x)=f(x)-\left[\frac{a}{2\ln 2}\right]^2\frac{d^2f}{dx^2}.\tag{52}$$

Transforming this result, we can set up the problem of finding such an approximate function for the apparatus function that the expansion (50) contains two terms, i.e., that all $b_n = 0$ except, for example, b_2 and b_4. If the following expression is given for $A(\omega)$:

$$A(\omega)=[1+\alpha_1^2\omega^2]^{-1}[1+\alpha_2^2\omega^2]^{-1},\tag{53}$$

then the exact solution of Eq. (1) will have the form

$$\varphi(x) = f(x) - (\alpha_1^2 + \alpha_2^2)\frac{d^2 f}{dx^2} + \alpha_1^2 \alpha_2^2 \frac{d^4 f}{dx^4}. \qquad (54)$$

The apparatus function which corresponds to (53) is (see, for example, reference 85)

$$a(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{ix\omega}\, d\omega}{(1+\alpha_1^2\omega^2)(1+\alpha_2^2\omega^2)}$$

$$= \frac{1}{2(\alpha_1^2 - \alpha_2^2)}\left[\alpha_1 e^{-|x|/\alpha_1} - \alpha_2 e^{-|x|/\alpha_2}\right]. \qquad (55)$$

A graph of this function, in contrast to (23), has an extremum at the point x = 0, while the presence of two parameters enables us to change the form for "adjustment" in the apparatus function of real instruments within certain limits.

The insufficiency of the solution of the form (51) is that calculation of the derivatives according to the curve measured experimentally, a curve obtained with a certain error, is actually very difficult and in essence reduces to the calculation of finite differences. Below (Sec. 3), it will be seen that this difficulty is not accidental but in principle takes place in any method of reduction which can be shown to be exact for some form of the apparatus function.

A natural generalization of the derivation of Eq. (51) is the following. Let $A^{-1}(\omega) - 1$ be expanded in a certain functional series

$$A^{-1}(\omega) - 1 = \sum_n b_n u_n(\omega). \qquad (56)$$

Substituting (56) in (49), and changing the order of summation and integration, we obtain

$$\varphi(x) = f(x) + \sum_n b_n \frac{1}{2\pi}\int_{-\infty}^{\infty} u_n(\omega) F(\omega) e^{ix\omega}\, d\omega. \qquad (57)$$

In analogy to the case considered above, we can say that the multiplication of the Fourier transform $F(\omega)$ of the function $f(\omega)$ by some function $u_n(\omega)$ means a definite operation on the same function $f(x)$. If $u_n(\omega) = \omega^n$, then this operation is an n-fold differentiation; if $u_n(\omega) = \exp\{-ina\omega\}$, then the corresponding operation consists in a shift along the x axis by a distance na, etc. Of course, multiplication of $F(\omega)$ by $A^{-1}(\omega)$ can also be regarded as a certain operation. The meaning of the approximate methods is also that the operator $A^{-1}(\omega) - 1$ is replaced by a set of operators $u_n(\omega)$, which define operations that are more suitable (in some fashion or other) than the operations determined by the same operator $A^{-1}(\omega) - 1$.

We shall consider other methods of solution from this point of view. Historically, the first method to be developed was that of finite differences. This method is obtained by the following expansion:

$$A^{-1}(\omega) - 1 = \sum_n b_n \sin^n \frac{a\omega}{2}. \qquad (58)$$

Calculating the first term of a series in (57),

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \sin\frac{a\omega}{2} F(\omega) e^{ix\omega}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{2i}\left[e^{i(x+a/2)\omega} - e^{i(x-a/2)\omega}\right] F(\omega)\, d\omega$$

$$= \frac{1}{2i}[f(x+a/2) - f(x-a/2) \equiv \frac{1}{2i}\Delta_a^1 f(x),$$

we conclude that multiplication $F(\omega)$ by $2i \sin\frac{a\omega}{2}$ corresponds to the operation of taking the finite difference of f(x) over the interval a, while multiplication by $(2i)^n \sin^n \frac{a\omega}{2}$ is an n-fold application of this operation, i.e., the taking of the finite difference $\Delta_3^n f(x)$ of order n in the interval a (see, for example, reference 82). Thus we obtain the following expression for $\varphi(x)$:

$$\varphi(x) = f(x) + \sum_n \frac{b_n}{(2i)^n}\Delta_a^n f(x). \qquad (59)$$

This method was advanced by Rayleigh in 1871[84] for the reduction of observed spectra for infinitesimally narrow slits of a monochromator; there he limited himself only to the first non-vanishing correction. Thereafter, the method was developed by Paschen and Runge,[87,88] who took into consideration higher approximations. In subsequent investigations, the case was considered of a monochromator with different widths of the input and exit slits,[89] the change of the dispersion over the range of the width of the slit[90] was considered and, finally, a generalization of the method of finite differences for other "non-slit" apparatus functions was given.[91]

The method of finite differences is applied at present to a wide variety of cases. Widespread use of this method of reduction is accounted for in a significant degree by the extraordinary simplicity of the calculation of corrections with the aid of a geometrical construction also given by Rayleigh. In reference 92, a correction was carried out according to coefficients for a parabolic interpolation of the observed distribution. This method, being equivalent to the construction of Rayleigh, is significantly less useful practically and is rarely employed.

It is interesting to note that the parameter a in Eqs. (58) and (59) was never specified and can be chosen arbitrarily; in particular, for a → 0, the method of finite differences transforms into the method of Eddington. In the expressions obtained by Paschen and Runge,[87,88] the finite difference is taken over an interval equal to the width of the slit s of the monochromator, while Rayleigh used an interval equal to $s/\sqrt{2}$.[86] In Sec. 3 we shall see that this is arbitrarily removed in a reasonable manner only upon calculation of random errors of measurement. If we represent $A^{-1}(\omega) - 1$ in the form of a trigonometric series

$$A^{-1}(\omega) - 1 = \sum_n b_n e^{-ian\omega}, \tag{60}$$

then the solution of Eq. (1) has the following form:

$$\varphi(x) = f(x) + \sum_n b_n f(x - an), \tag{61}$$

inasmuch as $u_n(\omega) = \exp\{-i\alpha n\omega\}$ is the "displacement operator." Thus it is easy to see that the latter two methods are quite related, since we can express $\sin^n \frac{a\omega}{2}$ in terms of multiple harmonics, and go from (58) to (60). From the practical point of view, these methods are different, since they define different operations on the real distribution $f(x)$ in the calculation of a finite number of terms: in (59) we shall limit ourselves to the finite difference of a particular order, say the second, while in (61) we shall limit ourselves to values of the functions at points that are a sufficiently large distance from the point x, for example, $\pm 2a$.

We can achieve the solution in the form (61) by different means. In many researches where such a form has been advanced, the author started out from the approximate substitution of the integral equation (1) by a system of linear algebraic equations with the aid of various formulas of approximate integration; then this system was solved in some fashion or other.[93-95] In a number of papers use has been made of trigonometric expansions for solution of Eq. (1). Such an expression can be obtained if we write

$$A^{-1}(\omega) - 1 = \sum_n b_n \delta\left(\omega - \frac{n}{a}\right). \tag{62}$$

then, obviously,

$$\varphi(x) = f(x) + \frac{1}{2\pi} \sum_n b_n F\left(\frac{n}{a}\right) e^{i\frac{n}{a}x}, \tag{63}$$

that is, the expansion (62) corresponds to replacing the integral (49) by a series or, in practice, by a sum of a finite number of terms. This method is usually applied in x-ray research,[96,97] and, as is pointed out in reference 98, it can be suitable for the investigation of spectral lines by the Fabry-Perot etalon (by virtue of the periodicity of its apparatus function.)

We shall now consider the method of successive approximations put forth by Berger and van Cittert,[99,100] and frequently applied in the reduction to an ideal instrument in various cases.[75,82,92,98] This method corresponds to an expansion of $A^{-1}(\omega) - 1$, in a geometric progression:[82,93,101]

$$\frac{1}{A(\omega)} - 1 = \frac{1 - A(\omega)}{1 - [1 - A(\omega)]} = \sum_{n=1}^{\infty} [1 - A(\omega)]^n. \tag{64}$$

In contrast to the cases considered previously, the operator is an integral operator. Actually, in accordance with the theorem given above on the representation of a function whose Fourier transform is equal to the derivative of the transform of two other functions, we have, calculating for example the first term of the series in (64),

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} [1 - A(\omega)] F(\omega) e^{ix\omega} d\omega = f(x) - \int_{-\infty}^{\infty} a(x - y) f(y)\, dy.$$

If we can consider the n first terms of the series (52), then we can represent the solution in the form

$$\varphi(x) = f(x) + \{n f(x) - \int_{-\infty}^{\infty} U_n(x - y) f(y)\, dy\}, \tag{65}$$

where the resolvent $U_n(x)$ is expressed in the following fashion:

$$\left.\begin{aligned} U_n(x) &= \frac{(n+1)n}{2!} a(x) - \frac{(n+1)n(n-1)}{3!} a_1(x) + \ldots + (-1)^n a_{n-1}(x); \\ a_1(x) &= \int_{-\infty}^{\infty} a(x - y) a(y)\, dy; \quad a_k(x) = \int_{-\infty}^{\infty} a(x - y) a_{k-1}(y)\, dy. \end{aligned}\right\} \tag{66}$$

For each particular instrument the resolvent can be calculated beforehand and the reduction leads to calculation of the integral in (65). Special integrators[99,102] have been prepared for computation of corrections by the method of successive approximations. These permitted rapid computation of integrals of the type (66).

In the practical employment of the method of successive approximations, the following notes can be useful. Under real conditions, the width of the slit of a spectral apparatus is frequently changed in correspondence with the widths of the lines and bands investigated, and for other reasons. In order that it not be necessary to recompute the resolvent for each value of the slit width, successive elimination of slit and non-slit distortions is carried out. In many cases, we can consider the former in terms of the simpler method of finite differences of (59). A second remark is connected with the fact that the roughness with which $f(x)$ and $a(x)$ are known gets worse in the higher approximations, i.e., for large n. Therefore, as the authors of the method point out, the solution will be satisfactory if we can limit ourselves to the first approximation.

In addition to the methods discussed above, there is a series of other methods of solution of Eq. (1), which, however, have not found application in spectroscopy. There are methods consisting of the simultaneous application of the operators of displacement and differentiation,[82] and also a method (which is interesting from the mathematical point of view) of expansion into the orthogonal polynomials of Hermite and Bernoulli.[103]

We pause on the problem of the choice of zeroth approximation. As was discussed above, in many cases we have unity for the zeroth approximation in the expansion of $A^{-1}(\omega)$; in this connection, the zeroth approximation for the solution of Eq. (1) is the observed distribution $f(x)$. The rational form of the expansion of $A^{-1}(\omega)$ is fundamentally determined by the properties of the apparatus function. In a number of cases, however, when the higher approximations have large values, it can be shown to be appropriate to take into consideration the character of the observed distribution. This can be done in the following way. Let us represent

$$f(x) = f_0(x) + f_1(x); \quad a(x) = a_0(x) + a_1(x), \tag{67}$$

where $f_0(x)$ and $a_0(x)$ are certain functions, given in analytical form, for which the solution $\varphi_0(x)$ of Eq. (1) is known, and which satisfactorily reproduce the curve of the functions $f(x)$ and $a(x)$, respectively, so that we can consider $a_1(x)$ and $f_1(x)$ to be small. For example, for $f_0(x)$ and $a_0(x)$, we can take the Gaussian or the dispersion curve. We get the function $\varphi_0(x)$ as a zeroth approximation. Substituting (67) in (1), we get the following equation

$$f_1(x) - \int_{-\infty}^{\infty} a_1(x-y)\,\varphi_0(y)\,dy = \int_{-\infty}^{\infty} a(x-y)\,\varphi_1(y)\,dy \tag{68}$$

relative to the function $\varphi_1(y) = \varphi(y) - \varphi_0(y)$. We can now solve this equation by the methods considered above; its solution will determine the correction to the zeroth approximation.

5. In concluding this section, let us note some special cases of reduction which are not included in the scheme set forth above. First, we have the interesting reduction method advanced in reference 104 for the case of the Fabry-Perot etalon. The apparatus function of the Fabry-Perot etalon, as was shown in reference 104, can be represented in the form of a superposition of equidistant contours of the dispersion shape with identical intensities:

$$a(x) = \frac{(1+r)/(1-r)}{1 + \frac{4r}{(1-r)^2}\sin^2 \pi x} = \sum_{-\infty}^{\infty} \frac{\alpha/\pi}{(x-m)^2+\alpha^2}: \quad \alpha = \frac{1}{2\pi}\ln\frac{1}{r}. \tag{69}$$

It then follows that for the dispersion form of the true distribution, the observed distribution is also expressed in terms of a series of dispersion curves, similar to (69), which can, after some transformations, lead to the following expression:

$$f(x) = \sum_{-\infty}^{\infty} \frac{\delta/\pi}{(x-m)^2+\delta^2} = \frac{\coth \pi\delta}{1 + \left[\frac{\sin \pi x}{\sinh \pi\delta}\right]^2}; \quad \delta = \gamma + \alpha, \tag{70}$$

where γ is the width of the true distribution. Thus, determining the parameters δ and α of the observed distribution and the apparatus function, we can compute the true line width. Equations (69) and (70) take into consideration the superposition of neighboring interference maxima and make more precise the usual method,[53] in which the apparatus function of the etalon is approximated only by a single dispersion curve. We note that the expansion (70) can also be shown to be useful in the treatment of the almost periodic structure of rotational spectra.

A second special case relates to the measurement of absorption spectra. Here Eq. (1) connects the true, $\varphi(x)$, and the observed, $f(x)$, transmission. The characteristic of the material is not the transmission but the absorption coefficient, which is determined according to Bouguer's law by the relation

$$k(x) = \frac{1}{d}\ln\frac{1}{\varphi(x)} \equiv \frac{1}{d}D(x), \tag{71}$$

where d is the thickness of the absorbing specimen and $D(x)$ is its optical density. Consequently, the true form of the band absorption, which is determined by the function $1 - \varphi(x) = 1 - e^{-dk(x)}$, will not dpend on d only for small values of $dk(x)$. In this connection, some authors have considered an equation in which the unknown function is the optical density directly, i.e., the following nonlinear integral equation:

$$D_{\text{obs}}(x) = -\ln\left\{\int_{-\infty}^{\infty} a(x-y)\,e^{-D(y)}dy\right\}, \tag{72}$$

where $D_{\text{obs}}(x) = \ln\frac{1}{f(x)}$ is the observed value of the optical density[35,105,106]. Expansion of the right hand side of the equation allows us to simplify this expression to a certain degree, representing $D_{\text{obs}}(x)$ in the form of a power series in the value of the true absorption.[35,107] For example, for the case of a single absorption line, (72) takes the following form

$$D_{\text{obs}}(x) = J_1(x)\,D_m - \beta_1(x)\,D_m^2 + \ldots, \tag{73}$$

where D_m is the true density at the maximum of the line, and the coefficients J_1, $\beta_1, \ldots$ must be considered in terms of the known apparatus function of the monochromator and the curve $k(x)$ of the true absorption coefficient, whose form is also assumed to be known. If these data change then, by numerical or graphical integration in (72) or, what is simpler, use of the series (73), we can tabulate known values of the absorption and line widths as functions of the observed function and band width and the width of the apparatus function. The results of detailed calculations of such a type were given in references 35 and 107 for the case of a triangle-shaped apparatus function and a dispersion of the form $k(x)$. In the general case, in which the form of the case of the true absorption coefficient is unknown and the problem consists in finding this coefficient, use of Eqs. (72) and (73) do not simplify but greatly complicates the calculations. Here it is more appropriate to solve Eq. (1) relative to the transmission $\varphi(x)$ by the universal methods previously considered, and to transform to the densities by taking the logarithm of the latter.

All the methods described in the present section refer to the determination of the total true distribution $\varphi(x)$. In a number of cases, especially for macroscopic measurements for analytical purposes, values of less than complete characteristics of the true spectrum are sufficient, for example the integral absorption coefficient, "effective width" of the line, intensity at the maximum of the line, etc. Not going into details, we refer only to references 1, 24, 75, 105, and 107, where the methodology is considered and such "non-total" reduction is actually carried out.

3. UNIQUENESS AND ACCURACY OF REDUCTION* TO AN IDEAL INSTRUMENT

1. The problem of the uniqueness of the reduction is fundamental in the theory of reduction. The importance of this problem is evident, inasmuch as only its complete evaluation permits us to judge objectively the experimental data and to make them the basis for any conclusions.

In the limiting case of absolutely accurate measurements, the problem of the uniqueness of the reduction reduces to a statement of the condition for the uniqueness of the solution of Eq. (1). It has been shown by Bracewell and Roberts[101] and Toraldo di Francia that Eq. (1) does not have a unique solution if the Fourier transform $A(\omega)$ of the apparatus function $a(x)$ vanishes at certain points (or in a certain region). Actually, if the function $A(\omega)$ vanishes at the points $\omega = \omega_j$, $A(\omega_j) = 0$, then $\Phi(\omega)$ is indeterminate, since Eq. (15) is satisfied not only by the function

$$\Phi_1(\omega) = F(\omega)\,A^{-1}(\omega), \quad \omega \neq \omega_j, \tag{74}$$

but also by functions of the form

$$\Phi_2(\omega) = \Phi_1(\omega) + \sum_j a_j \delta(\omega - \omega_j), \tag{75}$$

where a_j is an arbitrary number. If $A(\omega)$ vanishes in a region of frequencies $\omega_1 < \omega < \omega_2$, then

$$\left.\begin{aligned} &\Phi_2(\omega) = \Phi_1(\omega) + \sum_j a_j \delta(\omega - \omega_j) + b(\omega)\,s(\omega); \\ &s(\omega) = \begin{cases} 1, \omega_1 < \omega < \omega_2; \\ 0, \omega_1 > \omega; \omega > \omega_2, \end{cases} \end{aligned}\right\} \tag{76}$$

where $b(\omega)$ is an arbitrary function. Taking the inverse Fourier transform, we obtain the following equation $\varphi_2(\omega)$:[101]

$$\varphi_2(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_1(\omega)\,e^{ix\omega}\,d\omega + \frac{1}{2\pi} \sum_j a_j e^{i\omega j y} + \frac{1}{2\pi} \int_{\omega_1}^{\omega_2} d(\omega)\,e^{i\omega x}\,d\omega. \tag{77}$$

Thus, if the Fourier transform $\Phi(\omega)$ of the investigated distribution $\varphi(x)$ contains harmonics whose frequencies coincide with the zeroes of the Fourier transform $A(\omega)$ of the apparatus function, then these harmonics do not affect the observed distribution $f(x)$ and, consequently, can in no way be "recovered" from this distribution.

From this point of view let us consider specific expressions for the apparatus functions given in Sec. 2. If, for example, the apparatus function is determined only by the finiteness of the slit width [see (17) and (17a)], then its Fourier transform vanishes at the point

$$\omega_j = \pm \frac{2\pi}{s} j, \quad j = 1, 2, 3, \ldots$$

Consequently,

$$\varphi_2(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_1(\omega)\,e^{ix\omega}\,d\omega + \frac{1}{2\pi} \sum_{j=1}^{\infty} \left[a_j e^{i\frac{2\pi}{s}ix} + a_{-j} e^{-i\frac{2\pi}{s}jx}\right],$$

i.e., $\varphi_2(x)$ is determined with accuracy up to periodic functions whose period is equal to the slit width s and whose mean value is zero. However, this indeterminacy is not irremovable in principle. Actually, changing the width of the slit, we can change the period of the indeterminate term in $\varphi_2(x)$ and thereby determine experimentally its presence or absence. A similar situation holds for the purely slit apparatus function (18) of the monochromator.

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*We remind the reader that the term "reduce" is used here solely in the sense of "restore" or "bring back" rather than "contract" or "decrease" – Tr.

We now return to the other limiting case, when the apparatus function is determined only by diffraction. Unusually, we assume that the apparatus function for a slit-shaped aperture is given by Eq. (16), while its Fourier transform is given by (16a). Consequently, in this case,

$$\varphi_2(x) = \frac{1}{2\pi} \int_{-2\pi/s_0}^{2\pi/s_0} \Phi_1(\omega) e^{ix\omega} d\omega + \frac{1}{2\pi} \int_{-\infty}^{-2\pi/s_0} b(\omega) e^{ix\omega} d\omega + \frac{1}{2\pi} \int_{2\pi/s_0}^{\infty} b(\omega) e^{ix\omega} d\omega.$$

In connection with such a situation, the conclusion was reached in references 101 and 108 that unique reduction to an ideal instrument is impossible and, consequently, diffraction phenomena in the apparatus lead to a limitation on its possibilities as an instrument for the investigation even of absolutely precise measurements in the limiting case.

However, there is an inner contradiction in the reasoning leading to this conclusion. The fact is that Eq. (16a) for $A(\omega)$, which is obtained in the solution of the diffraction problem by the Kirchhoff method, is an approximate expression which is suitable only with a definite degree of accuracy. Consequently, in the investigation of the limiting case of the complete absence of random errors of measurement, Eq. (16a) cannot be used, and must be replaced by the exact expression. On the other hand, it was shown in reference 109 that in the strict analysis of the diffraction problem the exact Fourier transform of the apparatus function cannot vanish over the whole frequency range (see also reference 24). Thus, in contrast to the conclusions of references 101 and 108, it should be emphasized that the diffraction phenomena in a real apparatus do not lead to non-unique reduction in the case of absolutely accurate measurements.

2. However, the position is changed when we take into account random errors of measurement and, accordingly, we go from Eq. (1) to the more complete expression (27). In these cases, phenomena occur which are essentially equivalent to the non-uniqueness of the reduction. Van de Hulst noted in particular[103] that as the accuracy of the reduction method is increased, the reduced distribution ceases to be "smooth," and "peaks" and "troughs" connected with the measurement errors appear. Van de Hulst calls this phenomenon the "instability" of the solution, and emphasizes that it is inherent in the integral equation and is not due to any particular method of its solution. Unfortunately, confirmation of the instability of the solution in reference 103 is treated as an empirical fact to which no explanation or foundation is given.

The nature of the instability of the solution and the relation of this phenomenon to the uniqueness of the reduction can easily be clarified in the following manner.[24] As was observed above (Sec. 2), reduction to an ideal instrument is equivalent to multiplication of the Fourier transform of the observed distribution $F(\omega)$ by $A^{-1}(\omega)$, a quantity that is the inverse of the Fourier transform of the apparatus function. But this operation is unavoidably connected with a randomly-fluctuating component of $F(\omega)$, and each harmonic component of the noise power $\Psi(\omega)$ will be multiplied by $|A^{-1}(\omega)|^2$. Consequently, the error of the reduced distribution will be

$$\overline{\Delta\varphi^2} = \frac{1}{2\pi} \int_{\infty}^{\infty} \Psi(\omega) \, | A^{-1}(\omega) |^2 \, d\omega. \tag{78}$$

It is not difficult to see that this integral always diverges. Actually, in accord with (34), the mean spectral density of the power of the fluctuation $\Psi(\omega)$ is proportional to the square of the modulus of the frequency characteristic $A_2(\omega)$ of the recording instrument. On the other hand, this same frequency characteristic acts also on the observed distribution, i.e., $A(\omega)$ is the product of $A_2(\omega)$ by the Fourier transform of an apparatus function of optical origin [see (3)]. Denoting the latter quantity by $A_1(\omega)$, we have

$$\overline{\Delta\varphi^2} = \frac{C}{2\pi} \int_{-\infty}^{\infty} | A_1(\omega) |^{-2} \, d\omega. \tag{79}$$

Inasmuch as $|A_1(\omega)|$ tends to zero upon increase in ω, the integral diverges.

It follows directly from what has been said above that the question as to what sort of error $\overline{\Delta\varphi^2}$ is to be expected if the value of the error $\overline{\Delta f^2}$ in the observed distribution is given — a problem which is common to a whole number of physical measurements — has, in the general case of reduction to an ideal instrument, an unpleasant answer: the error can be arbitrary. This means, furthermore, that the presence of random errors leads essentially to a non-unique reduction. Elimination of this lack of uniqueness is possible only by gathering additional evidence on the investigated distribution, evidence obtained in an independent fashion.

In spectroscopic measurements (and in many other cases), in addition to the evidence obtained in experimental data in the form of recordings of the observed distribution $f(x)$, one frequently assumes additional information on $\varphi(x)$. For example, the form of the true energy distribution over the spectrum of $\varphi(x)$ may be known (single or double lines of Gaussian shapes — Doppler effect; a single line of dispersion shape — natural and

Lorentz damping, etc.). Here it remains to determine only the parameters of the true distribution (the intensity at the maximum, line width and position, distance between lines in the case of several lines, etc.,), and not the whole distribution, and we can speak about the accuracy of the measurement of these parameters in the usual sense. Mathematically, this is expressed by the fact that for a previously known form of $\varphi(x)$ we can determine the integral in (1), and the reduction will be carried not to a solution of the integral equation, but to a solution of an algebraic or transcendental equation, or of a system of equations relating the unknown parameters of the true distributions.[24,27]

In many cases we shall not assume sufficiently detailed information on the true distribution so as to impose so rigid an a priori limitation on $\varphi(x)$ as the postulation of its shape. However, it is shown that for "stabilization" of the solution, in order that the reduction be unique and that we can speak in the usual sense about the accuracy of measurement of the reduced quantities, it suffices to apply to $\varphi(x)$ significantly less stringent requirements, namely an assumption on the behavior of the Fourier transform of the true distribution for large values of ω.[24] Actually, the integral (79) diverges because of the high frequencies. Consequently, the mean error of the reduced distribution will be finite if the high-frequency harmonics of the observed distribution are not only not reduced but on the contrary are significantly suppressed in the process of reduction. But such a reduction will be approximate and, consequently, will be justified only when the Fourier transform $\Phi(\omega)$ of the true distribution does not contain high harmonics. Thus a sufficient condition of the uniqueness of the reduction is a postulate on the smallness of the amplitudes of the harmonics of the true distribution which lie outside of a certain finite frequency integral. If such a postulate is taken into account, then the problem of reduction to an ideal instrument has a definite meaning: it is necessary to restore elements inside of this frequency range and to suppress the harmonics outside of it as much as possible or, in the limiting case, not to restore them.

We shall not consider here the problem of the physical basis of the conditions just pointed out. In actual cases these can be very different. For example, the lower boundary of the width of a single line and, consequently, the upper boundary of the width of the Fourier transform of the true distribution can be controlled by the Doppler effect. For our purposes the problem of establishing the postulate is not significant; of principal value is the very necessity of such a form of the postulate.

3. The methods of reduction that are well known in the literature (see Sec. 2) essentially reflect the described state of affairs, i.e., in all of them, the reduction* is of importance only in some limited frequency interval. For example, the expansion $A^{-1}(\omega) - 1$ in powers of $\sin\frac{a\omega}{2}$, which corresponds to the method of finite differences, is periodic with period $\frac{4\pi}{a}$, as is not difficult to see. Consequently, for a given value of the parameter a it is not the whole Fourier transform of the observed distribution that is generally reduced, but only harmonics with low frequencies.[82,91] The same considerations apply to the expansion (60). In the expansion of the solution in a trigonometric series [see (63)] only a finite number of terms N is used. This is equivalent to the fact that, in corresponding expansion (62) for $A^{-1}(\omega) - 1$, it is assumed that $b_n = 0$ for all $n > N$, that is, once again the high frequency harmonics are not reduced.[96] So far as expansions in power series and the expression of $\varphi(x)$ in terms of $f(x)$ and $\frac{d^nf}{dx^n}$ are concerned, we should note that in practice use is made of approximate values of the derivatives, and this is equivalent either to their replacement by finite differences,[91] or to the bounding of the limits in the integrals $\int \omega^n F(\omega)\, e^{ix\omega}\, d\omega$,[84] that is, in this case also the problem becomes that of regular reduction in a finite frequency interval. In this connection we have not included those methods of reduction in which the apparatus function is approximated by analytical expressions. For example, we can consider the approximation of the true distribution of the apparatus function for Voigt profiles as the expansion $\ln A(\omega)$ and $\ln F(\omega)$ in powers of ω,[47] where the first two terms of the expansion are used; this is sufficient only at low frequencies.

For a smooth representation of the considerations given above it is appropriate to introduce a graphical illustration. In Fig. 6 there are shown schematically the graphs of the Fourier transforms

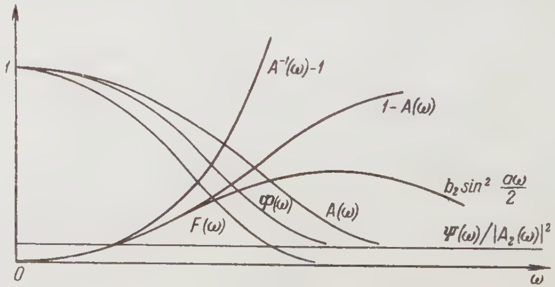

FIG. 6

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*See footnote on p. 260.

entering into the consideration of the functions: $\Phi(\omega)$, $A(\omega)$, $F(\omega)$, $A^{-1}(\omega) - 1$, and $\Psi(\omega)/|A_2(\omega)|^2$. Furthermore, the given curves corresponding to the first terms of the expansion of $A^{-1}(\omega) - 1$ in the methods of Rayleigh and of successive approximations $\left[b_2 \sin \frac{a\omega}{2} \text{ and } 1 - A(\omega)\right]$ From Fig. 6 it is seen first of all that in the reduction process the random errors of measurement are increased and that this increase is the more significant the wider the frequency range in which the observed distribution is reduced. Consequently, decrease of systematic apparatus distortions is achieved at the price of increase of random errors in the reduced distribution in comparison with the observed. Conversely, the tendency to transform random errors insignificantly means a bad reduction and, consequently, a large departure of the reduced distribution from the true, i.e., large remaining systematic errors.

From what we have said, the question naturally arises as to the estimate of such an "optimal" method of reduction, by which we would minimize the total mean-square deviation P^2 of the reduced distribution from the true, made up of the remaining systematic errors and the random errors arising in the reduction process. It is not difficult to write down this condition mathematically. If we denote by $G(\omega)$ the approximate expression for the Fourier transform of the apparatus function, that which is obtained in the reduction, then P^2 has the form[24]

$$P^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(\omega)|^{-2} \Psi(\omega)\, d\omega + \left| \frac{1}{2\pi} \int_{-\infty}^{\infty} [1 - A(\omega) G^{-1}(\omega)] \Phi(\omega) e^{ix\omega}\, d\omega \right|^2. \tag{80}$$

The first term of this expression is the random error, the second — the remaining systematic error. Consequently, estimating the optimal method of reduction means such a selection of the function $G(\omega)$ for which the expression (80) has a minimal value. In this case, as is seen from (80), $\Phi(\omega)$, $A(\omega)$ and $\Psi(\omega)$ must be fixed, i.e., the object of investigation and the instrument must be fixed.

Another statement of the problem is possible. We can fix $\Phi(\omega)$ and the method of reduction, i.e., the form of $G(\omega)$, and concern ourselves with the choice of the parameters of the instrument. Such an arrangement of the problem is of practical importance for the following reasons. In the first place, different approximation methods of reduction, as will be seen, lead to approximately identical values of P^2. On the other hand, in spectral instruments there are parameters which have a significant effect both on the apparatus function and on the value of the random errors of measurement, and at the same time can easily be varied by the experiment (widths of the slits, rate of scanning, time constant, thickness of the absorption layer etc.). Therefore it is very important to know for what values of these parameters we can carry out measurements in order that P^2 would be minimal.

By way of an illustration, let us consider (following reference 58) a choice of a slit width s of the monochromator, a time constant τ, and a scanning rate v in the measurement of the intensity at the maximum of a single line. For simplicity, we assume that the reduction to the ideal instrument is not carried out [i.e., $G(\omega) = 1$], that the apparatus function is determined only by the slits of the monochromator, and that s is much smaller than the width of the line. For a recording instrument which is equivalent to an RC network, and for slow scanning P^2 is expressed in the following way:

$$P^2 = \frac{c^2}{s^4\tau} + [as^2 + b(v\tau)^2]^2. \tag{81}$$

The first term in (81) describes the random errors of measurement, while the second gives the systematic error connected with the apparatus functions of the monochromator and the recording instrument. The coefficients a, b, and c are determined by the shape, width, and brightness of the line and also by the properties of the radiation detector. For a line of Gaussian shape of width γ and for approximation of the apparatus function by a Gaussian curve of width $a = s$ [see (21)] it is shown that

$$a = 1/2\gamma^2; \quad b = 4 \ln 2/\gamma^2. \tag{82}$$

The optimal values $s = s_m$ and $\tau = \tau_m$, which make P^2 a minimum, are found from the conditions $\partial P/\partial s = 0$, $\partial P/\partial \tau = 0$, whence we easily obtain the following expressions for s_m, τ_m, and $P_{\min}$:

$$\left.\begin{aligned} s_m^9 &= \frac{8}{5} \sqrt{\frac{b}{a^5}}\, vc^2 = 16\gamma^4 vc^2; \\ \tau_m v &= \frac{1}{2} \sqrt{\frac{a}{b}}\, s_m = 0.21 s_m; \\ P_{\min} &= 1.86 \left[\sqrt{b}\, a^2 vc^2\right]^{2/9} = 1.53 \left(\frac{v}{\gamma}\right)^{2/9} \left(\frac{c}{\gamma^2}\right)^{4/9}. \end{aligned}\right\} \tag{83}$$

On the right-hand sides of these expressions we substitute the values of a and b from (82) for functions of Gaussian shape.

As is easily seen from (81), the minimum of p in the parameter v is obtained for $v = 0$, since the error decreases monotonically with decrease of the scanning rate. Consequently, the choice of v is determined only by the general time measurement T and the frequency interval scanned. Inasmuch as this interval is obviously proportional to γ, then the ratio γ/v is proportional to T and in the same fashion turns out to be a given quantity since T must be fixed for each particular case. Consequently, $P_{\min}$ has the form

$$P_{\min}=\frac{P_0}{T^{2/9}M^{4/9}}, \tag{84}$$

where P_0 is a coefficient of proportionality which depends neither on the characteristics of the instrument nor on the width and brightness of the line. Thus $P_{\min}$ depends on the ratio of "signal to noise" $M=\gamma^2/c$ for $s=\gamma$ $\tau=1$ and on the general time of measurement T, that is, only on such characteristics of the object, apparatus and other conditions of the experiment on which, starting out from the same general considerations, we usually expect the errors to depend for a reasonable choice of the conditions of measurement.

From (81) and (83) we easily obtain the fact that, under optical conditions, the systematic error $bv^2\tau^2$ produced by the inertia of the recording instrument is four times smaller than the distortion due to the apparatus function of the monochromator. This important consequence is directly connected with the fact that random errors are proportional to $1/s^2\sqrt{\tau}$, that is, that the exponent for s is four times larger than the exponent for τ. Furthermore, inasmuch as s_m and $P_{\min}$ are very weakly dependent on the scanning rate (as $v^{1/9}$ and $v^{2/9}$, respectively) then it becomes clear that the choice of τ is not very critical, that is, the difference of τ and τv from their optimal values does not lead to any material increase in the error. From this it follows that in practical measurements and for the analysis of more complicated cases than the example considered (application of an exact or approximate reduction etc.,), we can assume that the inertia distortions must be negligibly small in comparison with the systematic errors from the apparatus function from the monochromator and we can consider τ as fixed.

The recommendations given above rarely contradict the results obtained in the researches of references 110 to 112, where such a problem of the choice of the parameters of the arrangement is discussed. This is connected with two circumstances. In the first place, errors due to apparatus function of the monochromator are not considered in references 110 and 111. In the second place, the parameters of the instrument are chosen in references 110 to 112 by starting from a completely different criterion, and this is very important. In references 110 and 111, for example, the requirement for minimizing the total mean-square error does not apply here, and the random and systematic error values are given independently. The operating conditions determined by this criterion are of course not optimal from the point of view of minimum P.

The scheme given above for the selection of the parameters of the instrument can in principle be applied in very different cases. At the present time such calculations are still very infrequent; however, they have already permitted us to make a number of important conclusions.

The question of the choice of slit widths of the monochromator and the thickness of the absorbing layer for the reduction of optical density at the maximum of a single line of Gaussian form has already been considered in references 56 and 76.* It was shown that the values of these parameters ought to be different in the employment of different methods of reduction. Furthermore, it turned out that for a typical infrared instrument, use of the first correction in the Rayleigh method of reduction (59) guarantees 1.5 — 2 times less accuracy in comparison with those cases in which the form of the line is known beforehand and the reduction completely eliminates the systematic errors.

The latter result permits us to make an important comparison of the various methods of reduction. Actually, if the first correction in the Rayleigh method leads to practically the same result as the "exact" method of reduction, then we must assume that all approximate methods will give (approximately) the same result. Thus random errors of measurement and their transforms in the reduction process smooth out differences in accuracy between the various reduction methods, and the choice of the reduction method should be based only on simplicity and ease of application.

It appears that this conclusion holds only for the correct choice of the slit width and for the other parameters of the system. Furthermore, it is obtained for a definite apparatus and form of the distribution under consideration. Therefore the application of the above results to other cases requires additional justification.

A resume of the present paragraph can be made in the following way. In the limiting case of absolutely accurate measurements, reduction to an ideal spectral instrument is unique, and in this sense, the presence of only the apparatus function for the real instrument does not lead to any limitation of its possibilities. Actually, random errors are finite and this leads to the impossibility of a unique calculation of the true distribution from the observed distribution $f(x)$. Practically, this nonuniqueness manifests itself in the fact that as the accuracy of the method of solution of Eq. (27) in the reduced distribution increases, there appears a certain false structure. To establish the nonreality of this structure, there is a paramount need of additional information on the distribution $f(x)$ under consideration, a distribution which is obtained independently of the method of the given experiment. In many cases, the additional information on $\varphi(x)$ permits us to consider the reduction to be unique and in the usual sense of the word permits us to speak about the error of computation of the true distribution.

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*The time constant is chosen fixed and the inertial distortion is not considered. By virtue of the reasons set forth above, this is entirely correct.

It is important to emphasize that, with the exception of those cases in which the form of the true distribution is reliably known and in which we deal only with measurements of a finite number of parameters — in all remaining cases, the presence of random errors of measurements leads to a situation in which it is impossible to establish the problem of obtaining data completely independent of the apparatus, and we can only speak of the estimation of such a method of reduction and such conditions of measurement in which the difference between the true and the reduced distributions is minimal (for example, in the sense of the mean-square deviation).

4. THE PROBLEM OF RESOLUTION

1. The resolution criterion given by Rayleigh[30] and the concept of the resolving power of a spectral instrument which was introduced by him were first applied only to the visual observation of two monochromatic lines in instruments whose apparatus function was determined only by the diffraction on the rectangular aperture of the diaphragm, that is, it has the form $\frac{1}{s_0}\left[\frac{\sin \pi x/s_0}{\pi x/s_0}\right]^2$. This criterion, as is well known, reduces to the following; two monochromatic lines of different intensity are considered resolved if the distance d between them is not less than $s_0 = \lambda f/D$. In this case the maximum of the intensity in the diffraction curve of one line coincides with the first minimum in the diffraction curve of the other line, while the intensity of the middle of the resulting curve is equal to $8/\pi^2 \cong 0.81$ of the maximum intensity.

Subsequently, there appeared a large number of papers in which the problem was considered of the development and the generalization of the Rayleigh concept of resolving power. Generalizations applied first of all to the application of the resolution criterion to instruments whose apparatus function differed from the diffraction one. Schuster[32,113] considered the case of a spectroscope in which the apparatus function was determined by the diffraction as a finite width of the slit. As a criterion of resolution he kept the condition that the intensity at the middle point of the total distribution of the two lines should be 81% of the maximum intensity.

For the ratio $\lambda/\Delta\lambda$, where $\Delta\lambda$ is the minimal resolved interval of wavelengths for finite slit of the width, Schuster introduced the special name of "purity" of the spectrum. However, this terminology is rarely employed, and most authors apply the term resolving power both in the case considered above and in other cases when various forms of the apparatus function are employed. A similar criterion of resolution was applied for the characteristics of the Fabry-Perot etalon,[2,53,114] which have the apparatus function (69). The small difference, which consists in the fact that the value of the "gap" in the total distribution from two monochromatic lines is taken here to be 20%, has no practical significance.

Further development of the resolution criterion is connected with the fact that the eyes, the photographic plate and other radiation detectors are capable of observing much smaller changes in the intensity than 20%, i.e., the change which is applied on the basis of the Rayleigh criterion (see, for example, reference 115). In connection with this fact, several visual resolving criteria have been suggested based on various premises. The simplest development is the postulation of a value of the gap different from 20%, for example 5%. Such a criterion was considered in reference 116 in an application to an apparatus function of the dispersion form. In references 52 and 117, the minimum resolving interval of wavelengths was assumed to be equal to the width of the apparatus function. For such a criterion, in the case of various shapes of the apparatus function, the value of the "gap" turns out to be different. For example, for the dispersion shape it is equal to 17%, that is, it differs insignificantly from the Rayleigh criterion, while for a diffraction distribution it has a gap of 2%, and in correspondence with this the resolving power for such a criterion is 13% larger than the Rayleigh value. We recall, finally, the criterion of Sparrow,[118] who considered that two lines are at the limit of resolution if the distance between them is such that the second derivative at the central point of the total distribution of the intensity of the two lines vanishes, that is, that the gap between

Apparatus function	$\frac{\Delta\lambda_R}{\alpha}$	$\frac{\Delta\lambda_5}{\alpha}$	$\frac{\Delta\lambda_S}{\alpha}$
Diffraction*	1.13	1.02	0.94
Triangular	1.20	1.05	1.00**
Gaussian	1.13	0.99	0.85
Dispersion	1.05	0.80	0.58

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*The quantities α, $\Delta\lambda_R$, $\Delta\lambda_S$ in the calculation of diffraction and finite slits were considered in references 24, 25, 32, 36, 118, and 121.

**For a triangular apparatus function, the second derivative at the central point of the total distribution is always equal to zero. In the table, in line with the basic idea of Sparrow, that value of $\Delta\lambda_S$ is given for which the gap only just vanishes, although in reference 25 the value of $1.156\ \alpha$ is given.

the lines only just vanishes. This criterion was applied for various shapes of the apparatus function[118–120,25] and, as an experimental investigation has shown,[118] very closely corresponds to visual determination of the presence of a gap between the lines. A comparison is given in the table of the resolving criteria stated above for the forms of the apparatus function met most frequently in practice. Here α is the width of the apparatus function and $\Delta\lambda_R$, $\Delta\lambda_S$, and $\Delta\lambda_5$ denote the minimal resolving intervals of wavelengths for the Rayleigh criterion, the Sparrow criterion and a criterion in which the minimum value of the gap is set equal to 5% of the maximum intensity, respectively.

In a number of researches there have been considered the dependence of the value of the gap on such factors as the difference in the intensity of two monochromatic lines,[118] the finiteness of the width of the line,[25] the level of "background" of constant intensity, etc. In these cases also, the authors call the ratio $\lambda/\Delta\lambda$ the resolving power, where $\Delta\lambda$ is the minimum distance between still resolved lines. Consideration of these circumstances is necessary in the analysis of concrete real instruments, the experimental determination of their limiting characteristics, and in other problems. However, analysis in these cases reduces to the establishment of those criteria of resolution which were considered above and do not lead in principle to new ideas on the resolution problem.

It must be remarked that inclusion of the random errors of measurement in the resolution problem is very important on the one hand, but on the other presents well known difficulties. Variation of the depth of the gap for a constant apparatus function cannot be considered a completely satisfactory receiver. The complexity of the problem lies in the fact that the value of the random errors of measurement and the width of the apparatus function are not independent. For example, in an expansion of the slit, the width of the line increase and the value of the gap quickly decreases. However, the errors of measurement decrease simultaneously and, consequently, a gap of given depth is recorded with greater reliability. Therefore, in the determination of the resolved intervals of wavelengths, it is necessary to take into consideration simultaneously both the random errors and the apparatus function and their mutual interaction. In application to visual methods of application and for diffraction slit apparatus function, the analysis of this important factor has been carried out qualitatively by Schuster[32,113] and van Cittert.[36] For photoelectric methods of recording, such an analysis is generally lacking at the present time. The first step in this direction was taken in reference 122 for the case of infrared absorption spectra. Those values of the absorption and width of the slits of the monochromator were computed for which the ratio of the depth of the gap to the mean square of measurement had a maximum value. In this case a triangular shape was taken for the apparatus function and a dispersion shape for the absorption coefficient. Calculations lead to a somewhat unexpected result. That is, the width of the slit, s_m, for which the best resolution (in the sense given above) takes place is almost entirely determined by the distance between the lines d from which it is shown that

$$s_m \cong \frac{1}{2} d$$

for lines of practically any width.

2. The characteristic feature of all the researches considered above on the resolving power of spectral instruments is that in the examination of the energy distribution of the two lines and in the comparison of this distribution with that of a single, strictly monochromatic, line, i.e., with an apparatus function, all attention was devoted to the single item of obtaining a distribution and to ascertaining whether it has a maximum or minimum. Such an approach is quite natural: if the apparatus function is a maximum at the middle point and the observed distribution is the minimum, then it is clear that this distribution is brought about by nonmonochromatic radiation. It is not difficult to see, however, that the inverse confirmation is not valid. In other words, such an approach assumes in principle that on the basis of the resolution criterion a certain qualitative difference should be established between the observed distribution and the apparatus function, a difference that is noted at first glance,

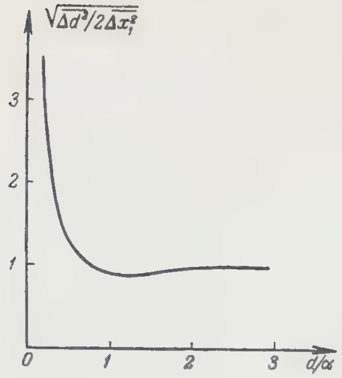

FIG. 7

without quantitative measurements of the entire energy distribution of the spectrum. Moreover, at all points of the curve there will be some quantitative difference which can be measured and used for establishing the spectral composition of the radiation under consideration.[123] Thus, the resolution criteria considered above are slanted primarily to qualitative observations and the magnitudes of the resolving powers obtained on the basis of such criteria only determine the order of magnitude of the distance between the lines which can be achieved for a given real instrument.

As an illustration, let us consider the following example. The observed distribution from two monochromatic lines for an absolutely accurate measurement has the form

$$f_1(x) = I_1 a(x - x_1) + I_2 a(x - x_2). \qquad (81')$$

For the sake of simplicity, we shall assume that the apparatus function is known with absolute accuracy, while the observed distribution $f(x)$ from two monochromatic lines is measured with some random roughness. This real recording is naturally approximated by the function $f_1(x)$ and the problem consequently reduces to the determination of the four parameters I_1, x_1, I_2, and x_2. We shall make a choice of these parameters by the method of least squares, i.e., we shall consider as the most probable values of I_1, x_1, I_2, x_2, those values for which the total mean square deviation of the real signal $f(x)$ from the approximating function $f_1(x)$ has a minimum value,* that is,

$$\int_{x_1}^{x_2} [f(x) - f_1(x)]^2 \, dx = \min.$$

From the point of view of the resolution problem there is interest in the error of measurement of the distance $b = x_2 - x_1$ between the lines as a function of the value of this distance and width of the apparatus function. Simple calculations carried out under the assumption of the Gaussian form of the apparatus function [see (21)] and equal intensities of the lines $(I_1 = I_2)$, lead to the following expression for the mean square error in the determination of d:

$$\overline{\Delta d^2} = 2\overline{\Delta x_1^2}$$
$$\times \frac{1 + \exp\left\{-2\ln 2\,\frac{d^2}{\alpha^2}\right\}}{1 - \exp\left\{-4\ln 2\,\frac{d^2}{\alpha^2}\right\} + 4\ln 2\,\frac{d^2}{\alpha^2}\exp\left\{-2\ln 2\,\frac{d^2}{\alpha^2}\right\}}, \qquad (82')$$

where $\overline{\Delta x_1^2}$ is the mean square error of the determination of the position of the maximum of a single line. If $d \gg \alpha$, i.e., the lines are not superimposed, then, as is to be expected, $\overline{\Delta d^2} = 2\overline{\Delta x_1^2}$. If $d \ll \alpha$, then (82′) takes the form

$$\overline{\Delta d^2} = 2\overline{\Delta x_1^2}\,\frac{1}{4\ln 2}\,\frac{\alpha^2}{d^2}, \qquad (83')$$

That is, the error increases rapidly in the case of small distances between the lines in comparison with the width of the apparatus function. The general form of the curve $\sqrt{\overline{\Delta d^2}/2\overline{\Delta x_1^2}}$ as a function of $\sqrt{\overline{\Delta d^2}}$ is given in Fig. 7. It can be seen that the sharp increase of $\sqrt{\overline{\Delta d^2}}$ begins only for $d < \alpha/2$, when there is no sort of "gap" at the center point of the observed distribution $f_1(x)$ (Fig. 8).

This smallest distance between lines, $d_{\min}$, which can still be measured in the case under consideration, is obtained from the condition $\sqrt{\overline{\Delta d^2}} = d_{\min}$. Employing Eq. (83′), we obtain:

$$\frac{d_{\min}^2}{\alpha^2} = \sqrt{\frac{1}{2\ln 2}\,\frac{\overline{\Delta x_1^2}}{\alpha^2}}. \qquad (84')$$

For example, if the error in the determination of the position of a single line amounts to one tenth of the width of the apparatus function, that is, $\sqrt{\overline{\Delta x_1^2}/\alpha^2} = 1/10$, then $d_{\min} = 0.29\,\alpha$, which is 3 or 4 times smaller than the smallest resolved

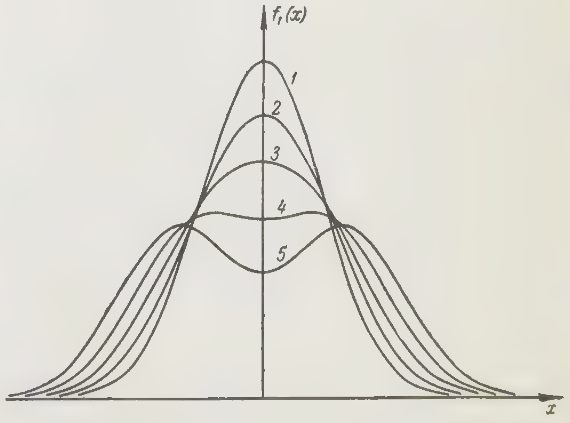

FIG. 8. Total distribution for the intensity for two monochromatic lines of equal intensity and for Gaussian shape of the apparatus function of width α: 1) d = 0; 2) d = 0.5α; 3) d = 0.72α; 4) d = 0.96α; 5) d = 1.20α.

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*Similar consideration for the case of a single line was given in references 24 and 26.

interval determined by different criteria of resolution (see the foregoing table). If the accuracy of measurement is high, then the difference between $d_{\min}$ and α will be still larger.

The example considered shows clearly that with quantitative measurements and subsequent mathematical treatment of the observed distribution the possibilities of real instruments are not limited by what the Rayleigh criterion (or any other sort of criterion based on the analysis of the behavior of the total distribution at a single points) gives, but can be more fruitful if one takes into consideration the difference between $f(x)$ and $a(x)$ at all points of the curve.

The non-correspondence of the Rayleigh concept of resolving power (and its modifications considered above) to experiments with quantitative measurements has lead in recent years to a wide generalization of the concept of resolution, which is directly connected with the application in optical measurements of the modulation method. We have in mind the researches of G. S. Gorelik and his colleagues[124–126] on the improvement of accuracy of measurement in a different type of interference experiments. His general considerations can be applied without difficulty to spectroscopic measurements.[124]

If in the papers referred to, the problem of resolution appears as a problem of finding in an observed distribution some feature of the true distribution — the extension of the light source, splitting of a spectral line when a magnetic field is applied to the light source, number of lines, etc. In other words, by resolving ability is meant the capacity to note, on a real instrument, a difference between the observed distribution and the apparatus function which arises as a result of the given difference of the true distributions from a δ-function.

It should first of all be noted that in such an approach we are not talking about any qualitative indication such as the absence of a minimum, but about a quantitative difference of two distributions. For example, if two spectral lines are considered, then the limit of resolution corresponds to such a distance between them that the departure of the observed distribution from the apparatus function is equal, say, to the mean square error of measurement.

Furthermore, by the problem of resolution is understood not only the possibility of establishing the presence of two monochromatic lines of equal intensity, but also many other problems connected with finding in an observed distribution some indication of the true distribution. In such an approach, it is especially clear that quantities of the type of the resolving power are not characteristics of the apparatus only but depend essentially on the distribution being measured and determine the sensitivity of the instrument only in the measurement of a given distribution, giving very little information as to what the situation would be in another case.

In the approach to the resolution problem that we have made, there is a special case of the general problem of reduction to a real instrument considered in Secs. 2 and 3.* Actually, for known form of the true distribution $\varphi(x)$, the reduction reduces to the calculation of parameters characterizing $\varphi(x)$ (the width of the line, its intensity, distance between lines, etc.), starting out from the observed distribution $f(x)$. As a result of the crudeness of measurement, the parameters of the true distribution are made known with some error; this error is the greater the wider the aperture function. For sufficiently gross measurements, or a sufficiently small value of the parameter of interest to us, the error of its determination is shown to be equal to the magnitude itself. This value of the measured parameter also characterizes the resolving ability of a real instrument in the investigation of a distribution of a given type. Thus the magnitude of the resolving power (in the sense considered) characterizes the conditions of the experiment (the apparatus and the object measured) in which the reduction to an ideal instrument no longer has meaning.

The connection shown between the problems of reduction and resolution permit us to explain one very widespread misunderstanding which amounts, roughly speaking, to the following. It is frequently desirable to make use of the value of the resolving power as an explanation of the limiting possibilities of an instrument in the case of differences of the true distributions of different and, generally speaking, unknown shapes. For example, one wants to know for what conditions one can distinguish on the given real instrument a set of three neighboring lines from a single line of finite width, and so forth.

Such an arrangement of the problem means in essence the use of the concept of the resolving power beyond the limits of its applicability. Actually it was shown in Sec. 3 that for finite errors of measurement the possibility of reduction to an ideal instrument depends essentially on additional information about the true distribution which was obtained independently of the given experimental procedures. It was explained in particular that only for a previously known form of the true distribution could

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*The example considered at the beginning of the present section is an excellent illustration of this.

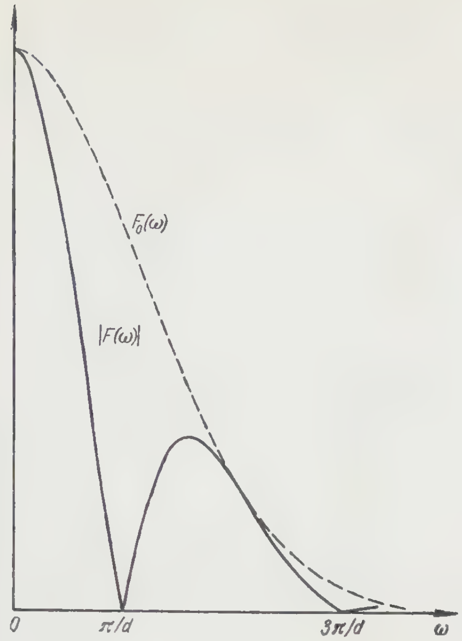

FIG. 9

one completely eliminate systematic distortions connected with the apparatus functions. In the case of the absence of such detailed information, reduction does not permit us to obtain data completely independent of the apparatus.

Everything pointed out of concerning reduction refers (perhaps with still greater firmness) also to the limiting possibilities of reduction and consequently to the resolving power. The concept of resolution under consideration relates exclusively to that case in which the form of the true distribution is known beforehand, and where we are speaking only about measurements of its parameters. In the absence of such information, this concept apparently loses its previous meaning and must be redetermined in a corresponding fashion. This redetermination will be different for information of a different type. At the present time, however, this question has not been discussed in the literature.

3. In conclusion, let us consider briefly several possibilities which the spectral approach opens up in the resolution problem. First of all, we have in mind the research of D. S. Rozhdestvenskiĭ,[115] in which, for analysis of the quality of spectral instruments, use is made of a harmonic energy distribution over the spectrum. The advantage of this method lies in the fact that it is comparatively easy experimentally to obtain the harmonic distribution by means of a combining of the spectral instrument with a double ray interferometer. In this case, without consideration of the effect of distortion factors of the first group (see Sec. 1), a point harmonic distribution exists. At the same time, employment of linear spectra for this purpose frequently becomes complicated (especially for instruments of comparatively high resolving power) by virtue of the presence of hyperfine line structure and other factors.

Essentially the Rozhdestvenskiĭ method consists of following the amplitude distribution while increasing frequency of the harmonic distribution (in practice, while increasing the difference of path between the rays of the interferometer) until frequencies are reached when the "visibility" of the interference picture vanishes. In other words, making use of the terminology given here, we are dealing with the discovery of the zeroes of the Fourier transform $A(\omega)$ of the apparatus function. For example, if the apparatus function is determined by diffraction then, in accordance with Eq. (16a), $A(\omega) = 0$ for $|\omega| \geq 2\pi/s_0$, that is, beginning with the period of the harmonic distribution equal to the Rayleigh minimum resolution distance. If $A(\omega)$ has no zeroes, then they extend to those frequencies in which the visibility becomes equal to the errors of measurement.

The principle set forth as the basis of the Rozhdestvenskiĭ method is employed in many widely different cases. A. I. Salishchev has carried out several measurements by the Rozhdestvenskiĭ method with the purpose of explaining the effects of defects of preparation of the instrument on its resolving power.[127] Rozhdestvenskiĭ himself has shown[127] that his method coincides in principle with the well-known method of Michelson[9] of measurement of the angular dimensions of stars. In an interesting paper[128] on the foundation of the same principle, the question was considered of the measurement of spectral width of slits and the resolving power of infrared spectrometers. The universally adopted method of measurement of resolving power of photo objectives by means of periodic patterns (see, for example reference 129) is also based on the method of Rozhdestvenskiĭ. This is connected with the fact that the effect of the apparatus function is primarily to decrease the amplitudes of the higher harmonics of the Fourier transform of the transmission function of the patterns. Therefore an arbitrary periodic pattern, with an arbitrary distribution of transmission over the range of the period, will give in practice a harmonic observed distribution of intensity at frequencies close to the limit of resolution, while the second and higher harmonic play virtually no role at all. Finally, it can be shown by the work of reference 130 (in which the resolving power of the photographic emulsion was determined directly by means of a harmonic distribution of illumination obtained by interference from two point sources).

An interesting application of the spectral approach to the problem of the resolution of two lines was given by G. G. Petrash. It is not difficult to show that the Fourier transform $F(\omega)$ of the observed distribution of two lines of identical intensity is

$$F(\omega) = F_0(\omega) \cos \omega d/2, \qquad (85)$$

where $F_0(\omega)$ is the Fourier transform of the distribution of a single line, d is the distance between the lines. As is easy to see, $F(\omega)$ vanishes at the points (Fig. 9).

$$\omega_k = (2k-1)\frac{\pi}{d}, \qquad k = 1, 2, \ldots, \qquad (86)$$

while in the vicinity of the lines, the values ω_k "move away" in the direction of larger and larger frequencies. Thus, if harmonic analysis of the intensity is carried out even when there is no "gap" at the central point, then we can decide on the existence of two lines by the presence of a zero in $F(\omega)$. Two circumstances prevent the zeroes of the function $F(\omega)$: decrease of the Fourier transform $F_0(\omega)$ of a single line upon increase in frequency and the presence of random errors of measurement. The limit of resolution corresponds to such a closeness of the lines in which the first maximum, after the frequency $\omega_1 = \pi/d$, where $F(\omega)$ first goes to zero, becomes equal to the error of the measurement. Consideration of concrete examples shows that such a method of finding two lines leads to a smaller resolved interval than the Sparrow criterion. For example, in the case of a dispersion line shape we can easily distinguish two lines if the distance d between them is equal to one half of their width γ, while the "gap" at the central point of the observed distribution vanishes for $d = 0.58\gamma$ (see the table on page 266).

We note that the Petrash criterion, as also the Rayleigh, Sparrow, and other criteria, is based on the observation of a single point, with only this difference, that in the latter cases this point is selected in the observed distribution, while in the former it is from its Fourier transform. However, the Petrash criterion possesses this important advantage that upon infinite increase in the accuracy of measurement, the minimum resolved interval tends to zero while in the other criteria this natural requirement is not satisfied.

It is interesting to note that the method of estimation of the resolving power of a spectral instrument considered here is essentially the generalization of that approach which is undertaken in the analysis of the resolving power of the well-known method of investigation of the line structure by means of a double beam interferometer employed by Michelson and other authors.[9,67,68] Actually, the visibility curve V of the interference bands according to which one judges the line structure is the modulus of the Fourier transform of the energy distribution over the spectrum. In the case of two identical lines, consequently, the visibility curve has the form

$$V(\omega) = |F(\omega)| = \left| F_0(\omega) \cos \frac{\Delta\lambda\omega}{2} \right|,$$

and the minimum observed difference of wavelengths $\Delta\lambda$ is determined by the condition that the visibility band beyond $\omega = 2\pi/\Delta\lambda$ does not exceed the random errors of measurement.

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Notes on Techniques

NEW DEMONSTRATION APPARATUS

V. I. EROFEEV, V. K. RUBTSOV, and S. I. USAGIN

Uspekhi Fiz. Nauk **66**, 519-521 (November, 1958)

CIRCUIT and construction details are given for the demonstration of wave phenomena on a screen. The design is based on the apparatus developed by P. N. Lebedev and I. F. Usagin.*

The apparatus described permits the display of a whole series of wave phenomena on the surface of a liquid to be displayed on a screen in a darkened auditorium. The experiments include: (a) launching and propagation of plane and spherical waves (b) the phenomenon of refraction at the interface between two media (c) reflection of waves from plane and spherical surfaces (d) Huygens' principle (e) interference of waves (f) diffraction of waves.

The general form of the apparatus is shown in Fig. 1. The principal parts of the apparatus are the tank, the electro-magnetic vibrator, and the projection system.

The tank is provided with a transparent glass bottom. Various obstacles are placed on the bottom to secure different effects. Capillary waves produced on the surface of the liquid (acetone) in the tank are projected onto a screen. The path of the rays of the projection system is shown in Fig. 2. A K-22 motion-picture projector bulb serves as the light source. The light rays from the source travel through the first condenser and, upon reflection from mirror 2, travel upward to lens 3 which focuses the rays on the aperture in disk 4. The shutter disk 4 has three equidistant apertures (at 120°) on a circle. Upon rotation of this disk, the tank is illuminated by the portion of the light passing through the shutter. The illumination of the tank is synchronized with the operation of electromagnetic vibrators; the commutator is set on the same axis with the shutter; in this way, the tank is illuminated stroboscopically, so that a stationary picture of the waves is portrayed on the screen. To obtain "traveling" waves, the apparatus is provided with a special mechanism

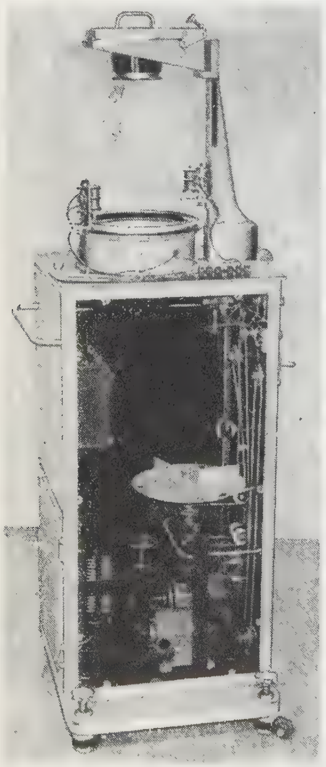

FIG. 1. General view of the apparatus.

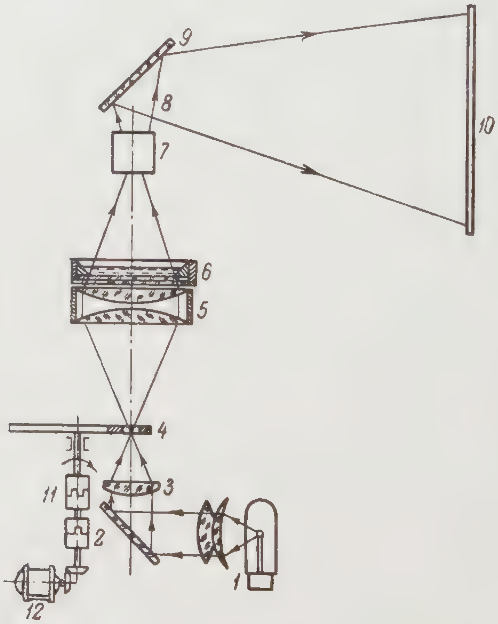

FIG. 2. Diagram of the projection apparatus: 1 – K-22 lamp with condenser; 2 – mirror with silvered surface; 3 – focusing lens; 4 – shutter; 5 – condenser; 6 – tank; 7 – objective I-13; 8 – small screen for obtaining picture in a darkened field; 9 – mirror with variable slope; 10 – screen; 11 – commutator; 12 – electric motor.

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*P. N. Lebedev and I. F. Usagin, "Apparatus for demonstrating Huyghens' principle," Uspekhi Fiz. Nauk 49, 469 (1953).

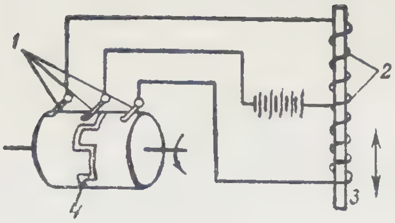

FIG. 3. Diagram of the commutator: 1 – brushes; 2 – coil of vibrator; 3 – core; 4 – insulator.

which destroys the synchronism between the disk and the vibrators. This is achieved by shifting the brushes of the commutator about the axis of the commutator in one direction or another.

The vibrator consists of a coil with two windings, with a core placed inside. A cap of appropriate shape is attached to the end of the core. The vibrator is fed by direct current which flows alternately first in one coil and then in the other; as a result, vibrations take place. The apparatus has two commutators for switching the current. The commutator circuit is shown in Fig. 3. One of the commutators is double, for the purpose of coherent operation of two vibrators.

Rotation of the commutators is produced by electric motors. The speed of the latter is regulated by rheostats ($R_1 = 400$ ohms). In this way for the frequency of commutation can be altered, and with it the frequency of the vibrators, which can be made as high as 2000 vibrations per minute. The vibrator circuit contains variable resistors ($R_2 = 50$ ohms) which change the vibration amplitude of the vibrators. The electrical circuit of the apparatus is shown in Fig. 4.

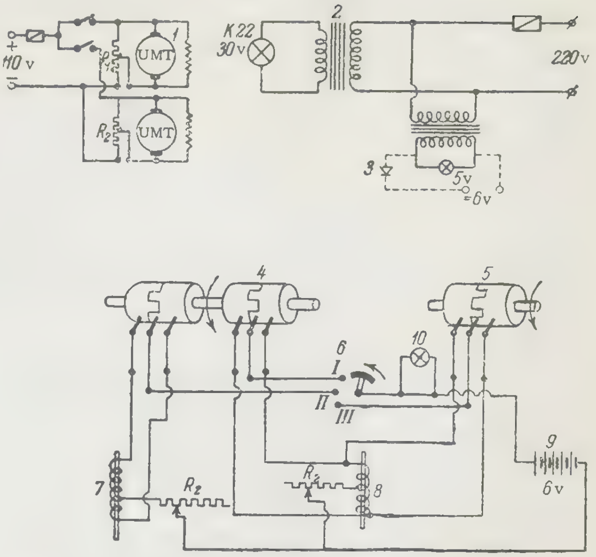

FIG. 4. Electrical circuits: 1 – UMT-21 motor; 2 – transformer; 3 – copper oxide rectifier; 4 – double commutator; 5 – commutator; 6 – four position switch; 7 – permanently fixed vibrator; 8 – movable vibrator; 9 – battery; 10 – lamp.

Translated by R. T. Beyer

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From the History of Physics

LIFE AND PHYSICAL DISCOVERIES OF TORRICELLI

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Following the resolution of the World Council for Peace, the 350th birthday of Evangelista Torricelli, the outstanding Italian scientist, was celebrated this year by the entire civilized world. Many Soviet organizations — The Physics and Mathematical Division of the U.S.S.R. Academy of Sciences, the Institute of History of Natural Sciences and Technology of the U.S.S.R. Academy of Sciences, the Union of Soviet Societies for Friendship and Cultural Relations with Foreign Countries, the "U.S.S.R. — Italy" Society, the Soviet Committee for the Maintenance of Peace — held a joint session on October 16, 1958 to commemorate this outstanding date. The introductory address was delivered by Academician N. N. Andreev; a paper "Torricelli as a Mathematician" was delivered by corresponding member of the Academy of Sciences of the U.S.S.R., B. N. Delone; Professor V. P. Zubov lectured on Torricelli's experiments in Florence.

EVANGELISTA Torricelli, one of the most remarkable scientists of the 17th century, passed through the world of science like a flashing meteor. This Italian mathematician, physicist and mechanist died in the very bloom of his creative forces and talent, at the very zenith of his fame, before reaching his 39th birthday.

Torricelli was born on October 15, 1608 near the small town of Faenza to a very poor family. He lost his father early and was brought up by his uncle, a Benedictine monk, of St. John's Monastery in Faenza. Later Evangelista studied philosophy, mathematics, and physics in a Jesuit school in his home town for two years. He showed his unusual mathematical capabilities early and in 1628, at his uncle's recommendation, he was sent to continue his education with an outstanding mathematician, one of the talented students of Galileo, the influential Abbot Benedetto Castelli, who at that time was tutor to the nephew of Pope Urban VIII at the Vatican.

Abbot Benedetto Castelli (1579-1643) entered the Monastic Order as a youth, but devoted himself entirely to scientific research. Along with great discoveries in various branches of mathematics, Castelli was the first to formulate the scientific foundations of hydraulics.

Castelli's students included Torricelli's contemporary, the outstanding physicist, astronomer and physiologist, Giovanni Borelli (1608-1670) and the great mathematician Bonaventura Cavalieri (1598-1647).

Castelli highly regarded young Torricelli and undertook the guidance of his education along with insuring his support by appointing him his personal secretary.

In 1632, an outstanding event occured in Italy's cultural life. In January of that year, Galileo Galilei published his outstanding book "Dialogue on the Two Most Important Systems of the World, That of Ptolemy and of Copernicus" (Dialogo sopra due Massimi Sistemi del Mondo Tolemaico e Copernicano).[2] Written in Italian so as to be accessible to any literate person, and presented in a highly artistic style, this book by Galilei rapidly found its way to the minds and hearts of large circles of the educated Italian society.

It should be noted that even long before the publication of the "Dialogue" there was a stubborn struggle in Italy between the adherents of Galilei, as the exponent of the revolutionary traditions of the Renaissance on the one hand, and the faithful representatives of the Catholic reaction on the other. This struggle flared up in 1610 when Galilei published his book "The Stellar Herald," where he first briefly reported his astronomical observations, made with the aid of the telescope he created, and where he first came out clearly in favor of the Copernican system. Even then the adherence of progress started to call themselves "Galileists." However, in 1616 the "Holy Congregation of the Sacred Cardinals" pronounced as "false and in complete opposition to the Holy Script the Pythagorean theory of the motion of the earth and the stationary sun, as taught by Nicolai Copernicus," and prohibited all the books containing this theory, lest it "gradually propagate further to the destruction of the Catholic truth."[3]

The appearance of the "Dialogue" in 1632 revived the struggle and the church obscurantists again started to demand insistently the prohibition of the seditious book by Galilei, who again allowed himself to discuss the problem of the truth of the Copernican system, a problem not subject to discussion as already resolved by the church. Apparently Galilei wrote a letter, since lost, asking aid from his former student and friend Castelli. But the letter did not find Castelli in Rome, and the latter's 24-year secretary took advantage of this occasion to strike up an acquaintanceship with the great Galilei. On September 11, 1632, Torricelli wrote Galilei a letter (reference 4, Vol. III, p. 35) that begins:

"Most worthy and honored Signor:

In the absence of our master, the most reverend father of mathematics, I, his most unworthy student and servant, who has the honor of being his secretary . . ." Toricelli acknowledges receipt of Galilei's letter to Abbot Castelli, and reports that he became acquainted with its contents in accordance with instructions given by his master. "I can assure your honor that the Father Abbot has always attempted to defend your honor's 'Dialogue' in all cases, both to the ruler of the Holy Court, with his entourage and to other ecclesiastic persons . . . I became fully acquainted with this matter. I myself am a mathematician, albeit young, a student of the very reverend father for six years. For the first two years I studied with the Jesuit Fathers. I have thoroughly and continuously studied, to this day, the book by your grace with great delight, at first in the house of the Father Abbot and then in Rome. Such delight can be experienced only by one who has thoroughly mastered geometry, Appolonius, Archimedes, and Theodosius and who has also studied Ptolemy and all the works by Ticho* and Longomontanus,† who finally, persuaded by the large number of coincidences, reaches Copernicus and who is in specialty and conviction a Galileist . . ."

Torricelli reports to Galilei in passing how unfavorably certain highly-placed papal officers look upon his Dialogue. In particular, he mentions malicious remarks by the Jesuit Scheiner, who later on was one of the initiators of Galilei's trial.

Torricelli finishes his letter with the following enthusiastic words: "I consider myself the happiest man in the world for being born in this century, when I can know and correspond with Galilei himself, the oracle of nature, and when I can have the favor of being related and guided by Ciampoli himself."*

We don't know how Galilei answered his ardent fan or whether he answered him at all. Torricelli's preserved records show a tremendous gap following this letter, covering almost 9 years — to January 1640.

There are no data indicating the cause of this gap. However, the historical events that took place during that period and certain meager data do give grounds for certain assumptions regarding this matter.

What happened after September 1632? The sale of the Dialogue was already prohibited at the end of this year. Galileo's trial in the beginning of 1633 began and ended, as is known, in his public denial (June 22, 1633) of the "Heretic Theory of Copernicus." The great scientist was officially made a "prisoner of the inquisition." He was deprived the right of printing anything or discussing any problems on the motion of the earth. From 1634 through 1641 Galilei was under a careful supervision of the inquisitors in his own villa Arcetri near Florence, where he was isolated from his friends and students. We note, however, that the embattled churchmen did not confine themselves to persecuting the author of the Dialogue alone. In November 1632 the well-known litterateur and patron of the arts, an ardent adherent of Galilei, the canon Giovanni Bastista Ciampoli, whose name was so reverently mentioned by the young Torricelli in his first letter to Galilei, was removed from his position as the secretary of Pope Urban VIII. It was none other than Ciampoli who helped Galilei in 1632 to obtain the censor's permission to publish the Dialogue, thus causing the anger of Urban VIII.

The disillusioned Ciampoli, as he himself indicated, was "cut off from the world in a deaf corner of the Appenines." The last years of this period he lived in the small town of Fabriano.[6]

Torricelli's letters of 1640-1641 are also dated in Fabriano. It seems thus that Torricelli spent at least two years in this dead corner, where the disgraced canon Ciampoli was forced to live. Apparently Torricelli performed secretarial duties there. In the letter of January 5, 1641 Torricelli writes from Fabriano to his friend Magiotti that he has finally recovered from his sickness, of which he "nearly died against his wishes," and during his sickness there were accumulated "more than 200 letters which I must answer for my patron." The name of the patron remains unknown.

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*Ticho Brahe.

†Probably in error for Regiomontanus.

*Cf. below.

In March 1641, finally free of the supervision of the inquisition, the blind and seriously ill 78 year old Galilei received a letter from Castelli, in which the latter reports that he plans to visit him enroute in Florence in order to show him his new book, and also "another book written by one of my students who, after learning the first principles of geometry, was my student for ten years and made such progress that he proved many theorems on mechanics, previously proved by your eminence, but in entirely different ways (reference 4, Vol. III, page 46).

Finally, on March 15, 1641 Torricelli sent Galilei a new letter, from which it is seen that Castelli has sent to his great teacher Torricelli's "De motu gravium naturaliter descendentium ..." (On the Motion of Heavy Bodies, Descending in their Natural Manner).

Torricelli writes in this letter (reference 4, Vol. III, pages 48-49):

" ... your emminence's papers are more worthy of admiration than of comments. I was seized with the greatest amazement from the very first day that I succeeded in seeing your books. However, your latest book on relative motion* has caused more boldness than amazement."

Torricelli reports that he desires to publish a small book in Rome or in some other city but wishes first to present it to Galilei for review. "I wrote these folios not because I intend to advance some doctrine of my own, but out of necessity to set down a memorandum for my small mind and out of desire to show my distant teacher with what enthusiasm I studied his theory in his absence." Torricelli then asks Galilei to consider this composition "as an essay like you, were you still a student, would write on poems from the Aeneid or on the speeches of Marcus Tullius."†

He begs to remain his servant "and although convinced that I am inferior to Magiotti and Nardi"‡ in my abilities, however, I excel them in paying immeasurable homage to the famous name of Galilei, a name universally known forever ... Will your emminence excuse my lack of education, my style, and the unlimited errors, particularly in the second part, which has not been copied but is a first draft, written rapidly, and I myself have not yet reread it ..."

Having visited Galilei in Arcetri, Castelli warmly recommended that he receive Torricelli and use him for the development, publication, and further advancement of his unfinished research. Galilei read Torricelli's book "On Motion," highly praised it, and asked Castelli that he send the young author to him. Torricelli could not move to Galilei immediately. Only by the end of September of that year did he set on his journey and arrived in Arcetri during the first days of October. Vincenzo Viviani, Galilei's biographer and student, who lived in Arcetri, reports that "Galilei immediately undertook to discuss with him one whole "Day" in the "Dialogues." Some material of the Dialogues was contained in his notes and some only in his thoughts, and Galilei intended to incorporate them in his Dialogues in the form of two "Days," to be appended to the previous four, published several years earlier in his book on the two new sciences of mechanics and local motion ... But Galilei died only some three months later, (January 6, 1642), after Torricelli barely started his work on the fifth Day. This sad event, which Torricelli did not expect at all so soon, left him in somewhat of a stupor ... and when he finally decided to return to Rome, he received instructions to await the orders of the Grand Duke who at that time was in Pizza, for he was appointed Mathematician to His Highness. Thus," continues Viviani, "the lectures on mathematics in the local lecture hall, which date back to antiquity but were interrupted for a long time, have been resumed."[7]

Let us recall that Torricelli's predecessor as professor of mathematics of the Florence Academy, starting with 1610, was Galilei, whose lectures were stopped by the orders of the inquisition in 1633. Thus Torricelli received the title previously held by Galilei, "Philosopher and First Mathematician of His Highness the Grand Duke of Tuscany" and simultaneously assumed the duties as Lecturer of Mathematics at the Florence Academy of Sciences (Accademia del Cimento). This was followed by the title of Lecturer on Military Fortification at the Florence Academy of Design (Accademia del Disegno). The material situation of the 32 year old Torricelli was suddenly improved. While previously the modest secretary could engage in theoretical investigations only in his free time, in Florence he finally found it possible to apply some of his ideas in the field of experiment. Here Torricelli apparently gradually completed work on Galilei's supplements to his "Dialogues" (Days V and VI). However he didn't succeed in getting them published. The "Fifth Day" was published by Viviani in 1674 and the "Sixth Day" only in 1718. The name of Torricelli is not mentioned in these, and exactly how much he has really contributed is not known.

As the Court's Natural Experimenter (Philosopher) and Mathematician Torricelli settled in apartments of the old palace of the Medici's in

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*He refers to the "Dialogues."[1]

†I.e., Cicero's.

‡Galilei's students.

Florence. His place became the meeting ground for outstanding artists and scientists. Torricelli struck a particular friendship with the known artist Salvatore Rosa and with the known hellenicist Carlo Datti. The catalogue of Torricelli's personal library, a catalogue preserved to this day, lists, along with various works on mathematics, physics, astronomy, mechanics and military affairs, also a considerable number of artistic literary works.

Torricelli's biographers emphasize that he himself was not a stranger to literature and wrote sharp epigrams and comedies, which however were lost. In one of these epigrams in Latin, he pokes fun of some constructor who built a bridge which collapsed before completion. This epigram ends in a bitter complaint on the evil times that give rise to stupid bridge builders — ponti-fices (the plural of pontifex). However, any one who has studied Latin knows very well that the ancient Latin work pontifex (high priest) is the official title of the Pope. Torricelli's epigram became very popular in Italy as an allusion to Pope Urban VIII, who vainly attempted to save the churches' failing power by repression and persecution.

In 1644 Torricelli published first some of his papers on geometry and mechanics, on which he worked sporadically all these years. His name became widely known in the entire scientific world. The greatest scientist in Italy and abroad acknowledged Torricelli as a first-class mathematician. Apparently Torricelli's name does not remain unnoted in the literary awards. The Academia della Crusca in Florence, an institution specially dedicated to cultivation of beauty and purity of Italian literary speech, elected Torricelli a member and invited him to deliver a series of lectures. The mathematicians Magiotti and Cavalieri, learning of the honor accorded to their colleague, made quite a few sarcastic remarks in personal letters written to him concerning the Academicians of della Crusca, who were quite unversed in mathematics and physical sciences (reference 4, Vol. III, pages 74-75). However, these remarks did not worry Torricelli and he delivered 12 popular lectures on a variety of topics at the Academy della Crusca. In one of these lectures, "In Praise of Mathematics," Torricelli did not hesitate to ridicule "philosophers who consider themselves born to science and knowledge" but who turn out to be quite ignorant of machinery and equipment that is perfectly well understood by simple uneducated persons. These lectures were published by the Academy only in 1715.[16] These lectures were preceded by a curious note, stating that the authorities of the Academy, having become acquainted with the text of the "lectures" of Evangelista Torricelli, "have observed no errors in language in these lectures." Consequently, these lectures have been acknowledged to be a model of irreproachably pure Italian speech. In this matter, too, Torricelli turned out to be a worthy successor of Galilei, whose Dialogues are considered today to be undying memorials of not only science but also of artistic prose. "The academic lectures" of Torricelli were unfortunately not translated into other languages and are now little known. They differ from Galilei's "Dialogues" in a certain florid style, but are nevertheless authentic works of art. Six of these 12 lectures are devoted to physical problems and deserve most serious attention both as a popular discussion of the views and of the most important physical research of Torricelli, which will be discussed later. At the same time, Torricelli's "Academic Lectures" are thoroughly saturated with the bold aggresive spirit of the renaissance, struggling against the "peripatetic philisophers" i.e., against the adherence of the obsolete ideas of Aristotle, officially recognized by the dogma of the Catholic Church as unique and infallible. Although nothing is mentioned of Copernicus in these lectures, nor of cosmology as a whole, Galilei's merits are ardently hailed many times. The lectures "On Lightness" and "On the Wind," as we shall see, are directly related to Torricelli's famous experiemnt which led him to the invention of the barometer in 1644.

The great Duke of Tuscany, personally present at the demonstration of this experiment, issued on this occasion a special decree in which it is stated: "Whereas Evangelista Torricelli, thanks to his valor and success in realizing this matter; whereas he discovered the truth; whereas he overcame the fear of vacuum; whereas he extended the boundaries of science, we announce glory to the immortal God and triumph to Evangelista Torricelli.[11]

During his sojourn in Florence, Torricelli developed a new method of making glass lenses for telescopes. The Torricelli lenses were acknowledged during that time to be unsurpassable and the method of their manufacture was kept in strict secrecy by the Grand Duke. One must not forget that in the 17th century spy glasses were a paramount military significance, for they made it possible to watch the enemy and to warn against a sudden attack. In one of his letters (May 1, 1644) to the French mathematician and physicist Mercenne (1588-1648) Torricelli describes his activity in Florence in the following words: "Two parts of my book were already printed, and the remaining two parts progress more slowly, since I am exceedingly busy either in the telescope-lens shop or else constructing from day to day various machines and setting up different physical experiments, both under the orders of my master the Grand Duke."

During this five year's stay in Florence, Torricelli was thus able to carry on tireless work in the field of physics and engineering. However the heavy load exhausted him more and more. In October 1647 Torricelli took to bed, plagued by "fever" and terrible headaches. The condition of the sick man became worse and the headaches became unbearable. Torricelli fell into a "senseless furious delirium," in the words of his friend, Sirenai, who wrote to Torricelli's brother on the impending catastrophe. On October 24, 1647 Evangelista Torricelli died.

Apparently, at the very beginning of his sickness Torricelli detailed his method of polishing lenses in a secret letter addressed to the Grand Duke, at the latter's request. In one of his conscious moments he dictated a detailed communication. He entrusted to his friends Cavalieri and Ricci the publication of all his unpublished manuscripts. He asked to be buried without any honors, but in the Basilica of the Church of St. Lawrence in Florence, "if the Canon Fathers will consider me worthy."

Not one of these wishes in Torricelli's last will were fulfilled. Cavalieri died a month after Torricelli, and Ricci could not find a publisher for the manuscript. The unpublished manuscripts of Torricelli's works and notes were handed down from person to person as late as the 18th century and were finally sold — to a retail shop for wrapping paper. A certain Clemento Nelli, who bought sausages in the shop and took home the wrapping, noted on it with astonishment an autograph by Galilei. He bought the rest of the manuscripts, hoping to publish them, but did not succeed. In 1818 the Torricelli manuscripts were purchased by the reigning Duke of Tuscany and in 1861 they were finally deposited in the National Library of Florence. Only in 1919 were these remaining manuscripts published. Apparently, the best preserved were the mathematical papers. A considerable portion of the papers on physics, mechanics, and engineering and all his literary works were irretrievably lost (reference 4, Vol. I, p. 15).

The canons of the Basilica of St. Lawrence apparently did not believe Torricelli "worthy" of burial in the church. He was buried somewhere near the church with other simple parishoners. The Grand Duke planned to erect a memorial stone on his grave but never got around to it. Torricelli's grave was thus lost.

Let us turn now to Torricelli's physical discoveries. It must be emphasized that they have not yet been studied in their entirety. The preserved researches of Torricelli cover the following branches of physics: particle and body mechanics, hydromechanics, physics of the atmosphere, geometrical optics, and the technology of lens manufacture. Torricelli's principal works on particle and body mechanics are contained in his "On the Motion of Heavy Bodies Descending in a Natural Way and of Projectiles" (De motu gravium naturaliter descentium et projectorum), in three "academic lectures" "On Impact" (Della percossa), and in unpublished manuscripts collected under the name "Various Data on Motion and Momenta" (De motu ac momentis varia).

The treatise "On the Motion of Heavy Bodies" is originally a strictly systematized treatment of Galilei's views, stated by him in popular form in his "Dialogues and Mathematical Proofs Concerning two new Branches of Science, pertaining to mechanics and local motion."[1] However, when published in 1644, Torricelli's essay contained a development of certain premises and an expansion of this essay. Thus, he develops problems of external ballistics to a greater extent than Galilei and gives more detailed tables of the superelevation of guns. The corresponding parameters of projectile trajectories which Galilei uses as an illustrative numerical example of parabolic motion, acquires a more practical character in Torricelli's work. The entire treatment of this problem is used by Torricelli clearly to help in a direct application of his tables to artillery.

Reviewing Torricelli's research on the motion of heavy bodies, the well-known Italian physicist R. Macolongo indicates[8,9] that Torricelli set out above all to prove Galilei's postulate that bodies that fall along slightly inclined planes from equal altitudes should have equal velocities. Torricelli's new proof of this postulate is based on the principle of the center of gravity, predicted by Galilei, but included by Torricelli in a more general principle: "Two heavy bodies cannot move by themselves jointly unless their common center of gravity descends." From this postulate Torricelli derives Galilei's famous theorem on the motion of two weights interconnected by a string running over a pulley. The next novelty introduced by Torricelli, as indicated by Marcolongo, is that no matter what the superelevation, the trajectory of a projectile is a parabola. Torricelli proves Galilei's entire theory of motion of projectiles in a simple manner and at the same time more rigorously, using the geometrical method in a consistent manner.

Many of Torricelli's papers, dealing more in mathematics than in mechanics, are devoted to the problem of finding the center of gravity of various plane figures and bodies. This problem was the topic of Torricelli's correspondence with Cavalieri and Ricci. Torricelli, as is known, developed the so-called "method of indivisibles," proposed by Cavalieri, and first treated this method in clear and understandable form. In this field, Torricelli along with Cavalieri, deserves being called the predecessor of Newton and Leibnitz in the discovery of infinitesimal analysis.[10]

As a rule, those studying Torricelli's papers on mechanics omit his popular lectures on percussion. Yet these lectures contain quite a number of deep physical ideas, which are of undoubted historical interest.

"The force of percussion, writes Torricelli, belongs among the most effective of all the discoveries of mechanics, and is probably the most hidden and the most obscure of all the secrets of nature." (reference 4, Vol. II, p. 5).

Torricelli then proceeds to repeat essentially the contents of the Sixth Day of the "Dialogues." But Torricelli goes deeper and develops further the individual problems raised by Galilei in his "Dialogues."

Thus, comparing the pressure of a heavy body at rest with the pressure arising upon impact of a falling body, Torricelli dwells in particular on certain properties of gravity. "The gravity of the bodies of nature is the source from which momenta continuously originate," writes Torricelli, "and a heavy body creates force at every instant . . ." Torricelli, uses, essentially, Galilei's terminology, defining a moment as "that force, that effort, that action with which the mover moves and the moved body resists." (reference 1, remark 22). Torricelli indicates that gravity is the perpetually discovered "source of momenta," which "at any given instants of time or (if you don't like the word instant) at any briefest time interval produces a moment equal to the absolute weight of a given body . . . and in fact, when heavy bodies are at rest," continues Torricelli, "all these momenta (impeti) . . . are annihilated by the supporting body, which, without concealing its opposing action, continuously suppresses all these nascent momenta. However, when the same heavy body fall in air, all these momenta are not suppressed, but are retained in it and multiply." (reference 4, Vol. II, p. 15).

This splendid description contains practically the entire concept of force as being the momentum acquired by the body within a certain brief time interval.

In his discussion of the momentum of a moving body, Galilei indicated that this momentum is determined by the weight and velocity of the moving body. Torricelli goes further. He replaces the word "weight" by a new definition — quantity of matter (la quantita della materia) (reference 4, Vol. II, p. 25), thus coming close to the concept of "mass" proposed by Newton 40 years later. In explaining the role of this "quantity of matter" in the reaction of a body to impact, Torricelli remarks: "There is no doubt that the matter itself is dead and serves only to prevent an acting force by opposing it. Matter is none other than Circe's vessel, sung about by the poets, which serves only as a container for the force, momenta, and impulse. The force and the impulses are such thin substrata, such imaginary quintessences, that they cannot be contained in any other vessel than the bodily nature of the natural solids. This is my opinion, "emphasized Torricelli," since the human force appears only in what he works upon and what he touches" (reference 4, Vol. II, p. 27). The above quotations are enough to show that Torricelli's lecture "On Percussion" contains material of interest to the historian, evidencing, in our opinion, that Torricelli was much closer than Galilei to the 18th century concepts of mechanics.

The second book of the treatise "On the Motion of Solid Bodies. . . and Projectiles" Torricelli includes a small section "on the motion of water" (reference 4, Vol. II, pp. 185-197). Here we have for the first time Torricelli's famous problem on the shape of a stream of liquid flowing from a vessel. The solution is based on the following assumption: Water which escapes abruptly has, at the point of escape, the same momentum (impetus), as would have some solid body or a single drop of water, were it to fall in a natural manner from the uppermost surface of the water to the opening from which it escapes." Proving next that the trajectory of such a drop will be a parabola, Torricelli notes that experiment "confirms to some degree our postulate." He then dwells on this experiment. A horizontal nipple with small vertical holes is placed in the bottom of a large vessel. If the holes are first sealed with the finger and then uncovered, it can be seen that the first drops breaking away from the hole rise to the level of the water in the vessel and describes a parabola. Torricelli correctly explains why the crest of the water jet lies just below the level of the water. He next indicates that if someone has the facility to verify this principle experimentally, it should be done not with water, but with mercury "since it has a greater internal gravity," i.e., a greater specific gravity. Then Torricelli describes "an experiment which confirmed almost all the details of our premises" (speculatiunculas). The height of the vessel, made in the form of a parallelopiped, exceeded "a geometrical pace" and the area of the bottom was not less than that of the palm. A round tube led out of the vessel. "The holes were in fact round, somewhat larger than the human pupil and were not made roughly, but drilled very carefully in copper thick plates, vertically mounted. The water which escaped vigorously always left perpendicular to this plane, from which it broke away, and it was therefore necessary that the outlets from our tube be horizontal."

The experiment has shown that a jet of water that breaks away horizontally curves into a parabola (reference 4, Vol. II, p. 188).

The important factor, naturally, is that Torricelli's premise is based on the law of conservation of momentum. Neither Torricelli nor Galilei formulated quantitatively the dependence of the momentum of the mass and velocity, and it can therefore hardly be said, as is done by several investigators, that Torricelli, like Daniel Bernoulli, began with the law of conservation of the "live force" (mv^2). Both Galilei and Torricelli were already very close to the concept of "live force," but its mathematical expression remained unknown to them.

However, Torricelli not only showed that a jet escaping laterally from a liquid-filled vessel assumed the form of a parabola, but also proved that the speed of the escaping liquid, meaning also its quantity was definitely related to the height of the column of liquid above the opening. He came to the conclusion that the speed is proportional to the square root of the height. Torricelli was not yet acquainted with the formula $v = \sqrt{2gh}$, first given by Johann and Daniel Bernoulli almost a century later. Torricelli uses $v = A\sqrt{h}$, where h is the height and A a certain constant; he thus gets $v : v_1 = \sqrt{h} : \sqrt{h_1}$.[12]

Along with research on the principle of hydromechanics, Torricelli is responsible for several outstanding projects in hydraulics, directly connected with the development of water-power structures on the control of the flow of the river channel. This set of problems, however, is outside the scope of this article.

We now turn to Torricelli's investigations in the field of the physics of the atmosphere. The principal problems in the physics of the atmosphere were then the problem of whether air has any weight and the problem of the origin of wind. Torricelli rendered great services in both problems. In all probability Torricelli wrote an unpublished report of his investigations, which was not preserved. The description of Torricelli's famous experiment with the barometer was actually preserved only in the form of a private letter addressed to Ricci (reference 4, Vol. III, p. 186). The problem of the weight of the atmosphere and of the origin and nature of wind is mentioned only in the popular lectures, delivered at the Academia della Crusca.[16]

At that time the problem of the weight of the atmosphere had a history dating back for several centuries. Aristotle stated that "everything in its place has weight, with the exception of fire and air." (’Εν τῆ ἀντοῦ χώρα, παντα βάρος ἔχει, πλην πνρὂς καιὁἀὴρ) (p 13). During these centuries some authors expressed themselves in favor of Aristotle, and others, to the contrary, were decisively in opposition. Thus, Galilei in his earlier work "On Motion" (De motu) indicated that "in spite of Aristotle's opinion, one cannot call something merely light or heavy . . . all bodies have weight, some more and some less, depending on whether their matter is compressed and packed together or diffused and rare."[13] This problem was analyzed in detail by Torricelli in his two lectures "On Lightness." Here he first criticized severely Aristotle's premise that absolutely heavy bodies tend to their "natural place below," where they come to a state of rest. On the other hand, absolutely light bodies like fire, tend, according to Aristotle, to go upward. "It is no news to me," said Torricelli, "that specially prepared mirrors gather the rays of light and that certain vessels and rooms concentrate the lines of sound in a single point. But that it is to me novel, unthinkable, and unheard of is that nature would endow the world with some internal principle of motion towards the center, towards a single point with a trend towards an eternal catastrophe" (reference 4, Vol. II, p. 44). "It is quite impossible," says Torricelli ironically, "that the elements of the earth and water could ever reach this cherished center and fall into it." Refuting Aristotle's idea as absolutely absurd, Torricelli insists on a formulation identical with that of Galilei: all bodies have weight.

The problem of vacuum has been closely connected with the problem of the weighability of air since ancient times. According to Aristotle, the concept of vacuum contains a logical contradiction, for "a place without a body located in it" cannot exist. For many centuries the greatest scientists have stated that vacuum is impossible in principle. Ibn-Sinna (Avicenna), for example, stated that if a vacuum would form anywhere on earth, the sky itself would split and descend to the earth in order to fill this vacuum. Thus there has gradually grown the conviction that nature experienced a sort of fear of vacuum (horror vacui). By vacuum was meant a space containing no air. It therefore became an axiom that it is impossible in principle to obtain a space without air. Galilei also adhered to this point of view in his "Dialogues" using as an example the sticking of two plates: "This experiment clearly shows that nature does not desire, for the shortest interval of time, the formation of a vacuum that would be produced between the plate until the instant when the surrounding air would occupy the corresponding space: . . . the resistance to the formation of vacuum, like that observed in the example of two plates in close contact, undoubtedly exist between different parts of one solid body and is at least one of the causes of its adhesion" (reference 1, pp. 63-64). This problem is considered further in the "Dialogues" in connection with the discussion of the impossibility of raising water with a pump to a height more than 18 cubits. Galilei was not only firmly convinced that air has a weight in principle, but was first to determine experimentally the specific gravity of air by means of a very clever experiment, described in the "Dialogues" (reference 1, p. 170-171). At the same time Galilei stated that the upper layers of the air need not press on the lower ones. The principal argument was an incorrect statement by Galilei that the pressure experienced by a solid in water independent of the height of the liquid column above it. He assumed that "water has no gravity at all in water . . . and if water in water weighs nothing, how can the lower layers be compressed by the upper ones?" Galilei used this erroneous statement to draw analogous conclusions regarding the atmosphere. "Note that all of the air in itself and above water weighs nothing . . . it is not surprising that the entire atmosphere weighs nothing, since the situation is exactly the same with water."[17] Galilei saw a weighty argument in favor of his theory in the fact that a fish does not feel the weight of the water and in the fact that neither animals nor humans feel the weight of the atmosphere. Denying a priori the possible existence of atmospheric pressure, Galilei was forced to look outside the atmosphere for the cause of the observed fact that no single piston pump could lift water to a height greater than 18 cubits.

Thus, Galilei ascribed a principal role in this fact to the circumstance that the nature so to speak opposes the appearance of vacuum. He concluded that the water column is acted upon, on the one hand, by the ill-famed resistance to the appearance of vacuum, which drags this column upward, while the weight of the column itself pulls it downward. As the water column lengthens "it breaks of its own gravity, the same as would occur with a string . . ." He furthermore says: "After determining the weight of water contained in 18 cubits of the pump pipe, no matter what the diameter of the latter, we can determine the value of the resistance to the formation of vacuum in a strong cylinder made of any material" (reference 1, page 72).

This theory of Galilei was refuted by some of his contemporaries. Thus Descartes, who was in the Netherlands in 1639, and who obtained from Galilei a personal copy of the "Dialogues," wrote to Mercenne: "What he ascribes to vacuum must be ascribed to none other than the weight of air. One can be assured that were the horror of vacuum to prevent the separation of two bodies, there would in general exist no force capable of separating them."

Shortly before Galilei's death the Parisian mathematicians sent him several remarks concerning his "Dialogues": "We believe that vacuum in no way prevents the separation of two polished spheres placed one on top of the other. This is rather done by the air, which presses on the top and on all sides" (reference 13, pages 99-100).

In 1644 Torricelli undertook in Florence an experimental investigation of air pressure. "I emphasize," wrote Torricelli on June 11, 1644 from Rome to his friend Ricci, "that I had no intention to make a philosophical experiment with respect to vacuum, nor merely produce this vacuum, but simply to produce an instrument that would show the variations in the air when it is heavier and denser and when it is lighter and thinner. Many have said that vacuum cannot be realized, and others that is is realizable with difficulty owing to a resistance on the part of Nature. I have observed the following: there is a principal reason for this. This reason, so to speak, prevents formation of vacuum. I believe it purposeless to attempt to ascribe to vacuum an action that is clearly caused by a different cause. Having made reliable and very simple calculations, I find that the reason proposed by myself (namely, the weight of the atmosphere) could in itself cause a greater resistance than it does produce, without resorting thereby to vacuum . . . We live submerged on the bottom of the sea of the elementary atmosphere. Experience shows without any doubt that this atmosphere has weight and the weight is furthermore such, that being at a maximum near the earth's surface, it equals approximately 1/400 of the weight of water . . . It is natural, the weight of the air is experienced at this bottom both by humans and animals, and that on the peak of a high mountain the air begins to be purer and much lighter in weight than 1/400 of the weight of water" (reference 4, Vol. III, pages 186-187).

Torricelli then describes in great detail the famous mercury-tube experiment which by now has become a conventional classroom demonstration. At first the experiment failed, but then Viviani, helping Torricelli in this research, overcame all the difficulty. Taking a tube two cubits long, the investigators filled it with mercury and covered the open end. When this end was uncovered, the mercury dropped and stopped at a level of $1^1/_2$ cubits. It was shown that this height was independent of the shape of the upper, sealed end. The instrument invented by Torricelli was named "barometer" by Mariott only in 1676.

It must be emphasized that the invention of the barometer was far from a chance result of Torricelli's investigations, but represented a logical and unavoidable conclusion of his work on the physics of the atmosphere. The extent to which Torricelli was far ahead of his time in this respect, can be readily seen from his lecture, "On the Wind." One must assume that in Torricelli's unpublished papers there are more detailed treatments of the investigations that served as the base for this popular lecture.

"The philosophers state," says Torricelli, "that the wind owes its origin to certain hazy evaporations from the moist earth. They noted that after rain the wind is usually stronger and lasts longer than ordinarily, and they therefore state that when the earth is covered with moisture the force of the sun rays and of the underground heat produces two sorts of evaporations, one moist, which is the source of the future rain, and the other dry, which produces the winds ... But if all the rains should produce two sorts of evaporations, one serving to produce wind and the other for future rains, is it not clear to anyone that the matter making up the rains will continuously diminish, and the matter producing the winds will continuously increase? But let us go further, for since the school of the philosophers has mastery over the transmutation of elements, it will soon find an answer to this objection.

I personally doubt above all the observations themselves. It is actually known that after the rain the winds are most frequently northerly ones. However with respect to southern winds this rule is not only inaccurate, but very frequently quite the opposite takes place. The Sirocco winds blow during the day almost always before the rain, until it begins, and die down after it stops raining. And yet, according to the opinion of the peripatetics, they should blow stronger after the wind than at any other time, since the moist earth is more amenable to evaporation of the elements contained in it. Furthermore, in this case there should be evaporated from the earth a larger number of fogs and clouds, when both causes act simultaneously, i.e., during the warm season and the earth is moist. And when is there a more favorable conjuncture for wind to arise than after rain, when indeed the southern winds arise? It is then that we have water-filled cracks in the trees, flooded meadows, and heavy streams. What can be more? Even the insides of the houses are so damp in those days, that marble begins to sweat. Isn't there enough moisture available during that time of the year from the noontime disease-laden oppressive heat, when the sirocco carries with it heavy air, which literally seems to blow from a furnace, and everything alive, afflicted by the heavy heat, can hardly keep on its feet? To the contrary, in final analysis, the strongest north winds come after some other rain there. Yet, the dry air, subjected to the action of the northern cold, should not have enough force to lift a large amount of vapor, if what the philosophers say is true, that both heat and moisture are needed to produce wind. What can we say about winds that arise spontaneously, not preceded by any rain? Not only the speculative philosophers, but even uneducated wayfarers are aware of these winds, which occur at an exactly fixed time and which prevail reliably ... " Torricelli then recalls the wind that regularly blows before the rise of the sun, the evening zephyrs, and the prevailing winds that blow in different countries during different times of the year.

Incidentally, he examines the opinion of the ancient scientists that "if a certain amount of water is converted into air in some manner, it expands by a factor of ten times and occupies ten times the volume." Torricelli speaks on this subject: Modern scientists, more inquisitive and thus more resourceful, have found through difficult experiments that if water is converted into air, it increases in volume not by ten times but by 400 times. Since this principle is now known, we see that not merely one rain, but even the entire ocean could not produce enough matter for the undefatiguable wind, that lasts sometimes eight or even ten days."

The moisture of rain, in Torricelli's opinion, is distributed in various ways and only a minute fraction of it evaporates. However, its volume is so large that, in expanding, it covers a great part of Europe. Were such a wind to blow, it could cover not only small Italy but also Spain and France and Germany, and all other countries, which all taken together amount to a rather large portion of the inhabitated world. The height of such a current or influx of air would reach at least three or four miles.

Refuting the speculations of the philosophers on the origin of the wind, Torricelli continues: "But isn't there some clear feature whereby to identify the true cause of winds, based on a single principle, which could be proved to exclude all others?

This principle is none other than the well-known and common principle of compression and rarefaction of air. The most venerable cathedral, Santa Maria del Fiore, and even more so the Rome Basilica, have the ability of ejecting, in the hottest summer days, quite a lot of fresh air through their own doors, at a time when the air is calm and there is no wind at all (on the outside). The reason for this is as follows: The air contained in the large building is for some reason cooler than the external air, which is heated by the strong direct and reflected rays of the sun. But if the air is cooler, it is also denser, meaning that it should also be heavier. This is why the air should indeed flow to the outside, and as much air flows in through the higher windows and flows out through the door. In the Rome cathedral during noon hours the freshness of the wind not only fails to caress, but even hurts. The wind from the doors of this cathedral is so strong that it causes a surprise.

Let us apply the same observation not to the closed volume but to the great extent of the open atmosphere... let us imagine that the whole northern hemisphere is quiet and completely at standstill, without a single gust of wind, without a single motion of the air. Let a sudden rain or some other cause, without changing anything in the remaining portion of the hemisphere, make Germany cooler than usual. Undoubtedly, the cooled air over this entire extensive country becomes denser. In order for it to become denser it is necessary that in the upper portion of the atmosphere above Germany there appear a certain emptiness, caused by the foregoing increased density. The air over the neighboring countries, being fluid, will try to fill this unexpectedly created emptiness. The upper portions of the atmosphere will then race, in the form of a wind, towards the cooler portion. But in the lowest region, i.e., the part of the air adjacent to the earth, an opposite flow will occur. In spite of the fact that Germany is already covered with dense air, the air there will become even denser, and, what is particularly distressing, will cause a motion of air everywhere... Thus, wind should be in the form of a circulation that cannot be avoided over any limited portion of the earth. The effect of this circulation can continue as long as its cause exists."

A study of Torricelli's lecture "On the Wind," fragments of which we have just cited, show that to him belongs not only the honor of discovering the origin of the wind, but that he as early as in 1644 outlined the principles of the general circulation of the earth's atmosphere.

One can thus assume that the invention of the barometer was obviously only a portion of Torricelli's intensive program of experimental meteorological investigations, interrupted by his death.

Historians of science frequently assign priority in the invention of the atmospheric circulation to various scientists of the 18th century, losing sight of the fact that Torricelli's "Academic Lectures," first published in 1715 (and now almost forgotten), should have been well known to the scientists of the first half of the 18th century.

Let us now turn to Torricelli's optical research Not one paper by Torricelli on geometrical optics has survived. However, in the salvaged correspondence we find mention of the fact that Torricelli, like many of his contemporaries, worked on this problem. Of particular interest from this point of view is a postcript to a letter addressed to Ricci on February 6, 1644: "Yesterday I was in high grace of the Grand Duke, who presented me with a chain worth 300 scudi. His Highness was extremely pleased with my invention concerning the preparation of lenses, accomplished by geometric considerations in conjunction with a study and knowledge of conic figures and the science of refraction" (reference 4, Vol. III, page 167).

In a letter of February 16 of the same year, Cavalieri warmly congratulates Torricelli for "he discovered something new in the problem of refraction and concerning lenses for telescopes" (ibid). And in a subsequent letter dated March 15 Cavalieri reports that both he and his students and friends await impatiently news on the nature of this discovery, which has enabled Torricelli to manufacture lenses of unusual quality. However, we know that this invention was kept secret by the grand duke, and Torricelli never reported it to any of his friends. At the same time it is known that the lenses made by Torricelli were very famous during his time. Naturally, certain master opticians attempted to surpass Torricelli and to compete with him. However, any time that Torricelli's friends reported the arrival of a new master, Torricelli invariably answered that his lenses could never be surpassed.

Since the only description of Torricelli's invention, submitted by him to the grand duke before his death, has been lost, the great Italian optician V. Ronchi attempted in 1924 to clarify this problem by directly investigating the remaining authentic Torricelli lenses and the notes made in his own writing. Ronchi has subjected Torricelli's lens (10 cm diameter, focal distance 5.7 m) to the ordinary modern meniscus-surface tests by comparing it at the National Optical Institute with a standard modern lens, using a special interferometric instrument. Here is what Ronchi writes on this subject: "The attached photograph shows interference fringes between the surface of the Torricelli lens and the model optical surface. The regualrity of the concentric lenses shows that the surface finish of Torricelli's lens has the maximum desired optical accuracy; in general, it is at the limit of optical perfection."[14]

Analyzing this striking result, Ronchi finally concludes that the "secret" of Torricelli's invention in lens manufacture lies in his inventing a meniscus shape that produces minimum spherical aberration. Actually the genuine Torricelli lenses are in themselves evidence that this condition was satisfied in their manufacture. This conclusion becomes the more likely, since Torricelli remarks in one of his letters in passing that in solving the problem of lens manufacture he used a method he developed for finding maxima and minima.

What is also remarkable is that Torricelli found some method of testing the correctness of his lenses.

Our brief survey of the life and the physical research of the great son of Italy, Evangelista Torricelli, makes no pretense whatever of completeness. We should like, in conclusion, to recall that this remarkable figure, one of the outstanding founders of the modern science and an active fighter for its triumph, still awaits a biographer.

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Translated by J. G. Adashko

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Meetings and Conferences

PLANCK CELEBRATIONS IN BERLIN AND LEIPZIG

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AT the end of April of this year the physicists of the whole world celebrated the hundredth anniversary of the birth of the outstanding theoretician and thinker, the founder of quantum theory, Max Planck. The German Academy of Sciences in Berlin (German Democratic Republic), the Physics Society of the German Democratic Republic, and the Union of Physics Societies of the German Federal Republic have organized on this occasion a joint celebration in Berlin on April 23 and 24. Timed with these was the annual Congress of Physicists of the German Democratic Republic and a theoretical conference held in Leipzig (April 27-30). Participating in the Planck anniversary congress and the Leipzig conference was a Soviet delegation, headed by N. N. Bogolyubov, and comprising, in addition to the author of this article, S. A. Azimov, A. F. Ioffe, A. V. Ioffe, G. P. Keres, V. I. Mamasakhlisov, A. B. Migdal, G. I. Rakhmaninov (secretary of the delegation), A. A. Smirnov, B. I. Stepanov, and Zh. S. Takibaev. In addition to participating in this congress, the members of our delegation inspected the Physics Institutes and delivered several scientific and popular papers.

In attendance at the anniversary celebrations, the congress, and the conferences were the entire membership of the physicists of the German Democratic Republic headed by Nobel laureate Gustav Hertz, the Director of the Physics Institute of the Leipzig University; R. Rompe, Secretary of the Division of Mathematics, Physics, and Engineering of the German Academy of Sciences (German Democratic Republic); Professor Barwich, Director of the Dresden Nuclear Center; Professor Herlich, Scientific Director of the Zeiss Enterprises, and others.

Of general interest was the extensive representation on the part of physicists from the German Federal Republic, including Nobel laureates W. Heisenberg, M. Laue, M. Born, O. Hahn, and also the outstanding theoreticians Hund, Bopp Hönl, Sauter, and Bagge. It was also pleasant to meet at the congress our close friends, physicists from China (Hu Ning) Poland (L. Infeld, W. Rubinowicz, and others) Hungary (L. Janossy, Budo), Bulgaria (Datsev), Rumania (Titeica).

Western science was represented by Nobel laureate P. A. M. Dirac (England), H. Møller (Denmark), Lise Meitner (Sweden), Nobel laureate J. Frank, V. Weiskopf, R. Courant (U.S.A.) and others. France was represented by a delegation comprising Prof. Bauer, Prof. J. P. Vigier, Professor M. A. Tonnela, and Dr. Kahan. Twelve countries were represented in Berlin.

The Planck celebrations were held in East Berlin on April 23 and in West Berlin on April 24. In the German Democratic Republic they had a national and state character. They were extensively covered in the national, local, and student press, and also on radio and television. The Peoples Parliament sent a special message to the Physics Society. The central organ "Neues Deutschland" carried a message from the Central Committee of the United Socialist Party of Germany. W. Ulbricht and Otto Grotewohl attended the celebration reception, held at the State Opera House.

An imposing session, which was broadcast, was organized in West Berlin by the Physics societies of the Federal Republic in the recently remodeled 1700-seat House of Congress. The press of various western countries paid considerable attention to the Planck anniversary. We note that the U.S.S.R. Academy of Sciences held a special session in Moscow in Planck's honor, and that scientific gatherings took place at Moscow and Tbilisi Universities and other Soviet scientific institutions.

Such a tremendous attention to the Planck anniversary is quite understandable, since Max Planck's discovery of the quantum of action and the formulation of the principles of the quantum theory of radiation was a momentous accomplishment, which opened the road to the recognition of atomic and nuclear phenomena and their further application. Furthermore, Planck stands before us as a deep thinker, who did not confine himself to the solution of individual problems, no matter how important, but attempted to interpret the entire structure of physical science, and led the struggle against the "Machistic" positivistic tendencies. Planck's extensive organizational and pedagogical activity is also well known. Finally, in our time, when humanity is threatened by atomic war, physicists and all men of good will cannot fail to be impressed by his passionate appeal for the utilization of the forces of nature only for the good of humanity, an appeal he voiced shortly before his death, frequently recalled in the speeches at the celebration.

Before proceeding to a more detailed description of the Planck celebration, it is appropriate to cite briefly some biographical highlights.

Max Planck was born on April 23, 1858 in Kiel, to a family with a long tradition of interest in science. From 1875 to 1877 he studied at the Munich University. It is known that Planck hesitated before choosing a profession for his interest in music was very great. Lise Meitner recalled in her address that Planck devoted much of his leisure time to music, gathering about him a large circle of scientists. Professor Westfahl recalled one remarkable home concert, in which pianist Planck and violinist Einstein were joined by singer Otto Hahn. In 1878 Planck attended lectures by Kirchhoff, Helmholtz, and Weierstrass in Berlin. In his memoirs, which were recently made available in Russian translation (see Usp. Fiz. Nauk, **64**, No. 4, April 1958) Planck notes that the courses in theoretical physics were not as useful as one might expect from the famous lecturers: Kirchhoff read, rather too glibly, a written text; to the contrary, Helmholtz, who prepared little for his lectures, was endlessly confused at the blackboard. Later on Planck struck up a deep friendship with Helmholtz, who was instrumental to the young scientist's being invited to join the Berlin University after Kirchhoff's death in the middle of 1892. In the preceding years, Planck's principal interests, starting with his doctoral dissertation (equivalent to the Soviet candidate dissertation) of 1879 ("Equilibrium State of Isotropic Bodies") were in the field of thermodynamics. Planck's early works, akin to physical chemistry, attracted no particular attention in science. Planck himself recalled that Clausius answered none of his scientific letters. The first decade of his professorship at Berlin was the blossoming time of Planck's creativity. Keeping close contact with experimenters who investigated the properties of equilibrium electromagnetic radiation at various temperatures, Planck announced his famous formula and soon derived it by introducing the quantum of action. Planck first lectured on his formula on October 19, 1900, at the session of the German Physical Society, as recalled with justified pride by Professor F. Trendelenburg, present President of the Union of West-German Physical Societies.

Planck's subsequent scientific activity was connected with the justification of the fundamental relations he derived and an analysis of the new world constant, with various problems in statistical physics, and also with the newly created relativistic mechanics. Planck himself understood the tremendous significance of his discovery. In his Nobel prize address and in many other speeches and articles he frequently returned to the arguments that led to the discovery of the quantum of action. Together with his autobiography, these papers by Planck represent material of exceeding value to the history of science and to the psychology of scientific creativity. In Berlin, Planck went through all stages of an advanced scientific career. In 1903-1904 he has the Dean of the so-called Faculty of Philosophy (which included the physical-mathematical division), and in October 1913 he was chosen Rector of the Berlin University. In 1926 Planck retired but continued to lecture, conduct seminars, and deliver numerous papers on general scientific themes. As is well known, Planck was not only an outstanding scientist, but a first-class teacher. He wrote a valuable text on theoretical physics, which was also translated into Russian. Along with his work at the university, Planck was active in the Berlin (then the "Prussian") Academy of Sciences to which he was elected in 1894, nominated by Helmholtz, Kundt, and Betzold. For 26 years he was the secretary of the Natural Science and Mathematics Division of the Academy. Planck was among those responsible for inviting Einstein to Berlin and for his election to the Academy, and was one of Einstein's close friends.

It must be emphasized that Planck was one of the outstanding scientific organizers not only within the framework of the university and the academy, but also as the President of the Kaiser Willhelm Society. This society, thanks to its relations with industry, included the better scientific centers of Germany. In its time the society included the Institutes of Otto Hahn, W. Heisenberg, and others. At the present time the society, which has been renamed after Max Planck, continues its activity in West Germany. One of the focal spots of the Berlin celebrations was the presentation of a bust of Planck to Otto Hahn, present President of the Max Planck Society, from the Academy of Sciences (German Democratic Republic) at the session held in the eastern sector of Berlin. The audience, who saw in this act a gesture of unity of scientists of both parts of Germany on the basis of Planck's humanistic legacy, reacted to this with a loud ovation.

Along with the scientific and organizational activity, mention must be made of Planck's work on the philosophy and history of physics. His struggle against the "Machistic" positivism, which was popular among many scientists at the end of the 19th century, is well known. Planck's services in the establishment of atomism are tremendous. He succeeded, in particular, in deriving on the basis of his formula the then best value, of the electronic charge 4.59×10^{-10}, which is quite close to the present-day value 4.80×10^{-10}, instead of the experimental $(6.15 \pm 1.5) \times 10^{-10}$, obtained by the better experimenters of that time. A recollection of this fact in the lively paper by Gustav Hertz, devoted to the experimental foundations of quantum theory, incited a heated reaction in the audience at the celebration session in West Berlin.

Many features of Planck's personality were outlined in beautiful addresses by Lise Meitner, at the relatively restricted session in the small hall of the Physics Society in Berlin, and by Professor W. Westfahl at the session in West Berlin. Along with his love for music, Planck was to his last days a passionate sportsman, interested in skiing, mountain climbing, and systematic long hikes. Lise Meitner spoke of an impression of a certain dryness and reserve that Planck exhibited at first acquaintance, and compared it to the live and jovial Boltzmann whose lectures she attended in Vienna before she began working in Berlin, first with Planck and then with Hahn. For all that, his closer friends found Planck to be a soft, responsive person of very high ethical principles. Meitner dwelled also in sufficient detail on the unique, nearly pantheistic religious feelings of Planck, quoting by Goethe on this topic.*

The last years of Max Planck's life were clouded by the coming to power of the Fascist Regime and by the second world war. In the speeches at the celebration sessions, H. Fruehauf, Vice-President of the Academy of Sciences (German Democratic Republic) and Lise Meitner touched in detail on Max Planck's manly and noble behavior during these difficult years. Planck's first son was killed in Verdun in the first world war. At that time Planck, not succumbing to the chauvinism that infected the country and many scientists, proposed to delay the expulsion of the French members of the Academy until after the war. His second son, a general, was executed in January 1945 for his participation in the anti-Hitler conspiracy. Planck's house in Berlin-Grünewald was destroyed by bombs, and Planck himself was saved during one of the air attacks from a buried shelter in Kassel.

Planck died in Göttinger on October 4, 1947. His words on the "danger of self-destruction that threatens all humanity should atomic bombs be used in considerable quantities in the next war" serves as a call to all men of goodwill.

Let us now dwell on some high spots of the Planck celebrations.

As already mentioned, they began in a session in the rebuilt opera house in Unter den Linden. The program included the greetings by the M. Vollmer, President of the Academy of Sciences (German Democratic Republic), a speech by H. Fruehauf on Planck's activity as the permanent secretary of the Academy, a paper by M. Laue on Planck's scientific work, and the presentation of a bust to O. Hahn. This was followed by a session of the Physics Society in the historical building in Kupfergraben, in which Magnus founded in 1842 the first physics institute in Germany, where Kirchhoff, Helmholtz, Dubois Raymond, Warburg, and many other great scientists worked. It was visited by many Russian physicists of the 19th century,—Umov, Golitsyn, and Lebedev. In the name of the City Council of Greater Berlin, Mayor Ebert transferred this building to the Physics Society of the German Democratic Republic. A. F. Ioffe, warmly greeted by the audience, transferred to the Society Planck's personal library, which was in safekeeping the the Soviet Union. As already mentioned, Lise Meitner delivered at this session her stirring recollections of Planck. This was followed by the celebration reception and banquet in the Apollo Hall of the State Opera, where greetings were read from the Academy of Sciences of the U.S.S.R. (N. N. Bogolyubov), the Royal Society (P. A. Dirac), the Academy of Sciences of the U.S.A. (V. Weisskopf), and other scientific institutions of Germany and foreign countries.

The crowded first day of the Planck celebration, which was the central one, concluded with the delegates attending Gluck's "Iphigenia in Aulis."

On the morning of the next day, April 25th, a celebration in the honor of Planck was held in the Humboldt Berlin University, at which a concert was given (Bach's Brandenburg Concerto No. 1 and the first part of a Mozart quartet), and which concluded with the unveiling of a memorial plaque. The rector and professors of the university paraded in academic procession.

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* It is interesting to note in this connection that W. Heisenberg, who lectured on April 28 in Kotbus (German Democratic Republic), at the invitation of the local Lutheran Society, on the ethical consequences of modern atomic physics, emphasized the need of unification of humanity on the basis of peaceful utilization of nuclear energy and other scientific accomplishments and indicated the similarity between such views and Christian morality. Apparently traditions of a unique religious feeling, in part pantheistic, in part of the Lutheran type, are in general quite strong in certain circles of the German intelligentsia.

Simultaneously, in one of the auditoria of the West Berlin so-called "free university," under the chairmanship of Professor Ludwig, a theoretical seminar was held at which Heisenberg answered many questions, submitted beforehand, on the development of nonlinear field theory.

The focal point of the celebrations during the second half of this day was in West Berlin, at a session consisting of the an introductory address, by the Chairman of the Union of the Physical Societies of West Germany, F. Trendelenburg (Erlangen), a lecture by Gustav Hertz (Leipzig) on the significance of the Planck theory to experimental physics, a lecture by W. Heisenberg (Göttingen) on the discoveries of Planck and philosophical problems of atomic theory, and a speech by W. Westfahl (West Berlin) on Planck as a person. At the start and the end of the session the symphony orchestra performed Planck's favorite compositions (Bach's Brandenburg Concerto No. 3 and Mozart's Adagio and Fugue in C Minor).

As is known, Laue, Heisenberg, Born, and Hahn headed the list of 17 German scientists of West Germany, who addressed an appeal for peaceful use of atomic energy and who refused to participate in military nuclear research. Shortly before the congress Laue made a similar appeal at one of the sessions in West Berlin, while Born did the same in Hameln. These appeals were also supported by the Union of Physics Societies.

Among the papers delivered during the Planck celebrations in Berlin, one that attracted much attention was W. Heisenberg's "Planck's Discovery and the Principal Problems of Atomic Theory," very interesting and full of deep yet controversial ideas.*

We do not plan here a critical analysis of Heisenberg's paper, and restrict ourselves only to a few remarks. What is curious is Heisenberg's refutation of orthodox "Machism" and positivism. Yet Heisenberg's emphasis of the advantages of Platonism and of the preeminence of form and symmetry laws is not convincing. In the final analysis, some sort of invariance requirement (symmetry) must be obeyed by the equation of motion of any primary matter, in this case spinor matter, although naturally one cannot exclude the appearance of new invariance requirements, which can change the equation somewhat and modify it, for example, by including the iso-group. What is surprising is the silence concerning Planck's negative approach to "Machism" and the absence of any mention of the modern philosophy of dialectic materialism in connection with quantum theory, in spite of Heisenberg's sufficiently complete coverage of the entire history of philosophy.

On the whole, the two days of the Berlin Planck celebrations must be considered as a perfectly organized international congress, the scientific, moral, and political significance of which will undoubtedly be noted in the annals of the scientific events of our time.

Let us now proceed to the Leipzig congress of the Physics Society of the German Democratic Republic and the associated Theoretical Conference. The congress opened on Sunday morning, April 27, in the magnificent auditorium of the recently constructed Physics Institute of the Karl Marx University. The auditorium, with 450 seats, was filled with delegates and students, who broke through the cordons of ushers. After the greetings by the rector, the floor was turned over to Heisenberg, who was greeted with a noisy ovation. The warmth of the greetings was due not only to the universally known value of Heisenberg's work and his position in the struggle for peace, but also in recollection of Heisenberg's many years of service as professor at Leipzig in the thirties. Heisenberg himself began his lecture with remarks on his pleasure of finding himself again in the reconstructed Physics Institute of the University, where he worked earlier and where he now meets many of his colleagues.

In his long paper Heisenberg expounded on the principal ideas and advances of unified nonlinear theory of matter. The present communication differed from an analogous paper delivered to the congress in Venice in October 1957 in the considerable assurance of success, owing to the inclusion of the iso-spin group, developed in 1957-1958 jointly with Pauli. At the same time, Heisenberg noted the existence of differences of opinion with Pauli and warned that the theory is far from complete and that many difficult and unexplained points are contained in it. As is known, news of the success of the nonlinear theory has produced in the past winter a unique sensation among physicists of the whole world, along with exaggerated hopes voiced in the periodical and popular press.

Let us summarize the principal premises of this paper, which was the central point of the whole session. The fundamental difficulties in the reconciliation between relativism and quantum theory is seen by Heisenberg to lie in the presence of a sharp boundary between the past and the future (light cone), called for by the theory of relativity, while the quantum uncertainty principle speak against the possibility of such a sharp boundary. As is known, many methods have been proposed to eliminate these divergences: the introduction of various cutoff multipliers, a change to non-local field theory, nonlinear generalization of the field theory, quantization of space-time, and a few others. All these mean essentially a departure outside the framework of ordinary relativistic quantum theory. It appeared at one time that the divergencies which manifest themselves in the singularity of the Greenians on the light cone could be eliminated in some manner by renormalization. However, Kallen and Lehmann (1952-1954) have proved, on the basis of quite general premises, that the divergence of renormalized Greenians cannot be lower than the singularity of the Green's functions of the free field. At the same time the Kallen-Lehmann theorem has proved it impossible to eliminate the divergence with the aid of the renormalization method. Along with the impossibility of obtaining finite values of field masses and field charges of particles, a weak spot in the ordinary theory was, until recently, the absence of a unified description of elementary particles. Such a situation has indicated the desirability of reviewing the most fundamental premises of the theory.

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*A translation of this paper was printed in Usp. Fiz. Nauk, Vol. 66, No. 2 (–Ed.)

By way of a solution to all these difficulties, many authors have proposed that the nonlinear spinor equation be made the basis of the field theory. As indicated many times, Heisenberg and his associates were able to attain most noticeable successes in this direction. First, the reciprocal transformation of the particles clearly indicates that they are excited states of some general substance. In accordance with the arguments of de Broglie, the simplest basic field, from which it is possible to construct all the others, should be a spinor field of Dirac particles with spin $s = \frac{1}{2}$. Actually, by combining spinors, it is possible to obtain wave functions with spin 0, 1, etc., while it is impossible to obtain "rotating" particles from the "non-rotating" Klein-Gordon spin-zero particles, although they are describable by a simpler single-component function. A clear example of the method of "joining" is the idea of construction of the neutrino theory of light by de Broglie (developed by Kronig, Jordan, A. A. Sokolov, and others). Progressing in a similar direction were the ideas of the models of complex particles (Fermi-Yang, Sakata, Mackey, M. A. Markov, Goldhaber, and others). By using nucleons, anti-nucleons, and hyperons as a base, it is possible to attempt to construct from these the pions, K mesons, and other particles. If we generalize these ideas and adopt the point of view of a unified theory, then, obviously, its base should be some sort of nonlinear generalization of the Dirac equation. In fact, to yield excited states, the fundamental world spinor field should interact with something, but in the unified theory it can interact only with itself! Later on we, jointly with A. M. Brodskiĭ, established the form of all possible nonlinear generalizations of Dirac's equation, not including the derivatives, on which Heisenberg indeed leans in his paper. Then, taking into account the invariance under Pauli and Salam-Touschek transformations (from the neutrino theory), Heisenberg arrives at the Lagrangian

$$L_{NL}=\psi^{+}\gamma_{\nu}\frac{\partial}{\partial x_{\nu}}\psi\pm\frac{l^{2}}{2}\sum_{\mu}(\psi^{+}\gamma_{\mu}\gamma_{5}\psi)^{2}$$

from which he obtains the fundamental nonlinear spinor equation of matter

$$\gamma_{\mu}\partial_{\mu}\psi\pm l^{2}(\overline{\psi}\gamma_{5}\gamma_{\nu}\psi)\gamma_{5}\gamma_{\nu}\psi=0$$

(the matrices γ_5 and γ_ν were absent from the original nonlinear term).

After establishing the fundamental nonlinear equation, it is necessary to consider the rules of quantization of the field. In this connection Heisenberg made a very bold and original step, by modifying the commutation rules through introducing a Dirac indefinite metric in Hilbert space so that these equations come into agreement with the new nonlinear equation. In this way Heisenberg hopes to eliminate simultaneously the infinities from the field theory, since now the new relations for anticommutation no longer give functions on the light cone, but simply vanish there. Heisenberg indicates that the indefinite metric of the Hilbert space II, which supplements the usual Hilbert space I, leads only to unobservable intermediate states. As noted by Heisenberg, the concept of probability will apparently be to some extent "supplementary" to the space-time description at very small distances (where the Hilbert II manifests itself). Undoubtedly, analysis of the new rules of quantization will require much more effort! Given a basic nonlinear spinor equation and new quantization rules, we can in principle, solve any problem.

Among the results of Heisenberg and his associates, let us recall their derivation of the fermion state with mass $k = 7.426/l$ (where $k = Mc/h$), determined by the interaction constant (when the calculations are made with the new nonlinear term, the coefficient is 7.08 in the first approximation and 6.67 in the second approximation)

$$\lambda=h'cl^{2}$$

and several excited states with masses

$$\begin{array}{rllll} \varkappa = & 0.33, & 0.95, & 1.74, & 3.32 \\ \text{spin } s = & 1, & 0, & 0, & 0 \\ \text{parity } \eta = & -1, & +1, & -1, & +1 \end{array}$$

the new calculations gave for the meson masses values that are very sensitive to the choice of the nucleon mass. It became also possible to obtain a value for the fine-structure constant in the form

$$\alpha = \frac{2\pi e^2}{hc} \cong \frac{1}{267}.$$

A variant of the approximate Tamm-Dancoff method was used in this case.

Among the new results of the Heisenberg version of the nonlinear spinor theory, let us emphasize the interpretation of the iso-spin (I), the strangeness $S = l - l_N$, the hypercharge or the iso-fermion number l, and the lepton number (l_N) ($Q = I_3 + \frac{1}{2}$). In this case the following conservation laws take place:

$$\left.\begin{array}{l} I_3 \text{ is conserved in modulo 2} \\ l \text{ is conserved in modulo 4} \\ I_N \text{ and } l_N \text{ are conserved without limitation} \end{array}\right\} \begin{array}{l} \text{in the “electric” approximation (thus predicting the decay of } \Lambda_0 \\ \text{into 4n, owing to electromagnetic forces).} \end{array}$$

For weak interactions: I_N is conserved in modulo 2 and l_N is conserved in modulo 4.

In the final analysis we obtain a classification for all the known elementary particles in the following form:

Nucleons $(p, n, \overline{p}, \overline{n})$:	$\Psi^+\Omega$, where Ω is a function of the vacuum state);
Leptons $(\overline{e}, \nu, \overline{\nu}, e_+)$:	$\Psi^+\Omega$ (lepton masses are zero if electromagnetic forces are neglected);
Pions (π^+, π^0, π^-):	$\Psi\Psi^+\Omega$;
Hyperons $(\Lambda_0, \Sigma^+, \Sigma^0, \Sigma^-)$:	$\Psi\Psi^+\Psi^+\Omega$;
Cascade hyperons (Ξ^-, Ξ^0):	$\Psi\Psi^+\Psi\Psi^+\Psi\Omega$;
Heavy mesons $(K^+, K^0, K^-, \overline{K}^0)$:	$\Psi\Psi^+\Psi\Psi^+\Omega$;
Light mesons (muons) (μ^+, μ^-):	$\Psi\Psi^+\Psi^+\Omega$;
Photons (γ):	$\Psi\Psi^+\Omega$.

Analogies with the Fermi-Yang and other models of complex particles are easily seen here. In spite of the convincing general arguments in favor of the nonlinear spinor equation as the basis of a unified description of matter, and of the undoubted successes of the Heisenberg version, based on the new quantization rules and on the alluring prospect of obtaining all particles, charges, and all coupling constants from a single theory, we naturally cannot yet speak of a closed formalism. What is striking is the absence of any exact agreement with experiment in the values of the masses and charges. Later on, several specific objections to particular points were presented. Fierz believes that the violation of causality in the smallest regions near the light cone should involve a violation of the same principle at large distances and that, independent of the character of the metric in the Hilbert space, new anti-commutation rules lead to a contradiction with causality. Pauli, who again departed from the nonlinear theory, judging from communications from the Geneva “Eight Rochester” Conference on High Energies (June-July, 1958), points out the danger of choosing the commutation rules in the Heisenberg form, which, apparently, involves the vanishing of the anti-commutator ψ at all points. Touschek remarks that the proposed Lagrangian of the nonlinear theory postulates a complex character for ψ, but would have a meaning also for the Majorana real spinor; in such a case the invariance with respect to the Pauli group and the connection with the iso-spin drop out. Pauli and Touschek believe it difficult to obtain fermions with integral iso-spin, and consider the degenerate vacuum, introduced by Heisenberg, to be an artificial assumption. Heisenberg himself warns against excessive hopes and has emphasized in Leipzig that the theory is still in the exploratory stage and a few years' work will be necessary to clarify matters. Yet his lecture had a tone of quiet optimism (he, too, did not consider the subsequent Geneva objections of Pauli as decisive).

Much interest was paid at the Leipzig Conference to gravitation, owing to the basic paper by Dirac, devoted to the Hamiltonian form of gravitational theory. The discussion of gravitation was continued at a special seminar, at which I had to preside at the suggestion of L. Infeld who, half in joke, desired to continue the tradition of the Padua Conference. Dirac emphasized the importance of presenting the theory in the Hamiltonian form, because of the great possibilities of application of tangency transformations and the clarification of the problem of what variables are physically realizable and which are connected with the coordinate systems. Yet the basic concept in the Hamiltonian theory is the state at a given instant of time, a relativistic generalization of which is the specification of the state on a three-dimensional space-like surface. Then the Hamiltonian equations of motion determine the character of variation of the dynamic variables, which decide the state when the surface is changed. In spite of the complete violation of the four-dimensional symmetry, it is convenient to choose a coordinate system in such a way, that the surfaces $x^0 = \text{const}$ are all space-like, and consider only states on such surfaces.

The work of Dirac is a continuation of the investigations of Pirani, Schild, Bergmann, Skinner, and Saukis (Physical Review **80**, 81 (1950), Physical Review **79**, 986 (1950), Physical Review **87** (1952)). Starting out with the Lagrangian of the gravitational field

$$L_g = J_g^{\mu\nu}(\Gamma^{\rho}_{\mu\nu}\Gamma^{\sigma}_{\rho\sigma} - \Gamma^{\sigma}_{\mu\rho}\Gamma^{\rho}_{\nu\sigma}), \quad (|g_{\mu\nu}| = -J^2),$$

Dirac determines the momentum $p^{\mu\nu}$ conjugate with $g^{\mu\nu}$, and obtains then an expression for the Hamiltonian

$$H_g = \int (g^{00^{-\frac{1}{2}}} H_{GL} + g_{\nu 0} e^{\nu\beta} H_{G0})\, d^3x,$$

where H_{GL}, H_{G0}, and analogous terms in the Hamiltonian that describes ordinary matter are all independent of $g_{\mu 0}$. By definition

$$H_G = \int (p^{\nu 0} g_{\nu 0,0} - L_G)\, d^3x.$$

It is important, that in addition to the variables that describe the ordinary matter, the Hamiltonian contains only six degrees of freedom. In the weak-field approximation, Dirac obtains by a new method gravitational waves (2 degrees of freedom: 12, 11 — 22) (corresponding after quantization to gravitons of spin s = 2) and an expression for the Newtonian energy of the gravitation of the particles together with the intrinsic energy of gravitation of each particle (degree of freedom: 11 + 22). On the other hand, the degrees of freedom 13, 23, and 33 do not enter into the Hamiltonian of the equation of motion. The exclusion of the degrees of freedom 13, 23, and 33 (11 + 22) can be made with the aid of the tangency transformation even without restriction to the case of slowly-moving particles, in analogy with the transformations that exclude the longitudinal waves in electrodynamics. In conclusion Dirac noted that the Hamiltonian of the gravitational theory is simpler than would be expected, starting with the ten quantities $g_{\mu\nu}$. However, such a simplification is attained at the expense of foregoing the four-dimensional symmetry. To the general enlivenment of the somewhat surprised audience, Dirac announced again that in his opinion it is necessary to forego the four-dimensional form of the description of the world ("One must go back from Einstein") and return to the three-dimensional one, since the four-dimensional symmetry is in his opinion not a basic property of the physical world! The physical state, Dirac notes, is not determined by the particular individual solution of the equations of motion (each of which has four-dimensional symmetry), but by a family of solutions. It is exactly such a family that corresponds to the wave function in quantum theory. On the other hand, individual solutions do not have a quantum analogue. In Dirac's opinion, the Hamiltonian method forces us to forego the four-dimensional symmetry, but, this is offset by the great possibilities of using tangency transformations. Undoubtedly, it is necessary to weigh carefully the technical advantages of the Hamiltonian method against the four-dimensional method and their physical consequences before dispensing with the four-dimensional symmetry, particularly considering the successes (and difficulties) of the covariant description in quantum field theory.

The fact that Dirac joined those "voting" for the predicted existence of gravitational waves has raised objections on the part of L. Infeld, who cited many new arguments in favor of his own theory, that there exists no energy-carrying gravitational radiation. In this connection, the problem was touched upon of the derivation of the gravitational radiation not only through the weak-approximation methods, but also through the method of Einstein-Grommer, Infeld, Hoffmann, and Fock, who derived the equation of motion of particles from the field equations themselves. Professor M. A. Tonnela (coworker of L. de Broglie) reported on a comparison of the Einstein-Infeld methods (motion of singular point) and that of V. A. Fock (motion of extensive particle). In the discussions at the sessions of the conference and at the seminar, Dirac has indicated that his theory leads to the possible transmutation of gravitons into ordinary matter (photons or electron-positrons), which we predicted. According to a well-aimed remark by Professor Rompe, the interesting discussions of gravitation had the "romantic" air of discussions of problems of the physics of tomorrow.

The third fundamental theoretical paper was delivered by N. N. Bogolyubov on his new method in the theory of superconductivity and superfluidity. The audience listened with great attention to the substantial communication and warmly greeted the lecturer, who was recently awarded the Lenin prize. As is known, in developing his method of canonical transformations, previously proposed (1947) for the formulation of a microscopic theory of superfluidity and to account for the interaction between bosons, and in now applying it to fermions, N. N. Bogolyubov comes close to Fröhlich's work on superconductivity, in which the substantial role of the interaction between the electrons and the phonons is emphasized, in connection with the discovery of the isotopic effect in superconductors. He also comes close to the work of Bardeen and Cooper and their associates. He shows here the essential correctness of the simplified picture of the existence of paired correlation ("attraction") between electrons that interact through phonons, particularly near the Fermi surface, leading to the Bose-Einstein condensation of similar formations with zero total momentum.

N. N. Bogolyubov and his associates (V. V. Tolmachev, D. V. Shirkov, S. V. Tyablikov, D. N. Zabarev, and Yu. A. Tserkovnikov) succeeded in giving a complete, mathematically correct theory of the no-longer mysterious phenomenon of superconductivity, with allowance, in particular, not only for single-fermion excitations, but also for collective excitations and for the role of Coulomb forces. This leads to interesting prospects of applying similar methods to atomic nuclei and of accounting for the unique "superfluidity" of the nucleonic fermions.

The paper by J. P. Vigier was devoted to the relativistic hydrodynamics developed by the associates of L. de Broglie (Loschak, Takabayashi, Halbwachs, and others) and D. Bohm. Briefly speaking, the matter concerns an attempt to follow not Dirac's path of the relativistic quantum equation of the point fermion, (for all its spectacular well-known success), but to return to a sort of starting point in the form of the work of Ya. I. Frenkel' on the classical theory of the rotating electron, and subject it to quantization after developing the relativistic theory of rotating formations. In this connection, we point to the work by Matison [Acta. Phys. Polonica **6**, 163 (1957)]; Møller [Ann. Inst. Henri Poincaré **11**, 251 (1949)], Weissenhoff [Acta. Phys. Polonica **9**, 7 (1947)], and Price [Proc. Royal Society, London **A195** (1948)]. In accordance with the Vigier-Bohm opinion, the Weissenhoff equation and those of others (connected by the condition $M_{\alpha\beta}u^{\beta} = 0$, where $M_{\alpha\beta}$ is the anti-symmetrical internal angular momentum tensor, and u^{β} is the four-velocity) correspond to a relative motion of the center of mass and the center of density of the matter. In the final analysis, quantization of rotating non-pointlike "drops" of a relativistic liquid leads Vigier et al. [Compt. rend. **241**, 692 (1955)] and Takabayashi [Phys. Rev. **102**, 282 (1956)] to a certain classical hydrodynamic model of the Dirac and Klein-Gordon-Kemmer equations, while the quantization of the excitations of such drops gives quantum numbers that can be compared with the strangeness numbers etc., especially in view of Tiomno's recent particle scheme. In spite of the preliminary character of such a theory, attention should be paid to several useful and interesting analogies and to the possibility of obtaining, even from this end, nonlinear equations of the same type that are considered in the nonlinear spinor theory.

Of considerable interest was the paper by L. Janossi on experiments, performed by him and his associates (S. Narai, P. Varga, A. Adam, and others) in the Optical Laboratory of the Central Physics Institute of the Hungarian Academy of Sciences, Budapest, located 30 meters underground. Janossi undertook to compare the interference patterns, obtained with the aid of a Michelson interferometer (length of arms 10 cm in some experiments, approximately 14 m (!) in others) in the case of the ordinary "high" intensity, and also in the case of the extremely low intensity, when in instrument contains, so to speak, less than one photon. Such experiments, which border on the well known experiments of S. I. Vavilov with photons and those of V. A. Fabrikant and associates on the interference of electrons at low beam intensities, are of obvious principal interest. Do photons (or electrons), simply speaking, interfere with each other or with themselves? The photons were registered with a special photomultiplier. As announced by Janossi to the general excitement of the audience, his own new unprecedently precise experiments, in spite of his assumptions, gave a result that agrees with the deductions of the quantum theory, i.e., his interference pattern is independent of the intensity. Heisenberg praised Janossi's experiment, noting that it is now possible really to realize several experiments, previously considered as only "hypothetical."

We shall not dwell in detail here on other numerous experimental papers, noting only a few of these. Many investigations concerned mass spectrography (K. G. Krebs, F. Bernhard, Chr. Keck); these included the important communication by M. Ardenne on a new precision mass spectrograph for high-molecular negative ions. A group of papers concerned magnetism (V. Holzmuller: Magnetic Processes in Ferrites; D. Unangst: Observation of Weiss Regions in Thin Single Crystals of Iron). A group of papers was devoted to x-ray structural analysis (G. Mueller and his associates, E. Schoene and his associates); the cycle of papers pertain to low temperatures (L. Beviloga, F. Eder).

Worthy of particular mention are the papers delivered by the associates of the Laboratory of the First German Reactor in Dresden, headed by G. Barwich. Prof. G. Barwich and N. Gessel delivered a paper on the solution of equations for the concentration of various decay products for many stationary and non-stationary cases in a homogeneous reactor (uranium plus heavy water). Another paper by G. Barwich was devoted to a survey of methods of isotope separation. G. Abel and W. Bredel (Dresden) considered the detection of neutrons with the aid of scintillation counters. From among the other papers on nucleonics, we note the report by P. Goerlich and his associates [A. Kross, G. I. Pohl, G. Reichel and L. Schmidt (Jena)] on the measurement of gamma spectra of Cs^{137} and Co^{60} with the aid of a new multiplier (M12FS) made by Zeiss. Among the theoretical papers, we note the report by Professor Heber (Jena) on individual and paired errors in the measurement of the size of field; the survey article by Professor B. Kokkel (Leipzig) on parity, a report by M. Meier (Bucharest-Dubna) on weak interactions in the theory of elementary particles of Schwinger, a paper by I. Zupeca (Zagreb) on the differential equation of the theory of electric conductivity at low temperatures.

Zellner (now in Dubna) considered in his paper the dispersion relations for the process $\pi + N \rightarrow 2\pi + N$ in the approximation of the nucleons at rest. A paper by F. Kaplun (Dubna) was devoted to the dispersion of relations for elastic scattering of pions by deuterons. K. S. Zeiten discussed the systematics of possible neutrino theories. A. B. Migdal lectured on the use of quantum-field-theory methods in the many-body problem. Dr. G. Kahan (Paris) reported on the quantum theory of the Faraday effect. G. Henl (Freiburg, Federal Republic of Germany) delivered a paper on the theory of potential in spherical space. In the author's absence a paper was delivered by T. Kakushadze (Tbilisi) on K_α and K_β satellites in x-ray spectra. V. Kofnik (Karlsruhe, Federal Republic of Germany) reported on the theory of anisotropic scattering of neutrons.

On the whole, it can be stated again that the conference in Leipzig was very successful and was able both to focus attention on several of the most urgent problems that now engage the universal physics fraternity (nonlinear spinor theory, and others), and to permit discussion of many interesting and important problems. The members of the Soviet delegation delivered many papers devoted to the development of physics in the U.S.S.R. More than a thousand persons gathered in Leipzig to hear N. N. Bogolyubov and D. D. Ivanenko on the Development of Soviet Science, on the Novosibirsk Scientific Center, and on the Progress of Soviet Nuclear and Accelerator Physics. Numerous remarks concerned individual details of the papers, but there were also other questions, for example, whether the Moscow University has a theology faculty (sic!). We note that at the Berlin University (German Democratic Republic) and certain others do have theological faculties. For example, at the May day parade in Berlin, the teachers and theology students marched with their flags. Many members of the delegation delivered papers on the state of science in some Union Republics, such as Georgia (Prof. V. I. Mamasakhlisov, corresponding member, Academy of Sciences, Georgian S.S.R.) and Estonia (Prof. G. P. Keres). Professor A. A. Smirnov (corresponding member, Academy of Sciences, Ukrainian S.S.R.) reported on progress in metal physics in the Ukraine; Dr. Zh. S. Takibaev lectured in Halle on research done on cosmic rays in Kazakhstan; Prof. B. I. Stepanov (active member Academy of Sciences, Belorussian S.S.R.) lectured in Jena on optical research in Minsk. I delivered a survey paper in Leipzig on the development of Soviet physics, and also lectured to the members of the Academy of Sciences in Berlin, at the Berlin University, on attempts to formulate a unified (principally nonlinear) theory of matter. I also lectured at the Jena and Halle universities on the results of the Leipzig conference. A. B. Migdal lectured in Dresden on the application of the method of superconductivity to the nucleus.

Members of the delegation spoke over the radio and appeared on television, and also gave several interviews. The scientists of the German Democratic Republic and the student youth showed tremendous interest in our reports, to which much space was devoted in the national, local, and student press. As a rule, a small circle of persons always remained after the lectures and the question sessions, and the discussions ceased only when it became necessary to present another paper or to proceed the next session of the conference. Such interest is undoubtedly due essentially to the present high rank of Soviet science.

Through the graciousness of the German colleagues, the members of the Soviet delegation became acquainted with the scientific centers of physical science in Germany. Its organization and state can be described in the briefest outline as follows. The key position in the organization of science is occupied by the German Academy of Sciences, the successor to the Prussian Academy, which in turn dates back to the time of Leibnitz

(1700). An exhaustive outline of the history of the academy was published during the days of the Planck celebrations. The academy is headed by a president, the well known physical chemist M. Vollmer, and four vice-presidents, including one of the oldest of its members, the physicist and biophysicist Walter Friedrich, who discovered the diffraction of x-rays by crystal lattices. Professor W. Friedrich is now known for his active social activities and his struggle for peace. The secretary of the Division of Mathematics, Physics, and Engineering is Professor Robert Rompe, who speaks a beautiful Russian, and who is the director of the Institute of Light Sources, some interests of which lie in the field of solid-state physics. The academy institutes that engage in the natural sciences, engineering, and medicine are unified into a single organization headed by a presedium (chairman G. Fruhauf, members R. Rompe, K. Schröder, E. Tilo, G. Hummel, and G. Neels) and a scientific secretariat, headed by the energetic Doctor Hans Wittbrodt.

The Academy of Sciences comprises the following: (1) The Heinrich Hertz Institute of Oscillation Processes (director, Professor Haganberg; the institute has two radioastronomical observatories, which have several small telescopes, and a new 36-meter radio telescope is under construction. (2) The Institute of Optics and Spectroscopy. (3) The Institute of Crystal Physics. These three institutes and observatories are located in one large country place outside of Berlin, in Adlershof. In addition, the Academy of Sciences includes: (4) The Institute of Sources of Light. (5) The Solid State Institute. (6) The Nuclear Institute (the latter located in Miersdorf below Berlin). (7) The Professor Steenbeck (known for his pioneer work on the betatron) Institute of Magnetic Materials in Jena. (8) The Low Temperature Laboratory in Dresden. (9) The Professor Seeliger Institute of Gas-Discharge Physics in Greifswald.

Of the six universities of the German Democratic Republic, physics is most advanced in Berlin (The Humboldt University, founded in 1809), Jena (The Friedrich Schiller University, founded in 1508) and Leipzig (The Karl Marx University, founded in 1409). To these should be added one of the important centers of German physics of the German Democratic Republic, in Dresden, home of the great Higher Technical School, in which a special nuclear faculty was organized, headed by Professor Macke a student of Heisenberg. In Dresden-Rosendorf there is in operation, under the leadership of Professor Barwich, the first nuclear reactor in all Germany, constructed with Soviet help, while the private Ardenne Laboratory is in operation in Dresden. Of great importance are the Laboratories of the well known Zeiss Company in Jena, which became a national enterprise and has again resumed not only its commercial activities, but also its scientific activities in the field of optics, spectroscopy, and precision instrument building, now under the scientific leadership of Professor P. Herlich, Doctor G. Schrade, and others. Some of the papers by the Zeiss staff are published in the excellently edited "Jena Annuals" and "Jena Survey." A new physics institute has been constructed by the University in Jena.

The Physical Society of the German Democratic Republic has been intensifying its activities, calling congresses and individual sessions. It is headed by G. Hertz (chairman) and an energetic young secretary, Dr. Buchner.

The principal physics centers of the German Democratic Republic in Berlin, Dresden, Jena, and also Leipzig are thus at the forefront. It is there that new institutes have been organized and the better staffs are concentrated. Reconstruction has not yet been completed in the two northern universities in Rostock and Greifswald, or in the Physics Institute in Halle, where Prof. Messerschmidt continues his research on variations of cosmic rays.

The higher schools of the German Democratic Republic are now going through a unique transition period. The higher institutions of learning have accepted representatives of a young generation of workers, and members of the labor classes of the population, who are full of enthusiasm. The higher institutions of learning are solidly turning towards the building of socialism. One of the causes that prevent a more rapid development of science is apparently the considerable shortage of staffs. There is a striking shortage of lecturers or young professors. In some way or other, strengthened by its traditions, German physics of the German Democratic Republic is now on the rise.

The trip to the congresses in East Germany enabled us to become acquainted with many cites of the German Democratic Republic. To complete the picture of the conditions of the progress of science we must mention briefly our impressions. The gloomy picture of a destroyed city, which is presented by Berlin, is well known. The city is still full of waste areas, many quarters full of destroyed buildings. The present central street of East Berlin, the Stalin Allee, has been reconstructed extensively and is of architectural interest. In Treptow Park there is a common grave for the Soviet soldiers; the statue of a soldier trampling on a Svastika, the work of Vuchetich, makes a very great impression. Treptow Park is frequently visited by Berlin residents from both sectors and by tourists. Immediately upon arrival, the members of the Soviet delegation visited this memorable place. Many interesting Berlin museums have already been reconstructed, namely the Museum of National Creative Work, the outstanding museum of excavations from Troy and Babylon, and a small picture gallery. The state opera and the comic opera, where we attended interesting performances — Mozart's "Abduction from the Seraglio" and one-act pieces of Milo and Bartok — serves as a center of attraction even for West Berlin. In Dresden, terribly bomb damaged right after the end of the war, now has the restored Zwinger, where a noted gallery is located. Leipzig (the center of the chemical industry) suffered much less, and the old city hall, with the market square in front of it, has been fully preserved. Fortunately, the Thomaskirche, to which the remains of Johann Sebastian Bach who was the cantor there, have now been transferred, has also been saved. The memorial to the great composer is in good harmony with the tremendous Gothic windows of the church. From the inscription on the gates we learn that Martin Luther preached in this church and that Richard Wagner was christened there. No far from the city hall in Leipzig there is a statue for young Goethe (with a bas-relief of his sweetheart), who was a student in the university here.

The cities of southern Germany have been almost completely preserved, including the great university and industrial center in Halle, with the well-known cathedral, adorned with four towers, which face the lively square with a statue of Handel, a native of this picturesque city, located on the banks of the Saale River. Very interesting is little Weimar with the museum rooms of Goethe, Schiller, and Liszt, which attract many tourists and excursion students. The first-floor rooms of the Goethe house, adorned with antique statues and pictures, are in keeping with his rank as a Minister of the Duchy, although tiny by present standards. A great impression is made by Goethe's more modest and by the multitude of mineralogical and zoological collections, along with the physical instruments on the second floor. The picture of a universal genius emerges, that of a writer, scientist, philosopher, and organizer. Schiller's house is an example of a much more modest "professorial lodging." As evidence of fascism's deepest insult to the national dignity and to general human morals, the remnants of the Buchenwald concentration camp, located on a hill about 10 km from Weimar, have been retained. Moving at a speed of 110 to 120 km/hr on a splendid auto road, probably the best in Europe (a wide divided highway running outside the cities and villages, without grade crossings), we could reach the most remote localities of the German Democratic Republic.

One recalls the well-known Wartburg castle on the hill near Eisenach, the center of the automobile industry. Here are retained the old halls, dating back to the 12th century, the room in which Luther translated the bible, with its desk and possessions. The gem of the valuable collections of the castle are several dozens of portraits by Kranach (Luther, his wife, Melanchton, and others). Very attractive is the quiet lyric Jena (pop. approximately 80,000), located on the hills of Thuringia. The university institutes are scattered over the entire city. Memorials to scientists are everywhere: on the Helmholtzweg, Goethe Allee, and on other streets. A surprise is the eleven-story building of the Zeiss main offices. Even a cursory visit in the German Democratic Republic, after an absence of 13 years, shows the great progress in the restoration and considerable progress in both industry and science.

A perfect ending to our two week trip in the German Democratic Republic was our attendance at the tremendous five hour first of May demonstration in East Berlin, in which residents of West Berlin also participated, carrying placards with names of their regions. The demonstrations were preceded by a brilliant parade of the army of the young democratic republic.

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Meetings and Conferences

FOURTH ALL-UNION ACOUSTICS CONFERENCE

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THE Fourth All-Union Acoustics Conference was held in Moscow from May 26 to June 4, 1958. The conference was called by the Commission on Acoustics, Academy of Sciences USSR, the Acoustics Institute, Academy of Sciences USSR, and the M. V. Lomonosov Moscow Order of Lenin and Order of Labor Red Banner State University "Lomonosov." More than 700 scientists and engineers took part in the conference. These came from 55 cities of the Soviet Union, and also included 40 foreign acousticians, coming from East Germany, China, Poland, the United States of America, West Germany, and Czechoslovakia.

A total of 170 papers on the most important current problems of physical acoustics were read at the conference. The meetings of the conference were divided into plenary sessions and seven specialized sections:

1. Sound propagation in inhomogeneous media.
2. Nonlinear acoustics.
3. Radiation and diffraction of sound.
4. Ultrasonics.
5. Electroacoustics.
6. Acoustical measurements.
7. Architectural and constructional acoustics.

The plenary sessions were devoted to review papers which delineated the most significant present trends in physical and technological acoustics. The meetings of the technical sections were given over to reports on original research completed in 1957-58.

The chairman of the Commission on Acoustics of the Academy of Sciences of the USSR, Academician **N. N. Andreev**, in opening the conference, noted the great importance of the meeting in summing up the researches of numerous Soviet acousticians, and pointed out the great problem confronting Soviet acousticians in the general development of science and technology in the USSR. He further took note of the participation in the conference of a large number of foreign acousticians; this, he said, should aid in further association between Soviet and foreign scientists, to their mutual advantage.

Corresponding Member of the Academy of Sciences of the USSR **L. M. Brekhovskikh** read an interesting general paper "On Surface Waves in Acoustics" at the first plenary session. Surface waves can be propagated without damping along a surface with elastic impedance iZ. In air acoustics, such a surface can be obtained by mounting a comb of fibers on a rigid wall; the distance between the fibers is made much shorter than the wavelength of the sound. In liquids, one can use a surface of porous rubber.* The attenuation coefficient in the direction normal to the surface is equal to $\alpha = \omega\rho/Z$. Cylindrical surface waves can be propagated along a fibered wall in air, or along an air cord (or cord of porous rubber) in a liquid. The speaker pointed out examples of the possible application of such surface waves in acoustics. By making a surface wave impinge at a certain angle on a band of sound absorbing material, one can measure the reflection coefficient as a function of the angle of incidence. By restricting a certain part of the surface by means of short solid walls, one can obtain an unusual "plane" reverberation chamber. Professor Brekhovskikh noted that it is possible that the results of measurement in such chambers may prove to be equal in merit to the results of measurements in the usual large "volume" reverberation chambers. Another advantage of plane chambers is their portability. Theory shows that to produce radiators or receivers of high directivity we can use cylindrical surface waves which propagate along rods of sufficiently small diameter. The half width of the principal lobe depends on the length of the rod l and is equal to $\pi\sqrt{\lambda/l}$.

V. A. Krasil'nikov considered "Certain Problems of Aerothermoacoustics," pointing out the great significance of problems of the propagation of radiation in a medium with random inhomogeneities in the index of refraction, produced by turbulence, and the production of sound (noise) from turbulence. Some substantial progress has been made in the study of propagation problems, due in part to the application of the basic results of the statistical theory of local isotropic turbulence in line with the ideas of A. N. Kolmogorov. The problem of sound generation, while possessing great practical importance, has been less studied. The principal problems in the solution of this question consist in the determination of the dependence of the intensity, frequency spectrum, and spatial distribution of the sound on the properties of the turbulent flow. The lecture included a brief critical review of the theoretical (Lighthull et al.) and experimental researches and, within the framework of the existing theoretical ideas, the problems of the interaction of sound fields (scattering of sound by sound) were considered, as well as some problems of thermoacoustics.

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*Surface waves cannot be propagated independently along a solid, nonrigid surface without a source distributed over the entire surface, since in such a case the inertial impedance prevails over the elastic. In the propagation of surface Rayleigh waves, the inertia and the elasticity are comparable because of the presence of shear rigidity.

In his paper, "Lateral Amplitude Diffusion in Wave Diffraction, Propagation and Reflection," **G. D. Malyuzhinets** (at the second plenary session of the conference, May 28) considered the behavior of approximately traveling waves which possess a sufficiently defined direction of propagation at each point and a certain smooth distribution over the wave front. Just as the propagation in the direction of the ray is according to the wave law, an equalization of amplitude takes place, according to the laws of diffusion, in the transverse direction, i.e., along the front. The coefficient of this transverse diffusion, which is complex, is proportional to the wavelength. As a consequence of the complex coefficient, wave diffusion, in contrast to the usual case, is accompanied by oscillations and phase shifts. As was shown by the author in 1946, the phenomenon of transverse diffusion in the general case is described mathematically by differential equations of the parabolic type in radial coordinates that are defined by the Fermat principle. In addition to computational advantages, the principle of transverse diffusion gives a clear interpretation to a wide class of wave phenomena, allowing us to describe qualitatively the features of wave field before carrying out a calculation or experiment. This also applies to those cases in which Huygens' principle is of no avail for a clear representation. In particular, the process of amplitude attenuation on a wave front that slides along the absorbing surface is analogous to the cooling of a heated plate which is kept cold at its edge.

M. A. Isakovich gave a review of certain problems of statistical acoustics, which have been treated in recent years at the Acoustics Institute, Academy of Sciences, USSSR. These refer to the question of the scattering from randomly rough surfaces, the problem of the scattering and radiation of waves in acoustically inhomogeneous and statistically vibrating surfaces, the problem of the investigation of elastic surfaces found under the action of statistically distributed lateral forces, and also a number of problems on the propagation of sound in non-regular channels.

In the paper of **Uno Ingard** (USA) on "Propagation of Sound in the Atmosphere," laboratory and field measurements of sound propagation in an inhomogenous and turbulent atmosphere were reviewed and discussed. The field measurements were carried out over a wide range of atmospheric conditions, and included studies of the shadow zone, brought about by the temperature or wind distribution, attenuation resulting from the scattering on turbulent inhomogeneities, the energy spectrum of atmospheric turbulence, etc. The laboratory investigations included measurement of the sound field in a shadow zone brought about by a temperature inhomogeneity, the dependence of the shadow zones on the boundary conditions, and also a study of the propagation of collimated beams and spherical waves in a turbulent medium. The laboratory measurements in the shadow zone agreed well with diffraction theory; however, the results of the field measurements in the shadow zone could not be explained by diffraction alone. The data of field measurements on attenuation and brought about by scattering in the case of the use of sound forces with comparatively small directivity, agreed with calculations based on data on scattering in the reverse direction. From this point of view, one could decide on the dependence of the energy spectrum of turbulence on the wind velocity, and the comparatively weak dependence on the wind velocity of the measured attenuation.

In the following plenary session, problems of signal transmission and architectural acoustics were discussed.

I. E. Goron and **A. V. Rimskiĭ-Korsakov** reported on "Investigations of the Significance of Distortion and Interference in a Radio-broadcast Channel." Comparing the transmission through a channel with a measured distortion (or interference), and transmission through a non-distorting channel, they determined threshold amounts of distortions (or interference) of different forms. Statistical analysis was carried out and the reliability of results obtained in various musical and speech programs with a large number of experts was determined. Thresholds were found for the limitation of the band of transmitted frequencies, the compression of the dynamic range, nonlinear dispersion noise and interference, and also for several simultaneous distortions and interferences.

The investigation allows one to make a classification of the quality of broadcast channels and to specify standards for permissible distortion and interference.

The old concepts of estimating the quality of acoustical auditoria by the reverberation process have undergone a revision in recent years. New hypotheses have been employed which connect high quality of hearing with different characteristics of the sound field in an auditorium, in particular, with the spatial and other inhomogeneities of the field. Judging, in the purely physical aspect, the "Frequency and Spatial Irregularity in the Establishment of an Acoustic Field in Rooms," **R. Bolt** (USA) observed that the fluctuations of the sound pressure at a certain point in a room (in which a sound field from a sinusoidal sound source has been established) is brought about by fluctuations in the position of the sound source and the frequency of the radiation and can be represented as an irregular superposition of pressures connected with the different normal modes. The resulting frequency irregularity F (db/cps) increases with frequency at low frequencies and tends to a constant limit at high frequencies. In buildings with a high degree of symmetry, F has (at a certain frequency) a clearly indicated maximum. Below this frequency, F decreases upon increase of absorption in the room and diffuseness of the field; above, it depends only on the absorption. The lecturer pointed out that the previous theory which gives the variation of F in regions of high and low frequencies, coincides with the present theory in the intermediate region. Theoretical rules for the spatial non-regularity, S db/half wavelength, were discussed and representative data for their measurement were given.

Professor **E. Meyer** (West Germany) reported on "New Studies of Architectural Acoustics at Göttingen." Electroacoustical experiments are being carried out in which the separate reflections of sound from surfaces of a room, together with reverberations in the proper sense of the word (and without them), are imitated. The threshold of discrimination of reverberation times of damped noise processes was also investigated. The amount and intensity of the most powerful reflections (from the surface of a room) were investigated; these reflections are characterized by a small retardation time. Furthermore, the speaker reported on the results of a measurement of the diffuseness of the direction in stationary excitation and of the dependence of the diffuseness on time in excitation by sound pulses, which permits one to note several objective criteria characterizing the acoustical properties of rooms.

In recent years the problem of the struggle against noise has acquired vital significance. Besides the further extension of already known methods of reducing vibration and the noise brought about by it, researches have been carried out on new ways for the solution of this problem. One of the new methods consists of placing on the vibrating parts of machines and structures layers of materials which possess an increased internal damping. This increases the active loss of mechanical vibrations of these parts. The amplitude of the vibration is decreased in the resonance region and the damping time of vibrations after impact are decreased. As a result, the level of the radiated airborne sound is decreased. **B. D. Tartakovskiĭ** pointed out the fundamental theoretical and experimental researches on vibratory absorbtive materials and structures, compared the different methods for their measurement, particularly the methods applied in the Acoustics Institute, and showed the possible regions of applicability of such materials and structures.

From the general theory of phase transitions of second order, it follows that the time for establishing the state of thermodynamic equilibrium (the relaxation time) ought to increase significantly, in a system close to the critical point. This should lead to an increased absorption of sound. Calculation of this time has enabled L. D. Landau and I. M. Khalatnikov to give a quantitative explanation of the anomalous absorption of first sound in liquid helium near its critical point at 2.19°K. Carrying out a thermodynamical generalization of the results referred to above, **I. A. Yakovlev**, **T. S. Velichkina**, and K. N. **Baranskiĭ** investigated relaxational absorption in a solid body which undergoes a phase transition of the second order. An effect of a substantial increase in the absorption of a transverse wave in piezoelectric salts close to the upper Curie point was observed. The results of the investigations were in excellent agreement with the theory of the phenomenon developed by Landau for this particular case.

A "magnetoacoustic effect" was shown by **L. L. Myasnikov** to arise in the interaction of sound (ultrasonic) waves in a conducting or semi-conducting solid or liquid medium with a magnetic field, as a consequence of which an alternating electromagnetic field is formed. In metals, the magnetoacoustic effect also appears in the dependence on the phase velocity and the damping of sound on the frequency and magnetic induction. It is found that in all alloys of nickel and manganese with iron, the magnetic dispersion of sound depends upon the amount of iron content; this permits us to suggest a new method of magnetic analysis. In the megacycle range, the magnetoacoustic effect takes place only in a thin surface layer. Therefore, it is easy to obtain it and to observe it with the help of surface ultrasonic waves. The lecturer spoke about several possible practical applications of the magnetoacoustical effect in measurements, defectoscopy, etc.

L. D. Rozenberg summed up investigations of the physical mechanism of ultrasonic cleaning of surfaces of solid bodies from contamination (surface films). As a result of a series of investigations, it was made clear that the role of vibrational acceleration in the cleaning is insignificant: in uniform accelerations, the cleaning proceeds in a liquid medium and does not take place in a gas. It was established by means of high-speed motion-picture photography that the cleaning takes place through the catastrophic disruption and scaling off of the surface film. The disruption takes place in the case of partial or complete annihilation by cavitation bubbles, as a consequence of which shock waves arise. The magnitude of the cavitation energy depends on the temperature; it has a maximum that depends on the liquid employed in the cleaning. Corrosion increases with temperature, which is explained by the increase in the number of cavitation bubbles. The corrosion depends on the magnitude and nature of the dissolved gas and increases with decrease in the solubility of the gas. Therefore, if water in contact with air is used for cleaning, the erosion is greater than in the case of the use of organic solvents.

At the last plenary session, Bruel (Holland) gave a review paper on the development of electroacoustical methods of measurement and the handling of electroacoustical measuring apparatus. The lecturer spoke in detail about developments pioneered and carried out at the scientific investigatory laboratories of the firm of Bruel and Kjaer, which in recent years has become one of the centers of production of acoustical apparatus in western Europe. The paper was accompanied by demonstrations of complicated measuring apparatus and was listened to by the audience with great interest.

The section "Propagation of Sound in Inhomogeneous Media," considered problems of the propagation of waves in statistically and regularly inhomogenous media, peculiarities of interference and focusing of waves, which are propagated statistically in a inhomogeneous medium. A second important group of papers were devoted to waveguides of various types, the propagation of waves in the presence of reflecting boundaries, and reflections from rough surfaces. The majority of the papers given were grouped in accordance with the general trends developing in the several scientific schools. These refer to: (a) investigations completed at the Acoustical Institute on propagation in wave guides and channels and in statistically inhomogenous media; (b) researches at the Leningrad State University on ray theory; (c) researches on the propagation and focusing in statistically inhomogenous media completed in the Yaroslavl Pedagogical Institute under the direction of L. A. Chernov; (d) the works of the group of V. I. Tatarskiĭ in the Institute of Atmospheric Physics on the study of the effect of turbulence on the propagation of waves.

In the papers of **B. M. Babich**, and also **A. S. Alekseev** and **B. Ya. Gelchinskiĭ**, there were descriptions of the ray method of calculation of the intensity of wave fronts, of successive approximations in this method, and of its application to the problem of propagation in the presence of boundaries. The determination of the intensity and shape of frontal waves in an elastic medium by the ray method was also discussed.

In analogy with the well-known methods of J. Hadamard and S. L. Sobolev of the solution of the Cauchy problem for parabolic equations, one could carry out the calculation of the intensity of wave fronts by considering the equations describing the wave processes to be satisfied on characteristic surfaces. Taking "wave front" to mean either the surface of discontinuity of the solution or its derivative or (in the case of higher frequencies) the surface of equal phases of vibration, and employing, close to the wave front $[t = \tau(x, y, z)]$, the expansion

$$U(x, y, z, t) = \sum_{n=0}^{\infty} U_n(x, y, z) f_n(t-\tau),$$

where

$$\frac{d}{dt} f_n(t) = f_{n-1}(t), \qquad f_n(t) \gg f_{n+1}(t),$$

we can obtain from the differential equations that are satisfied by V, differential equations from which we can determine U_n step by step. The quantity U_0 represents the intensity of the wave front (having at high frequencies the sense of amplitude). In the case of a point source of pulsed vibrations, one can demonstrate the convergence of the expansions carried out in the usual sense, based on certain results of Hadamard. In the case of caustics, the type of discontinuity [described by the function $f_0(t)$] may vary. However, in first approximation, one can find the character of the discontinuity and the intensity of the wave which passes the caustic.

For the solution of problems connected with the change in the type of discontinuity (for example, reflection of a wave behind the limiting angle), it is possible to apply the complex form of the ray method.

For this purpose, the field of an elastic wave excited by a point source that is applied sharply is represented in the neighborhood of the surface of the front in the form

$$U(x, y, z, t) = \operatorname{Im} \sum_{n=0}^{\infty} U_n(x, y, z) f_n(t-\tau),$$

where $U_n(x, y, z)$ is a complex vector; $\tau(x, y, z)$ is the eikonal of the wave under consideration, and $f_n(t-\tau)$ is a complex function which has a discontinuity in the n-th derivative at the surface of the front $t = \tau$. Recurrent differential relations for U_n (wherein their values are determined in the case of media with arbitrary inhomogeneity) follow from the Lamé equations of motion.

Considering the setting up of boundary problems in the ray method, **A. S. Alekseev** and **B. Ya. Gelchinskiĭ** have found that in the illuminated regions the boundary conditions must be satisfied at the points of intersection of the wave fronts with the surface of separation. From the condition of compensation of discontinuities of identical order, they obtained a system of algebraic equations for the complex amplitudes of n-th order. Using the ray method, we can compute the intensity of the head wave on the plane boundary of separation when the incident wave has an arbitrarily curved front. Depending on the ratio of the velocities of propagation of longitudinal and transverse waves in the boundary media, and on the type of head wave, the boundary conditions are not satisfied without introducing the so-called "inhomogeneous boundary" waves which have discontinuities only on the boundary of separation. The speakers clarified the number and character of the inhomogeneous waves and generalized the results obtained to the case in which one of the media is one-dimensionally inhomogeneous.

In a number of papers, propagation was considered in media and wave guides with regular inhomogeneities. As is well-known, in the calculation of a pulse generated in an inhomogeneous medium by a point source, the problem can be solved by finding the spectrum of the whole field for the particular case of an initial δ pulse, by multiplication of the result by the spectrum of the given initial pulse and integration over the frequency. In the case of layered inhomogeneous media, the expression obtained for the spectrum is quite complicated and calculation of the integral is difficult. **Yu. L. Gazaryan** has shown that the exact formulas for the high-frequency spectrum of the incident pulse can be replaced by approximate values which give its asymptotic value in the frequency. Application of this method to the computation of the pulse propagated along a pencil of rays which forms a caustic (under the assumption that the points of contact of the rays with the caustic are not singular) leads, for an outgoing pulse with time dependence in the form $p(t)$, to an expression of the form $\int_{-\infty}^{\infty} p'(\tau) F(t, \tau) d\tau$, which permits us to compute the approximate value of the pulse both at distant and at short-range distances from the caustic in the exposed region and close to the caustic in the shadow region.

N. B. Tsepelev has found a solution of the problem of the "Propagation of Waves in a Medium with an Inhomogeneous Transition Layer" of thickness H characterized by a velocity distribution $v(z) = v_0 e^{\epsilon z}$ (the velocities of propagation in the homogeneous half spaces are v_0 and $v_0 e^{\epsilon H}$). Investigating the solution by the method of contour integration, the speaker separated the principal parts of the solution describing the field of the displacement in the vicinity of the propagating wave fronts, and determined the reflection properties of the boundary on which the discontinuity of the derivative of the velocity and temperature takes place. The possibility was shown of a partial generalization of the method for the equations of elasticity theory.

A. N. Barkhatov and **I. I. Shmelev** investigated experimentally "Focusing of Sound in a Waveguide" formed by a layered inhomogeneous medium, in which the function describing the vertical distribution of the index of refraction has a maximum at some altitude. By means of diffusion between the several mixed liquids in a tank, two types of wave guides (which have a width many times larger than the wavelength) were obtained and investigated, one on the surface and one below the surface of the liquid. Measurements made on pulses in the frequency range from 500 kcs to 2.7 Mcs. At short distances (in the range of applicability of the ray theory) the pressure on the axis of the wave guide falls off by a spherical law, while at larger distances it falls off approximately in cylindrical fashion. The investigated structure of the foci for horizontal and vertical cuts at different positions of the source were compared with the ray and wave theories.

E. P. Masterov considered wave propagation of sound in a medium the square of the refractive index of which changes according to a biexponential law. He obtained an equation for the sound waves, found in integral representation of a field of a point source for harmonic radiation, and investigated the solution for the case of an unbounded medium.

N. S. Ageeva analyzed "The Propagation of Sound

Pulses in a Sound Channel," comparing the forms of the signal on the recorder with the ray picture calculated according to the conditions of experiment. The signal, which has initially the form of double and triple pulses, takes on a more complicated shape at large distances (up to 20 km). Controlled experiments with a short pulse showed that the complicated signal consists of individual pulses which correspond to different pulses at the point of reception at an instant of time that coincides with the calculated values. The amplitudes of these pulses agree with the calculation which takes into account the focusing factor and the directivity pattern of the source. With increase in distance between the source and the receiver (located at one horizontal position close to the axis of the channel), the total length of the signal increases. In this case, in correspondence with the ray calculation, the amplitudes of the first pulses, corresponding to the rays which pass earlier, are less than the amplitudes of the subsequent pulses. The shape of the signal is stable, which indicates the small effect of the micro-inhomogeneity of the medium (thermal fluctuations) on the direct field.

It is known that the propagation of normal waves in a medium within variable depth is similar to the propagation of waves in an inhomogeneous medium. Therefore, by preparing a layer with a specially chosen profile, we can obtain different focusing effects.

V. K. Kuznetsov investigated experimentally the effect of focusing normal waves by different inhomogeneities, built into a plane layer, representing the results of the measurement in the form of a continuous recording of the amplitude and phase of the sound field on a film of a loop oscillograph. Satisfactory agreement with calculations was obtained.

For the clarification of the applicability of the ray approach to the investigation of wave guides in a layered inhomogeneous medium, **V. A. Polyanskaya** calculated approximately and exactly the group velocities for some plane wave guides with continuous change in the velocity of sound. It was shown that for a definite distribution of the velocity of sound, the monotonic rise of the group velocity was broken by the appearance of extrema due to focusing in the medium and leading to some dependence of the field on the distance

S. Kalinski and **E. Kurlandski** (Poland) considered "Several Problems of the Propagation of Waves in Anisotropic Elastic and Non-elastic Media."

I. A. Urosovskiĭ solved the problem of "The Scattering of Sound on a Surface of Sinusoidal Shape Characterized by a Periodically-varying Normal Impedance." Assuming that the pressure p_i in an incident monochromatic wave depends only on the coordinates x and z (plane problem) while an uneven surface has the form $z = \xi(x)$, the lecturer found that the relation

$$\frac{\partial p}{\partial n}[x\xi(x)] = ik\eta(x)\, p[x,\, \xi(x)],$$

where $p(x, \xi)$ is the total pressure on the surface, n is the unit normal to the surface, k is the wave number, and $\eta(x)$ is a specified function. For a sufficiently sloping surface we can obtain the value of the total pressure on the surface with the aid of the inverse Fourier transform of the function

$$E(m) = \int_{-\infty}^{+\infty} p[x,\, \xi(x)]\, e^{imx}\, dx.$$

The value of $p(x, z)$ is an arbitrary point under the surfaces expressed by $p[x, \xi(x)]$ with the aid of the Green's function. The reflection of a plane wave was considered in detail.

Up to the present time the ray representation is used in seismic exploration for the construction of a geological profile. However, the wavelengths are not small in comparison with the thickness of the exploration layers, and the approximation given by such a construction is far from sufficient. By recording the vibrations produced by an explosion on magnetic tape, it is possible, **M. A. Antokol'skiĭ** pointed out, to amplify the regular wave fronts in comparison to the incoherent vibrations and to obtain additional information on the layers. Reproduction of the magnetic recordings permits us to investigate the decay of the intensity of the vibrations produced by the explosion.

A group of papers was devoted to the propagation and focusing of waves in statistically inhomogeneous media.

V. I. Tatarskiĭ considered "The Fluctuations of Amplitude and Phase of Plane Waves Propagating in a Turbulent Medium with Slowly Changing Average Characteristics." The correlation functions of the fluctuation of the index of refraction in such a medium depend not only on the difference in the coordinates at the points of observation $\mathbf{r}_1 - \mathbf{r}_2$, but also on the pair of points in space which is characterized by the vector $\mathbf{r} = \frac{1}{r}(\mathbf{r}_1 + \mathbf{r}_2)$. The form of the functions describing the local properties of a turbulent medium does not depend on the position of a pair of points of observation in space. Propagation of waves in such a medium is described with the aid of a linearized eikonal equation, in which the second derivatives with respect to the coordinates are preserved (the Rytov method of smooth perturbations). The lecturer expressed the spatical two-dimensional spectral density of the fluctuations of the logarithm of the amplitude and the phase of the wave in the plane $x = l$ (the incident plane wave is propagated along the x axis) in terms of the spectral density of fluctuations of the index of refraction. The special case in which the fluctuations of the index of refraction of the medium are described by the "two-thirds law" was considered in detail.

A. S. Gurvich, V. I. Tatarskiĭ, and **L. R. Tsvang** measured the quantity $\sigma^2(\overline{I-I})^2/I^2$ (where i is the density of luminous flux) with the help of specially constructed apparatus. They also measured the spatial correlation function, the probability distribution function, and the frequency spectrum of the fluctuations and the luminous flux from the light source placed in the ground layer of the atmosphere at distances from 250 to 2500 m in the frequency range 0.05 — 1000 cps for different (controlled) meteorological conditions. The radius of the correlation function of the intensity is determined by the quantity $\sqrt{\lambda L}$, while the probability distribution function of the fluctuations is satisfactorily described by a logarithmic normal law. The form of the frequency spectrum is in qualitative agreement with the calculated spectral density of the fluctuations. The frequency spectrum of the fluctuations is approximately uniform in the range from 0.05 to 10 cps; above, up to 200 cycles, the spectral density of the fluctuations falls off.

Investigating "Propagation of a Modulated Wave in a Statistically Inhomogeneous Medium," **V. A. Zverev** considered the effect of statistical inhomogeneities (whose correlation radius is much greater than the wavelength) on the phase (PR) and amplitude (AR) relations of a simple third-harmonic wave. In geometric approximation ($L \ll l^2/\lambda$) the effects, which consists in the change of PR and AR relations, are proportional to the magnitudes of the fluctuations of phase and amplitude of the wave. In the case of large distances ($L \gg l^2/\lambda$) there appears an additional factor p equal to the ratio λ/l and l/λ. Since the expressions, for the change of PR and AR contains the sound pressure p in different degrees, they can be used for the experimental examination of p, and consequently for the radius of correlation. **V. A. Zverev** showed a block diagram for the simultaneous recording of PR and AR which allowed him to measure the pressure p.

In order to bring about agreement between the equation of a wave propagating in a medium with large scale random inhomogeneities and the conservation of energy, it is necessary to introduce the normalizing factor $\exp(-L^2)$, which depends on the mean square fluctuation of the level. The normalizing factor for this requirement is determined non-uniquely, with accuracy up to an arbitrary phase factor $e^{i\varphi}$. With the aim of clearing up the ambiguity, **L. A. Chernov** and **T. A. Shirokova** applied a method for the determination of the "Normal field of a wave in a statistically inhomogeneous medium," which is based on a consideration of the second approximation in Rytov's method. They showed that the phase φ in the general case is not equal to zero. (However, this is not important for energy considerations, inasmuch as the energy is determined by the square of the amplitude.)

M. N. Krom and **L. A. Chernov** reported that the "Dependencies of the Diffraction Image in a Lens on the Magnitude of the Fluctuations of the Incident Wave" lead to the appearance of pulsations in the diffraction image and to deformation of the central diffraction image. They cleared up the effect of the dimensions of the objective on the relative fluctuations at the focus without limitation on the smallness of the excitation in the incident wave.

É. A. Blyakhman applied the spectral method for the investigation of the effect of a focusing system on the pulsation of the field of the wave which traverses a medium containing random inhomogeneities. For chaotic motion of the inhomogeneities, values of the spectral density were obtained in limiting cases of large and small fluctuations of the field of the incident wave and the dimensions of the lens that were large and small in comparison with the scale of the inhomogeneities. It was shown that for regular motion of the inhomogeneities in the special form of the correlation function of the pulsation of the field in the incident wave the lens acts as a filter separating out the characteristic frequencies.

V. N. Karavainikov computed the coefficient of mutual correlation between the fluctuation of amplitudes and phase and also the correlation of longitudinal and transverse auto-correlation of the fluctuation of amplitudes and phases in a spherical wave.

"Distribution Functions for Fluctuations of the Field in the Shadow Zone" was reported by **B. E. Kinber** and **L. G. Solov'ev.** The fluctuations of the field propagated across a statistically inhomogenous medium in the shadow zone are determined by two factors. From the change of the gradient of the index of refraction, the scale of which is commensurate with the distance between the source and the receiver, the scattering angle changes, which produces slow fluctuations of the field. The field of isolated inhomogeneities, whose scale exceeds the wavelength but is much smaller than the distance between the receiver and the source, combines with the random inhomogeneities which produces a rapid fluctuation of the field. The differential and integral distribution of the resulting signal was considered under the assumption of a statistical inhomogeneity of the slow and fast fluctuations in the reception at one and at two receivers. This distribution is well approximated by a Gaussian distribution of a quadratic polynomial for the logarithm of the signal.

A. D. Lapin solved the problem "Of the Scattering of Sound Waves in Irregular Waveguides" with consideration of multiple scattering, that is, in the case in which the first approximation of the method of small perturbation turns out to be inadequate. For each of the plane waves of which the incident normal wave is composed he applies the method of Rytov and finds the scattered field from these plane waves under the condition that the total field satisfied the boundary conditions on the walls of the wave guide. By way of irregularities, he considered small fluctuations of the parameters of the medium filling the wave guide and also roughness and inhomogeneity of the walls of the guide.

Ordinarily the problem of a long line with random inhomogeneities is solved by the method of successive approximations. Actually only the first approximation is found, in which the coefficients of reflection are added vectorially, while the attenuation of the initial incident wave resulting from reflection from the inhomogeneities and multiple reflection are not considered. **M. E. Gertsenshteǐn** and **V. B. Vasil'ev** obtained an exact equation for the probability density of the reflection coefficient for a lossless line with random inhomogeneities located at a great distance from one another. The equation takes into account multiple reflections and the attenuation of the initial wave, while to each new inhomogeneity there corresponds a fractional linear transformation of the curved diagram. Inasmuch as the inhomogeneities are statistically independent, the problem reduces to finding the resulting random fractional linear transformations. It turns out that in summing up a large number of random small vectors the probability density $w(x, y)$, as a function of the coordinates x, y, of the vector of total reflection coefficient, satisfies the diffusion equation.

In connection with problems of the investigation of noise, interest has been aroused in the study of elastic walls which vibrate under the action of statistical distributions of forces. **M. A. Isakovich** has considered the field generated in the Fraunhoffer region of a plane elastic surface bounded by a liquid and containing vibrations under the actions of statistically distributed surface forces. Taking the radius of correlation of these forces to be small in comparison with the dimensions of the surface under investigation, he solved the problem with the aid of an expansion of the surface forces in a double Fourier integral over the surface by application of the Rayleigh method. This permitted him to consider the reaction of the medium to the vibrating surface and to solve the dynamical part of the problem, representing the normal displacements of the surface also in the form of a double Fourier integral. Then the desired field was represented in the form of a single-term Green's function, whence he obtained for the Fraunhoffer region characteristics of the directivity of the radiation, expressed in terms of the correlation function of the surface forces. It was shown in particular that the radiation in the direction of the normal to the surface does not depend on the elastic properties of the surface; computed formulas were given for special cases of an elastic surface comprising a tightly-drawn membrane and in the form of an elastic disc subject to flexural vibrations. The results obtained can also be applied to the problem of a slightly curved elastic surface.

L. M. Brekhovskikh considered the Rayleigh surface wave that propagates along a rough free surface of a solid body. Assuming that the depth of the inhomogeneities (regular or statistical) is small in comparison with the length of the Rayleigh wave, he solved the problem by the method of successive approximations. The first approximation corresponds to the propagation of the wave along the plane free surface. In the first approximation the effect of inhomogeneities is equivalent to the action of an additional stresses distributed over the plane surface. These stresses produce scattered waves which are transverse and longitudinal, and which go out from the surface, and also new surface waves, which propagate along the surface with velocities that differ from the velocity of the Rayleigh waves. The waves diverging from the surface produce a damping of the original wave. Being superimposed on the first wave, they create, in the case of propagation of a monochromatic wave, a picture of spatial beats and in the propagation of a pulse they distort its shape.

A large part of the papers of the sessions on "Nonlinear Acoustics" were devoted to problems of the propagation and absorption of waves of finite amplitude in gasses and liquids.

Considering the propagation of plane waves of finite amplitude in a viscous medium, **V. A. Burov** combined the Navier-Stokes equation and the equation of continuity (under the assumption of smallness of the Mach number and of the change of shape and amplitude of the wave in a single wavelength and also of the adiabatic equation of states) to form a quasi-linear equation. Solving this equation for harmonic boundary conditions he found that at a certain distance the wave took on a saw-toothed shape and had a large absorption coefficient in comparison with that of small amplitude waves.

Considering "The Propagation of Spherical Waves of Finite Amplitude in a Viscous Thermally Conducting Medium" in Lagrangian coordinates, **K. A. Naugol'nykh** got rid of the nonlinearity of motion and simplified the giving of boundary conditions. He solved the equation, which describes with accuracy up to terms of second order in the Mach number, by the method of Krylov-Bogolyubov. K. A. Naugol'nykh found that, for sufficiently high intensities, an initial sinusoidal wave changes to a sawtooth form at a finite distance from the source, in spite of the fact that in addition to viscous absorption there is a great decrease of the amplitude as a result of the divergence.

This was experimentally verified by **E. V. Romanenko**, who demonstrated that in spite of the large decrease in the amplitude of the wave because of spreading, the wave took on a markedly sawtoothed shape while the second harmonic increased in magnitude to thirty per cent of the first harmonic. The measurements were carried out in the range 0.8 — 1.2 Mcs and at intensities up to 300 w/cm^2 in a pulsed mode with miniature piezoelectric receivers and quartz plates. The role of cavitation in the distortion of a (plane) wave was also investigated. It was shown that increase in pressure up to 18 atmos does not affect the shape of the wave.

However, observing the shape of a plane sawtooth wave at a frequency of one megacycle and intensity 35 w/cm^2 in tap water, **V. A. Burov, L. K. Zarembo, V. A. Krasil'nikov**, and **V. V. Shklovskaya-Kordi** established the fact that a decrease in the amplitude of the original takes place in time (~ 10 to 30 sec) and came to the conclusion that this "limitation" is a consequence of cavitation. They also measured at a frequency of 1.5 and 4.5 Mcs the mean phase velocity of propagation of a wave of finite amplitude in a solution of methyl alcohol and water, which is characterized by a small temperature coefficient of velocity (two orders of magnitude smaller than the pure liquids). The velocity increases smoothly with the intensity slowly at the beginning and then in a jump coincident with the "limitation." The general increase in the velocity was of the order of tens of centimeters per second.

Measuring the intensity of ultrasonic waves of finite amplitude in water at frequencies of 1.5 to 6 Mcs, **D. V. Khaminov** found that the absorption coefficient as the function of the initial intensity (which reached 1 w/cm^2 at the source) increased in water at the absence of cavitation by a factor of 5 — 40. The frequency dependence of the absorption coefficients satisfies the law $\alpha_{\max} \sim f$. For a given frequency, the relation $\alpha_{\max} l = \text{const}$ was observed approximately, where l is the distance for stabilization of the wave shape.

Investigating the diffraction of light by ultrasonic waves of finite amplitude, **I. G. Mikhailov** and **V. A. Shutilov** discovered a sharp asymmetry in the distribution of light around the diffraction maximum. Considering the diffraction of light with account of phase modulation by sound waves of sawtooth form, they constructed models of these waves according to the photographs of the spectra for different distances from the source and computed their intensity. The results obtained agreed, within the limits of experimental error, with the results of measurement of the absolute intensity of ultrasonic waves. By allowing the ultrasonic wave to pass through glass plates of thickness of one fourth, one third etc., of the wave, the authors separated from the spectrum of the distorted sound wave the various harmonic components, observing in this way the corresponding double, triple, etc., distance between the diffraction maximum.

Eikhenval'd, has already considered the acoustic field of finite amplitude which arises in a non-viscous thermally conducting gas between rigid walls for an initial sinusoidal propagation velocity of the particles. **Z. A. Gol'dberg** generalized these results to the case of an arbitrary medium with viscosity and heat conduction, which leads to an exponential damping in time. Considering the wave field established in the case in which one of the walls is rigid while the other vibrates, the author established the dependence of the amplitude of quantities of second order on the spatial coordinates and the presence in the second approximation of two terms, one of which leads to a symmetric and the other to an asymmetric (relative to the nodes) distortion of the form of the field.

A. L. Polyakova considered the propagation of sound of finite amplitude in a relaxing medium as a non-equilibrium process and obtained, on the basis of thermodynamics of irreversible processes, an expression for the energy dissipated in this process (the nonlinearity of the equations of hydrodynamics was preserved in this case). Considering the shape of the wave in an actual relaxing process to be Riemann, and expanding in a series, she found corrections of the second order to the absorption coefficient of the wave (according to the ratio of the dissipated energy averaged over time to the total energy stored in the wave).

Similar problems were taken up by **E. V. Stupochenko** and **I. P. Stakhanov,** who considered the theory of stationary flows of liquid (acoustic wind, Dvorzhin's phenomenon) in relaxing media. On the basis of the calculation, they set up the equations of motion of a viscous liquid supplemented by the "reaction" equation that describes the change in time of the parameter ξ (which characterizes the departure of the medium from the state of thermodynamic equilibrium), and the equation for the increase of entropy as a result of irreversible processes connected with change in ξ. Averaging the terms of the equation over times which are long in comparison with the period of the sound, and small in comparison with the characteristic time of the cumulative perturbation, the authors solved the equations which describe the quasi-stationary flows in second approximation in the case of cylindrical sound fields (for an arbitrary, radially-symmetric distribution of the sound intensity). In this case, account is taken of the attenuation of sound which is propagated along the axis of the cylinder; this is especially significant for frequencies close to the reciprocal of the relaxation time. In the limiting case of small relaxation times, the theory just developed describes the acoustical wind in a medium with volume relaxation in which, however, the attenuation of sound is taken into account.

The theory of secondary flows was also considered by **P. N. Kubanskiĭ,** He concerned himself with the mechanism of vortex formation in a resonator with a flange for the flow of air past its neck. The transverse flow which moves around the neck of the resonator forms a turbulent stream about it which experiences a lateral pressure produced by the flow in the cavity of the resonator. With increase in the velocity of flow, the force acting on the stream increases, the flow is diverted beyond the limit of the neck and gives an exit to the vortex from the cavity of the resonator. The self-oscillations that arise in such a case are characterized by large amplitudes of vibration of the stream; therefore secondary external streams arise in the neck of the resonator. The author assumed that the vortex formation in the neck of the resonator represents a superposition of vortices emitted periodically from the cavity of the resonator and which are generated by the vibrations of the stream.

Measuring the absorption of ultrasonic waves of infinitely small amplitude in solutions of rubbers and polymers, **I. G. Mikhailov** and **N. M. Federova** found that it differs only slightly from the absorption in the pure solvent although the macroscopic viscosity of the solution can exceed the viscosity of the solvent by a significant amount. A small additional absorption in the solutions is produced by losses in friction arising in the motion of the solvent relative to the polymer chains. In the transition to ultrasonic waves of finite amplitude, the forces of friction are so increased that there arises a disruption of the polymer chain. At this moment, the magnitude and character of the absorption in the polymer solution changes sharply. Evidently the process of disruption of the polymer possesses an activation character connected with the jump across the potential barrier; therefore, the threshold voltage on the quartz (for which the disruption of the structure of the solution takes place) is proportional to U/RT (U is the energy of coupling of the links of the chain, T is the temperature, and R is the gas constant). Investigating the temperature dependence of the threshold voltage, the authors found the magnitude of U for certain solutions of polymers, indicating in this way the possibility of use of ultrasonic waves of finite amplitude for the study of the interaction of polymer molecules in solution.

For a number of practical problems in hydroacoustics, there is an urgent need of increasing the intensity of the source. But the intensity of ordinary sound sources is sharply limited by the increase in nonlinear effects. New possibilities are presented by the phenomenon of sound production by sparks and corona discharges in water, investigated by **N. A. Roĭ**, **D. P. Frolov** and **A. L. Polyakova**. Studying the conditions of sparks of maximum duration, the electrical and acoustical characteristics of spark discharge, they found it possible to obtain high electro-acoustical efficiency. Applying the corona discharges they could spread out the sound formation over a large volume, decreasing the slopes of the fronts of the acoustic pulses and at the same time lowering the dissipation of the energy of the pulses in the medium. However, the electro-acoustical efficiency of the corona discharges are not large and research is needed towards their increase.

Several papers were devoted to problems of propagation of shock disturbances in various media. For the medium whose internal energy is determined by the specific volume and entropy, the form and properties of the shock adiabatic could be established (using differential relations which hold along the Hugoniot adiabatic), if the sign of the second derivative of the depression with respect to its specific volume is constant throughout the whole region. **G. Ya. Galin** succeeded (by obtaining additional evidence on the shock adiabatics) in establishing the form of the Hugoniot adiabatics and investigating the properties of the shock transition for media with arbitrary equations of state (when the derivative $\partial^2 p/\partial s^2$ changes sign along the Poisson adiabatic). He clarified the propagation of the disturbance in such ideal compressible media and showed, in particular, the possibility of the existence of more than just a single solution. For the choice of the latter, use was made of considerations of the stability of the surfaces of strong discontinuity.

N. N. Kochina investigated the problem of a powerful point explosion in an ideal compressible medium. If the internal energy of the medium has the form

$$\varepsilon(p, \rho) = \frac{p}{\rho}\,\varphi\left(\frac{\rho}{\rho_0}\right)$$

(where ρ_0 is a certain density), the problem is self-similar and reduces to the solution of an ordinary differential equation of first order, the form of which depends on the function $\varphi(\rho/\rho_0)$ and its derivative. Obtaining a special exact solution [which holds for an arbitrary function $\varphi(\rho/\rho_0)$], Kochina solved as examples some problems involving a strong shock, in particular, a shock in a medium similar to water, and found estimates for the time during which we can consider the shock to be strong.

Solving the equations of gas dynamics, which describe the propagation of waves of small amplitude (under the supposition that the width of the disturbance region is small in comparison with the characteristic dimensions of the problem), **K. E. Gubkin** found that the nonlinear character of the motion appears in significant fashion when the wave has traveled a distance which is appreciably greater than the width of the disturbed region. Account of nonlinear factors leads to a change in the profile of the wave and the formation of discontinuities in it, and also to an additional damping of the shock front. At large distances, the profile of the pressure wave behind the front is always close to linear, independent of the initial shape.

S. I. Sidorkina considered the formation of a shock wave by a flat massive plate which is also excited by a shock wave. The author made clear the fact that in a nonstationary shock the maximum intensity of the secondary wave is necessarily smaller than the primary, and obtained results of the approximate calculation of the motion of the plate in the gas behind it.

The research of the section on "Radiation and Diffraction of Sound" was devoted principally to the development and to the comparison between the different exact and approximate mathematical methods of the solution of stationary and nonstationary wave problems. In connection with the development of the acoustics of solids, a great deal of attention was given to vector wave equations, in addition to the consideration of the scalar wave equation.

In a group of papers presented by **G. I. Petrashen'** and his coworkers, there was put forth and developed a general method of the exact consideration of diffraction in corners which generalizes the Smirnov-Soboleva method, and the method of integral representation of Sommerfeld, developed in the researches of G. D. Malyuzhinets and his school. **G. I. Petrashen'** and **B. G. Nikolaev** noted that this method permits us to solve "naturally" and uniquely all the known classical stationary and nonstationary problems of diffraction from a doubly bounded angle, and also leads to the final result in a series of cases not considered previously.

Reporting on the results of the calculation of diffraction fields in stationary and nonstationary acoustical problems, **B. G. Nikolev** and **M. V. Vasil'eva** developed detailed formulas for computations and considered the qualitative and quantitative laws in the change of the field in the diffraction region. Detailed consideration was given to the case of a plane incident wave from plane and point sources.

An "Approximate Consideration of the Wave Field Close to a Randomly Rough Surface," was applied by **G. D. Malyuzhinets** to the approximation of transverse diffusion (change of the elliptical equation to the parabolic); he showed that the result of statistical averaging of the field of random functions over the ensemble coincides with the solution of the simplest problem involving the reflections of a wave created by the same source from an infinite bounding plane, characterized by some normal impedance. The magnitude of this "average" impedance depends on the spectrum of the random function and can be computed or determined by experiment. The effect of attenuation with increasing distance from the source can be estimated in terms of the "average field." The field close to the random surface can be represented as the sum of this field and the random one, corresponding to the effect of scattering and leading to fluctuations.

A new, simple and exact method of solution was considered by **M. D. Khaskind** for the problems of propagation and diffraction of sinusoidal waves in a half-space the boundary conditions on whose plane boundary are of the third kind. Using the general solution obtained, the author determined a simple form for the function of the radiation of cylincrical, spherical, and other sources and gave a method for the determination of exact solutions of problems of radiation and diffraction of sound waves by a vertical plate which partially encloses the half-space, by a plate containing a closed region, by a horizontal half-plane which encloses the part of the boundary of the liquid, etc.

The paper of **A. S. Goryainov** also contained an exact solution of one of the diffraction problems — the diffraction of a plane sound wave on a rigid infinite cone (and, correspondingly, of a plane electromagnetic wave on a conducting cone). The solution, which contains a diffraction series, is transformed into a contour integral, in which the incident wave is separated out. The contour integrals, which contain only the field reflected from the surface of the strip, are evaluated by numerical methods. The speaker compared the results of the exact theory with geometrical optics, and also with some approximate methods in the special case of axial incidence of the plane wave. For the case of reflection from it, a simple formula was obtained which gives excellent agreement with the exact theory.

Approximate solutions of diffraction problems were considered by **A. A. Fedorov** and **I. N. Kanevskiĭ**. The first of these, using the method of V. A. Fock, and assuming that the diffracting body of revolution was sufficiently large ($ka \gg 1$, a being a characteristic dimension of the body), summed the diffraction series asymptotically, the series representing the exact solution of the problem. Putting the computed formulas in the form of a product of Bessel functions and tabulated functions, and carrying out numerical computations for $ka = 5$ and 10, the author found that satisfactory agreement with the calculations from the exact theory is obtained for a sphere for $ka \gg 5$. The field in the distant zone is the result of the interference of the wave reflected according to geometrical optics and the waves which bend around the sphere by the meridional path (diffraction waves). The relations obtained for certain physical assumptions relative to the phase and amplitude of the diffraction waves can be generalized for spheroids.

To date, the results of experimental investigations of the field at the focus of sound focusing systems have been compared with the calculation of a field which does not consider the diffraction of the sound itself on the receiver. In the same way, in the calculations of sound concentrators, no consideration was given to diffraction on irradiated objects placed at the focus. Considering the diffraction of a converging cylindrical infinite front on an infinite cylinder (placed coaxially with the front), and also on a sphere (the center of which is located on the axis of the front), I. N. Kanevskiĭ found expressions for the potential, pressure and velocity of the resulting field, computed the distribution of the pressure and the velocity of the surface of a rigid cylinder and obtained asymptotic expressions for the intensity and total scattered power of the wave. A comparison carried out for the case of the diffraction of a plane wave by a cylinder and a sphere permitted him to make clear the characteristic peculiarities of the problem.

In discussions arising during the talks, **G. D. Malyuzhinets** interpreted some results with the help of a representation of the transverse diffusion of the amplitude over the wave front.

The next three papers were devoted to vibrations of elastic bodies. In the paper of **S. V. Kaliski** (Poland), the difficult problem of the vibrations of a finite elastic cylinder was first subjected to an exact analysis. In this case, the method of solution was based on a knowledge of the problem for a completely regular infinite system of algebraic equations.

V. V. Tyutekin took up the diffraction of a plane longitudinal wave propagated in an isotropic elastic medium on a cylindrical cavity of infinite length for an arbitrary direction of the incident wave. The scattered field was given in terms of scalar and vector potentials. The components of the vector potential were determined with the aid of three vector functions, each of which satisfies the vector wave equation. The solution is an unbounded sum of cylindrical waves of different orders, in which the zero term corresponds to pulsating vibrations of the walls of the cavity for different angles of incidence. In this way the author solved the problem of the scattering of a plane sound wave incident, at an arbitrary angle, on an elastic cylinder of arbitrary radius. It was shown that if one of the characteristic values of the wave number inside the cylinder coincides with the projection of the wave number in the medium on the axis of the cylinder, an intense scattering of the sound takes place.

L. Ya. Tutin discussed flexural vibrations of an infinite strip with supported edges and showed that if the width of the strip is less than half the width of the flexural wave, an undamped traveling flexural wave is not possible. For infinite length of a strip with free edges, he determined the dependence of the velocity of propagation of flexural waves on the ratio of the width of the strip to the flexural wavelength.

In connection with problems of the elimination of vibrations and the noises produced by them, there was great interest in the physical investigation of the interaction of the vibrations of the elastic plates and shells with an acoustic field in the surrounding medium, and a series of papers was given on the subject.

L. A. Molotkov and **G. I. Petrashen'** gave a paper on "Certain Dynamical Properties of Thin Elastic Layers." They considered free, thin, plane-parallel layers immersed in a liquid, and in contact with an elastic medium. On the basis of an exact solution, they clarified the question of the character of external dynamical influences, for which the layer can be replaced by an equivalent membrane, plate, or other degenerate system. The type of the degenerate system depends essentially on the medium in which the layer is found, and also on the character of the influence. For example, a thin elastic layer can be regarded as a plate if it is located in a cavity, but not so if it is immersed in a liquid.

L. M. Lyamshev considered the scattering of a statistical sound field from a thin elastic homogeneous shell when this field is defined by a certain distribution of random sources in a homogeneous medium, where the shell is found; he also considered the radiation of the statistical sound field of the vibrating elastic shell located in a homogeneous medium when the vibrations of the shell are defined by certain surface statistical forces. Both problems are solved by an application of the reciprocity theorem to the statistical field and the auxiliary diffraction (nonstatistical) field defined by a source located at the point in space at which it is necessary to determine the intensity of the statistical field. In this case the results obtained previously are used for the calculation of diffraction on the shells. The author computed the spectral intensity of the statistical field in the case of spherical and cylindrical shells and thin plates at distances that are large in comparison with the dimensions of the body.

An "Exact Solution of the Plane Wave Diffraction Problem on a Semi-infinite Elastic Plate" was obtained in the form of a Sommerfeld integral by **G. D. Malyuzhinets** by a generalization of the method used previously in the problem of diffraction on a wedge with boundary conditions of the third kind (with given edge impedances). The differential equations of flexural and longitudinal vibrations of a thin plate (with account of the external force distribution) serve as boundary conditions for the sound pressure p which satisfies the wave equation $\nabla^2 p + k^2 p = 0$ outside the plate. The vibrations of the plate in turn satisfy the boundary conditions of general form (with consideration of possible external influence) on the edge $x = 0$. Giving the amplitudes of the waves incident on the edge from infinity, waves which are propagated both in the surrounding medium and along the plate itself, the author found a solution which is continuous in the region and on the boundary, and which also satisfies the condition of extinction. As special cases he considered: diffraction on the plate of a plane wave incident at an arbitrary angle, reflection from the edge of the plate and radiation into the surrounding space of a flexural or longitudinal wave which passes along the plate from infinity, and propagation and radiation of a wave which is excited in the plate for a given reaction on the edge.

V. S. Buldyrev, applying the method of contour integration developed in the researches of G. I. Petrashen' and his coworkers, solved the nonstationary problem of the diffraction of sound waves on a membrane of a cylindrical shell. He found that the excitation of a reflected wave in the vicinity of its front is independent, in first approximation, on the parameters of the shell, and coincides with the excitation of a wave reflected from an absolutely rigid cylinder. As a consequence of the velocity dispersion in the shell, the disturbance before the front of the head wave is different from zero. The dependence of the intensity of the head wave at its front on the different parameters of the problem was made clear, and the possibility of replacement of thin elastic layers bounding the medium (in which the propagation of the waves take place) by models of the type of membranes and shells were discussed.

Too few experimental investigations make a significant contribution to the study of diffraction phenomena. Continuing their long-standing research on the visualization of ultrasonic fields, **S. N. Rzhevkin** and **V. I. Makarov** investigated, both experimentally and theoretically, the process of the excitation of ultrasonic waves in plates and cylindrical shells in the exposure of them from a liquid medium. The importance of discrete "excitation bands" on the shells was made clear and the possibility of simultaneous excitation of several types of waves in a solid layer was established. It was shown that the sound field in a cavity enclosed by a shell, excited by ultrasound, forms a region limited by a caustic surface, inside which there is no sound. The sound field outside the shell forms a stable interference picture of the bands, perpendicular to the shell (in the establishment of standing waves in the shell).

In the last paper of this group, given by **L. Ya.**

Gutin, the approximate relation between the mean square amplitude of the vibrational velocity and the radiated power for a complicated form of the excitation of rectangular plates with supported borders was given. It was first established that the coefficient of radiation of such a plate is almost identical for the excitation at resonant frequencies in the frequency range whose lower limit is determined by the ratio of the smallest dimension of the plate to the wavelength, and the upper limit, for the frequency at which the length of a free flexural wave in the plate is equal to the wavelength in the surrounding medium ("critical frequency"). It was then assumed that in the expansion of the force F in eigenfunctions $F = F_{mn}\psi_{mn}$, all F_{mn} are constant, and that intervals of frequencies between the exciting forces and the eigenfrequencies of the plate are equally probable.

The section on ultrasonics covered physical investigations of the propagation and absorption on nonlinear (together with the section on nonlinear acoustics) and linear ultrasonic vibrations in gases, liquids and solids, and investigations of a physicotechnical character, connected in large or small measure with the application of ultrasonic vibrations in technology. Papers of a purely technical and applied character were not given, since such researches were discussed in a series of specialized conferences.

The first session was devoted to problems of the acoustics of solids.

K. N. Baranskiĭ told of measurements of the attenuation of longitudinal vibrations with frequencies from 2×10^8 to 7.5×10^8 cps in (X-cut) quartz plates, excited in the higher harmonics of their base frequency. In this case a method was developed and applied which makes use of the phenomenon of the diffraction of light on ultrasound and which permits one to determine the temporal absorption coefficient from an investigation of periodic processes of establishment and attenuation of elastic vibrations in a quartz plate in its excitation by rectangular pulses of a high-frequency electric field. The form of the diffracted light pulses obtained were studied by a photomultiplier and an electronic oscilloscope. Values of the absorption coefficient obtained by this method, at a frequency of 5×10^8 cps, varied for different plates from 0.1 to 0.2 cm^{-1}. The values obtained agreed in order of magnitude with the theory developed by I. G. Shaposhnikov and S. N. Tkachenko.

The velocity of longitudinal and transverse vibrations in binary solid solutions of nickel-chromium and nickel-titanium for different amounts of plastic deformation and concentration of the chromium was measured by **T. Ya. Benieva**, who used a pulse method.

E. Ranachowski and **B. Lesniak** (Poland) studied experimentally the relation between the dielectric loss factor and the absorption of ultrasound in bakelite resin and polystyrene. They made an attempt to explain the mechanism of this loss in connection with this phenomenon.

Z. Pawlowski (Poland) considered the possibility of measurement of the elastic modulus of polycrystals by means of the determination of the angle of total reflection of ultrasonic waves. The results of the measurement were compared with data obtained by statistical methods.

O. I. Silaeva and **O. G. Shamina** gave reports on the peculiarities of measurements of the velocity and absorption of longitudinal and transverse waves on different laboratory specimens (plates, rods, massive blocks). It was established that the absorption coefficient of longitudinal waves in rods, plates and unbounded media have different values, which agrees with the theoretical data of G. I. Gurevich.

The special characteristics of propagation of surface waves was the subject of widespread discussion in the ultrasonics section. This is explained by their increasing role in the control of surface defects. Considering the problem of the possibility of formation and propagation of Rayleigh waves in a plane elastic layer with free boundaries, **I. A. Viktorov** showed that among the totality of normal waves excited by the source of Rayleigh waves placed on one of the free surfaces of the elastic layer, there is a component which is similar to a Rayleigh wave in the region close to the source. The lecturer investigated experimentally the origin and behavior of this component and obtained satisfactory agreement with calculations. He also computed the spatial coefficients of attenuation of the Rayleigh waves in aluminum and steel in terms of the corresponding coefficients of longitudinal and transverse waves and compared them with the experimental values. Qualitative agreement was obtained.

K. N. Vinogradov and **G. K. Ul'yanov** measured the phase velocity and attenuation of ultrasonic surface waves in certain metals, alloys and glasses. The vibrations were excited by the wedge method. In the measurement of the phase velocity, the receiver consisted of a noncontact magnetoacoustic receiver moved along the specimen. The attenuation was measured by the pulse method in a frequency range up to 5 megacycles with the help of wedge transducers. It was shown that the attenuation depends strongly on the purity of the preparation and the protective covering.

N. N. Egorov spoke on some peculiarities of the propagation of ultrasound along the surface of a double layered solid medium (the thickness of the upper layer was approximately equal to the wavelength). Since the attenuation of the ultrasound turned out to be strongly dependent on d/λ and for constant λ is (in a certain interval of thickness) a linear function of the thickness, it became possible to make an ultrasonic instrument for the measurement of the depth of the surface of hardened, cemented and certain other surface-hardened layers.

Some data on the diffraction of surface ultrasonic waves on a solid wedge, which are useful for the determination of the profile of a sample in an interferometric measurement of the velocity of propagation of surface waves, were given by **I. P. Denisov**.

A great deal of attention in the section was paid to the problem of the propagation and absorption of ultrasound in liquid media and alloys. **V. F. Nozdrev, N. I. Koshkin,** and **M. A. Gorbunov** gave a review paper on studies of the physico-chemical properties of complicated thermodynamical systems by ultrasonic means.

The investigation of binary liquid-crystal phase systems in the case of organic liquids showed the interdependence of the absorption coefficient and the peculiarities of the crystalline structure of the material. The absorption in the transition region is connected with the behavior of other physico-chemical parameters (heat capacity, coefficient of volume expansion, index of refraction, etc). The character of the dependence of the absorption on temperature in the transition region was explained qualitatively by them, beginning with the theoretical considerations of Ya. I. Frenkel' on the presence of quasi-crystalline regions. Investigations of the propagation and absorption of ultrasonic waves in the liquid-vapor system along the saturation curve and in the critical region permitted them to make clear the relation between the velocities of sound in the liquid and saturated vapor under conditions of dynamic equilibrium. The data on acoustical measurements were used for the calculation of the heat capacity and other physico-chemical parameters of the liquid.

Considering the problem of the absorption of ultrasound as the result of the excitation of two or more discrete energy levels, **R. Beyer** (USA) showed that the total absorption coefficient could be expressed as the sum of coefficients defined by the separate relaxation processes with normal modes of relaxation. Studying the ratio of the frequencies of these normal modes to the reactions rate corresponding to the excitation of a single level, he found that in many cases the frequency of the relaxation of lowest excited level is the normal frequency.

Analyzing new comparative data on dielectric relaxation and on ultrasonic shear and bulk relaxations in viscous associated liquids, **T. A. Litovitz** (USA) came to the conclusion that the ultrasonic relaxation times were frequently close in value. So far as the relations of the ultrasonic relaxation times and the dielectric relaxations are concerned, for some molecules they are close together, while for others of the same structure, this is not the case.

I. G. Mikhailov analyzed current experimental data on the absorption of ultrasonic waves in ethyl acetate, including measurements made in the author's laboratory by a pulse method in the frequency range of 5 — 25 megacycles. The most reliable experimental data agree well with the theoretical relaxational dependence of the absorption for a single relaxation time of the bulk viscosity.

This point of view was disputed in debates by **B. B. Kudryavtsev** and others. The discussion centering on this question was continued in a specially organized seminar.

B. B. Kudryavtsev gave a review paper on the application of ultrasonic measurements to the study of liquids. Setting forth briefly the phenomenological and molecular-kinetic theory of sound propagation in liquids, the lecturer demonstrated by several examples the possibility of the use of acoustical measurements in the study of the nature and properties of liquids.

The paper by **I. Z. Fisher** was devoted to the problem of establishing the connection of the sound velocity in liquids (and compressed gasses) with the molecular characteristics of the material. He analyzed two models, achieving an exact solution of the problem of the velocity of sound for arbitrary temperatures and pressures; a one-dimensional model of a liquid with an arbitrary law of interaction between the particles, and a three-dimensional model of a system of non-interacting spheres. Using the results and methods of contemporary statistical mechanics, he succeeded in both cases in expressing the sound velocity exactly in terms of thermodynamical parameters and the molecular characteristics of the system (mass and dimensions of the particles, potential of the intermolecular forces), which allowed him to give an exaustive analysis of the solution.

V. F. Nozdrev and **B. I. Kal'yanov** investigated the velocity and absorption of ultrasound in liquids by the pulse method for constant density as a function of temperature only. This permitted them to simplify the theoretical analysis of the relaxational processes that have been discovered and investigated experimentally.

M. S. Pesin and **I. L. Fabelinskiĭ** spoke about a continuation of investigations begun previously of the dispersion of the velocity of hyper-sound (with frequency $f = 10^{10}$ cps) in liquids with large volume viscosity coefficient; the measurements were carried out by the method of studying the fine structure of the Rayleigh scattering line. Dispersion of the velocity of sound was discovered (10 — 15%) in liquid methylene chloride, methylene bromide, and chloroform. The relaxation time of the second viscosity coefficient and the coefficient of absorption of these liquids were determined and discussed from the viewpoint of the relaxational theory of the absorption of sound in a liquid.

The widespread application of acoustics in the study of the physical properties of material was illustrated by a series of papers on the propagation of sonic vibrations in mixtures of different kinds and the connection of acoustical and other characteristics of these materials in the critical regions and at points of transition.

M. V. Bessonov talked about some peculiarities of the propagation of ultrasound in molten salts, in fused resin and in fused lead (frequency of 2.4 megacycles). For cooling in the region of vitrification, a sharp change takes place in the sound velocity. The measured values of the velocity were applied to the calculation of the coefficients of thermal conductivity by Bridgman's formula, in which satisfactory agreement with the results of direct measurements of these quantities was obtained.

N. F. Otpushchennikov measured the velocity of ultrasonic waves in the transition from the liquid state to the solid in naphthalene, hyposulfite, phenol and in low-melting-point alloys, and found that in the region 3 — 6°C from the beginning of crystallization of the material, the sound velocity changed anomalously (not by a linear law) with temperature. This shows that the process of crystallization is not a process which takes place only at the temperature of solidification and that, in a certain precrystallizing region, a process takes place involving the preparation of the material for crystallization. The latter conclusion is confirmed by other nonacoustical investigations of the physical properties of a number of materials close to the temperature of melting and crystallization.

V. I. Ilgunas and **E. P. Yaronis** studied the magnetic dispersion of ultrasound in mercury, liquid sodium and calcium with the use of interferometers. The interferometers were placed in a transverse magnetic field with an induction of 6 to 9 kilogausses at frequencies of 2.5 to 4 megacycles. For a roughness of measurement of ± 3 millimicrons, no effects of the magnetic field on the ultrasonic velocity were observed, which contradicts the theory of N. S. Anderson. Theoretical calculation of the magnetoacoustic effect, carried out by the authors by the method of hydrodynamic theory of sound with consideration of thermodynamic and electromagnetic factors, showed that the change in the sound velocity in the liquid metals investigated (in the limits of ultrasonic frequencies) gave a magnitude of the effect only of the order of 10^{-5} to 10^{-9} m/sec.

Z. L. Khodov spoke on measurements of the ultrasonic velocity in binary alloys Bi-Cd and Bi-Pb. The measurements were made by the pulse method at temperatures exceeding the melting point by 5 — 10°. It was established that the concentration dependence of the ultrasonic velocity is the same as for the alloys Bi-Sn (studied earlier), and is expressed by two straight lines that intersect at a point corresponding to the eutectic composition of the alloy. The concentration dependence of the coefficient of adiabatic compressibility (computed from the known values of the velocity and density of the alloys) is represented by two curves that intersect at this same point. The concentration dependence of the velocity for alloys of the system Bi-Pb of eutectic mixture and with an excess of lead is represented by a straight line.

The experimental investigation of ultrasonic absorption in suspensions is of great theoretical and practical interest. In spite of this fact, few investigations have been carried out. This is explained by the very great experimental difficulties, one of which is the difficulty in obtaining a monodisperse dispersion which does not coagulate during the measuring process. **I. G. Mikhailova** and **K. N. Marenina** have investigated the absorption of sound in the frequency range 0.6 — 18 megacycles in suspensions (not subject to coagulation) of quartz sand, lycopodium, wood, and paper pulp and other materials in water, kerosene, and alcohol. In qualitative agreement with conclusions of theory, a linear dependence was established between the coefficient of ultrasonic absorption in the mixture and the concentration of the suspension; a more complicated dependence was found for the absorption on the dimensions of the particles and on the frequency of the sound. The results obtained were used to produce instruments for the ultrasonic control of suspensions (for example, for the control of paper pulp fed into a paper-producing machine).

Observing the attraction of bubbles of gas in an ultrasonic field **Kornfel'd** and **Suvorov** have assumed that it is determined by Bjerknes forces (which are directly proportional to the product of the volume velocities of two pulsating spheres and the density of the liquid, and inversely proportional to the square of the separation distance). V. F. Kazantsev found the theoretical time dependence of the distance between bubbles for a given sound pressure, frequency, bubble radius and density, and verified it experimentally in the frequency region 7 — 13 kilocycles. The motion of the bubbles was studied with the help of a velocity plotting. Experimental and theoretical dependencies were found to be in good agreement.

Considering the universality of the pulse method for the measurement of ultrasonic velocity in liquids, **G. N. Feofanov** worked out a method for increasing the accuracy of this method. It is based on the observation of the interference of the previous pulse through the liquid with the reference signal of the same frequency, which was coherent with the vibrations of the source sending out the pulses. The reference signal can be introduced in various parts of the circuit, which makes it possible to create several variants of the pulse interferometer. The accuracy of the measurement is not less than the accuracy of measurement by the usual ultrasonic interferometer.

I. I. Ol'khovskiĭ considered the problem of propagation of ultrasound in monatomic gasses. In applying the method of moments for the solution of the kinetic equation of Boltzmann (for which no limitations are assumed on the relation between the frequency of the sound and the collision frequency of the gas particles) and generalized hydrodynamical equations (which are valid for the description of fast processes), the author made clear the effect of field intensity and thermal conductivity on the translational dispersion of plane waves in monatomic gases. In the particular case the results coincide with the theory of relaxation processes of Mandel'shtam and Leontovich. They also solved the problem of the effect of the boundary on the dispersion of ultrasound, while the obtained results were shown to be in satisfactory agreement with the experimental data of Meyer and Sesler.

In the classical linear theory of propagation of acoustical waves of Stokes-Rayleigh, the ray was considered in very simplified form (according to Newton's law) which was not at all connected with the optical properties of the medium. Later, this theory was repeated in unchanged form. **V. A. Prokof'ev** considered this problem, making use of the theory of propagation of small disturbances (pressure waves and thermal waves) on the basis of the hydrodynamical equations of Navier-Stokes and the equations of the theory of the radiation field. He established the fact that the characteristics of forced plane harmonic waves depend on two dimensionless radiation numbers, which describe the processes of emission and absorption of radiant energy. For special cases (long and short waves, barotropic medium, etc.) equations are obtained for the velocity and absorption coefficients of the waves. In the propagation of infrasonic and low frequency sound waves, radiation plays a role that is dominant in comparison with viscosity and thermal conductivity. For sufficiently high frequencies, it can be completely neglected. At the same time the insufficiency of the conclusions derived earlier (Stokes, Rayleigh, etc.) on the role of radiation in the propagation of acoustical waves can be shown.

The ever widening application of ultrasonic control of construction materials found its reflection in a number of papers in the section. **E. Ranachowski** (Poland) pointed out the connection of several properties of ceramic materials with the velocity of ultrasound and its absorption, and decided on the possibility of the use of this connection for the control of the quality of ceramic materials and the introduction of a rational process for their annealing.

V. Koltonski (Poland) advanced theoretical and experimental conditions for the propagation of ultrasound in magmatic, sedimentary, and metamorphic rocks, and established the dependence of the parameters of the propagation of sound of the frequency of the wave and the structure of the rock. He also investigated the effect of typical inhomogeneities of the rocks on the propagation of ultrasound in them and showed the possibility of a practical application of the ultrasonic method for the qualitative control of rocks directly in stone quarries.

D. S. Shreĭber gave a short report on the characteristics of propagation and scattering of polarized transverse vibrations in solid media, and pointed out that they possess certain advantages from the viewpoint of their application in ultrasonic defectoscopy. In the use of identical polarized source and receiver, less obstacles are presented by scattering of the ultrasonic vibrations on fine inhomogeneities.

Investigating the effect of ultrasonics on the phase transition in solid metals and alloys, **L. N.**

Larikov and **I. G. Polotskiĭ** came to the conclusion that the observed effect of ultrasound on natural aging of certain alloys is not the result of the direct action of the ultrasound, but arises from the higher temperature which results from the absorption of the ultrasonic energy by the alloy. Measurements were carried out with 750-kcs ultrasound at a limiting power of 10 w/cm^2.

L. A. Sergeev investigated the velocity of ultrasound in petroleum, and water saturated with methane, for estimating the sound reflecting capability of petroleum layers. Measurements were performed by the pulse method at 200 kilocycles in a high-pressure bomb. In water and petroleum different changes were observed in the velocity of ultrasound as functions of pressure and temperature. With higher temperature and saturation pressure (in proportion to the transition to the deeper layers) the difference in velocities for water and petroleum increased; at 72.5°C and 26°C, the pressure amounted to 30%. This, and also the difference of velocities of transverse and longitudinal waves in compressed sands permits one to undertake the exploration of petroleum deposits by ultrasonic methods.

For more reliable determination of the stratification of the sea bottom, the method of ultrasonic echo location applied in underwater seismic exploration for petroleum was improved by **L. A. Sergeev.** For this purpose, along with the ordinary recording on sounding diagrams of the time of propagation of signals, he determined the forms of the reflected signals and the repeated echo. The reading of the soundings was controlled and supplemented by electrical plotting of the bottom.

It is known that one can rid gases of aerosols by sonic coagulation of these particles. However, in addition to aerosols, the atmosphere frequently contains contaminating impurities. One such harmful impurity is sulfur dioxide, which is generated in certain blast furnaces in the treatment of ore contaminated with sulfur compounds, in the combustion of low calorie coals with a high content of sulfur in boiler furnaces, etc. **I. V. Slavik** (Czechoslovakia) advanced and developed an original ultrasonic method for cleaning air in blast furnaces from harmful contaminations. For this purpose he used gaseous reagents which, combining with the harmful impurities of the gases, formed solid particles that were then coagulated with the help of ultrasonic vibrations. The possibility of a wide region of application of ultrasonic coagulation aroused great interest.

Papers were read on the sources of ultrasound employed in techniques of sonic coagulation of aerosols. Investigating a generator of the Hartman type, **P. Lesniak** (Poland) determined the characteristics of the structure of the gas current which is emitted under pressure from the nozzle as a function of the velocity of flow. **M. L. Varlamov** and others presented a paper on the improvement of a generator of the Hartman type (a change in the distance between the nozzles and the resonator and the depth of the resonator while preserving axial alignment), and on the investigation of its sound field with the help of a small dimensional thermoelectric probe. Successful experiments were carried out by them on the coagulation of a fog produced by the cooling of SO_3 which is formed by contact oxidation of gaseous sulfur.

During recent years the problem of radiation and detection of ultrasonic vibrations has spilled over into the region of electroacoustics, obtaining equal weight with the problems of the detection and radiation of sonic frequencies. A small number of papers were devoted to this subject explaining the piezoelectric transducers in the ultrasonic section.

Many published researches have been devoted to the calculation of ultrasonic transducers within the framework of linear theory. However, in the majority of cases, the system considered were either single element or symmetric. **V. N. Tyulin** carried out a step-by-step theoretical analysis of many element non-symmetric transducers and determined their characteristics (frequency characteristics, power radiation, sensitivity in the receiving regime).

Some considerations on the effect of a matching layer on the frequency characteristics of piezoelectric transducers of the piston type, obtained as a result of the solution of the equations of the transducers in the general case were put forth by **K. V. Goncharov.**

Rod ultrasonic concentrators, widely used in the ultrasonic treatment of the solids, have been considered up to the present time without analysis of the effect of loading. In this connection, for example, it has been recognized that multi-step concentrators are better than exponential. Considering both exponential and multistep concentrators for a given loading of the circuit, **L. O. Markarov** showed that the first is more advantageous, since the change in the reactive loading (for example, as a result of deterioration of the instruments) brings about in them a smaller disorder in the vibrating system.

The section on acoustical measurements heard papers on new methods for measurement of the treatment and improvement of instruments for acoustical measurements and on increase in the accuracy of standard methods of acoustical measurements.

Applying sawtooth modulation to an acoustical source, **V. A. Zverev** and **A. I. Kalachev** determined the location and velocity of reflected sounds from objects according to the form of the frequency spectrum obtained as a result of detection of the received signal. The speakers showed the possibility of the application of this method to the measurement of sound absorption of partitions, measurements in defectoscopy, etc.

V. A. Zverev and **E. F. Orlov** described an arrangement for the analysis of spectra of optical photographs with the use of optical filters which excluded the effect of instability of the velocity of motion of the photograph and allows a simplification of the electrical part of the analyzer spectrum. Optical filters (made with an arbitrary initial frequency characteristic) can be applied successfully to the spectral analysis of a large number of recorded oscillograms of different types of fluctuations, etc.

The possibility and advantage of the application of tunable ferrite filters as a selective system of analyzers and spectrometers of sonic and ultrasonic range were discussed in the paper of **B. M. Beskarovaĭnyĭ, V. M. Vol'f, V. S. Gorbenko, M. I. Karnovskiĭ, B. I. Shotskiĭ,** and **A. A. Yur'ev**. As a result of the investigation, a spectrometer and analyzer of sound frequencies was constructed with an attenuation of 35 db when detuned a half octave.

Yu. M. Il'yashuk reported on the development of a miniature one-third octave spectrometer (25 transmission bands in the 36 to 11,000 cps range) with a fall off of 60 db per octave at the bounding frequencies of the band. Semiautomatic switching of the transmission bands permitted automatic recording of the results of measurement.

D. Z. Lopashev undertook to carry out several corrections in the readings of an objective noise meter which permits these readings to be closer to the true level of loudness of the noise.

An interesting paper on an oscillographic measurement of the two-dimensional probability density distribution of a random process was given by **Lyu Yung-Uzun** (China). The vertical and horizontal displacements of the beam of a cathode-ray tube correspond to the signal and to its retarded signal. The degree of darkening of a film exposed throughout the duration of the signal corresponds to the desired probability. For quantitative analysis, the degree of darkening was investigated by a densitometer.

V. I. Solov'ev spoke on the possibility of the application of correlation methods to the study of diffraction images of ultrasonic fields and for the measurement of signals of threshold levels. The speaker described circuits and principles of operation of an instrument which permits one to reduce the fluctuation interference by 35 db and correspondingly to amplify the range of threshold measurements and also discussed apparatus for the investigation of fine structure of ultrasonic fields in undeadened rooms.

V. P. Glotov and **R. A. Vadov** reported on measurements of small losses in liquids at frequencies from 3 to 15 kilocycles by determination of the rate of damping of separate normal modes of a cylindrical tank filled with the liquid under investigation and excited by a sound pulse. The rate of damping depends upon the amount of loss in the liquid and on the amount of loss in the tank. Measures were taken by the authors for a uniform reduction of the losses of the tank and for their calculation.

V. P. Glotov also reported on measurements of shear and volume viscosity in solids by a resonant-reverberation acoustical method (by means of the speed of damping of characteristic spherical and radio vibrations of samples prepared in the form of solid spheres). The error of the measurement amounted to about 5%.

By placing solid objects of special shape in a water tank and making use of the transformation of longitudinal waves into shear waves in these objects, **K. V. Goncharov** measured the damping of shear waves in them.

In the reports of **A. N. Krishtalevich** and **A. N. Rivin,** problems were considered of the increase in accuracy of the calibration of measuring microphones.

J. Merhaut (Czechoslovakia) considered error factors that can affect the accuracy of the absolute calibration of measuring microphones by the method of the pistonphone. A new type of pistonphone permits absolute calibration of microphones with an accuracy of 0.1 db.

L. B. Langans reported on a setup for the calibration of sound pressure receivers in the audio-frequency region in water. The error of measurement did not exceed ± 0.4 db.

I. L. Krasil'shchik reported on calculations of the characteristics of reference measuring hydrophones, in which a piezoceramic hollow sphere of barium titanate with calcium impurity was used as the sensitive element. The minimum pressure measured by these hydrophones in the frequency range from 50 cycles to 100 kilocycles amounted to 10 — 50 dynes/cm^2 in the case of a signal level exceeding that of the noises of the hydrophonic instrument by a factor of 5.

In the section on electroacoustics, the three most important problems of the reproduction and detection of sound vibrations were considered:

(a) Improvement of the quality of loudspeakers and microphones with the aim of bringing the quality of the recorded sound closer to the natural sound.

(b) Development of effective piezoelectric and other types of ultrasonic transducers which satisfy the conditions for their application in scientific-research and industrial devices and instruments.

(c) The study of the physiological properties of the human ear and speech.

Inasmuch as architectural and building acoustics is also closely concerned with the latter problem, the speakers on physiological acoustics met in joint session with the section on electro- and architectural acoustics.

Significant improvement of the sound quality of radio and telephone receivers has been obtained in recent years by split amplification and improvement of the low and high audio frequencies. High frequency speakers added to the lateral walls of receivers create a certain sensation of stereophonic sound.

D. Kh. Shifman, **A. I. Dneprovskiĭ**, **V. I. Samarskiĭ**, and **E. A. Timofeeva** spoke on further measures for the improvement of the quality of the sound of receivers. In particular, distributed acoustical systems were prepared consisting of low frequency (40 — 4000 cps) and medium frequency (300 — 6000 cps) speakers located on the front wall of the room, and two high-frequency loud speakers (3000 — 15000 cps) located on the side walls of the room. Such an arrangement of sound sources creates the illusion of sound of a large orchestra in the center of the living room. High quality and high frequencies loudspeakers of the horn and diffusion type were developed and also condenser high frequency speakers.

Recently, a tendency has been noted toward a deeper mathematical analysis of electro-mechanical processes going on in sources. An example of investigations of this type was provided by the paper of **J. Kacprowski**, (Poland) who considered the coefficient of transmission of an exponential horn. Use of the theory of quadrupoles permitted the author to obtain a series of fertile analogues between vibrational processes in a horn and processes involving transmission lines, etc. He showed that the horn can be regarded as a transformer with a constant and frequency-independent transmission coefficient only under certain conditions.

S. T. Ter-Osipyants noted that the inadequacies of sound recording by contemporary dynamic loudspeakers can be removed by the introduction of damping in the vibratory and radiating sections of the speaker system. This was established by him for a particular distribution of porous sound absorbing material close to the speaker. Active resistance was introduced into the highly vibrating layer of air in this case of such a form that the active linear loading was applied almost to all the vibrating elements of the speaker. As a consequence of this, multiple resonances both of the diffuser and of the volume of the cabinet were damped in a very wide range of frequencies. This significantly improved the timbre of the sound of the loudspeakers.

In the paper of **V. Murevskiĭ** on the results of the development of a high quality system of controlled loudspeakers which reproduce the frequency spectra from 40 cycles to 15 kilocycles, it was also pointed out that the damping by sound absorbing material in a reliable fashion by means of internal cavities of a labyrinth in the housing of low frequency apparatus permitted a reduction of the resonance maxima.

G. V. Butakov spoke on "Some Investigations of Mechanical-acoustical Systems of Electrostatic Sound Receivers." The method of electric modeling permitted him to explain the effect of any of the parameters of a complicated vibratory system (for example the combination capsule of the Brownmull and Weber type). The results obtained by such a method are in excellent agreement with the experimental data of real systems.

A. D. Tkachenko spoke on a new method of excitation of a microphone in speech transmission by means of the use of vibrations of the side faces which act [by means of the enclosed air space (acoustical coupling)] on the diaphragm of the microphone which is pressed toward the face close to the ear. In comparison to ordinary microphones located in front of the mouth, this method (which is practical with the use of electro-magnetic microphones) possess a number of advantages.

V. S. Grigro'ev, **L. P. Nikitina**, and **Yu. A. Ukhanov** reported on investigations of a model of an electromechanical transducer, based on the pondermotive interaction of a magnetic field with the displacement current in a dielectric (which is determined by the variable displacement of the charges of the dielectric). With values of the dielectric susceptibility of the order of tenths of units, the effectiveness of such a transducer is close to the effectiveness of the ordinary electrodynamic transducer (which is based on the use of conduction current). The application of dielectrics with large dielectric susceptibility permits one to increase the efficiency of the transducers being investigated.

Investigating the properties of nickel ferrite and a series of nickel-zinc ferrites, **I. P. Golyamina** demonstrated the expediency of their application in magnetostrictive ultrasonic transducers. Ferrites can be applied also to power sources; however, the efficiency of the latter depends on the power consumed in this case. In the paper the problems of the technology of production of acoustical ferrites was discussed, in particular, the application of die casting instead of dry pressing.

The method of hot die casting was also applied for the preparation of barium-titanate ceramics. (**A. V. Sosnov** and **M. A. Ugryumova**). They obtained ceramic materials which were volume-homogeneous and accurate in dimensions (thin cylinders, cones, spheres), which could not be prepared by the ordinary semi-dry pressing.

B. A. Finagin spoke on the advisability of the use in a number of cases of nonthickness vibrations of quartz and tourmaline sources of different shape. In comparison with the thickness vibrations ordinarily used, it appears possible to increase the intensity of the sound field for the same voltage on the source and to obtain an intense ultrasonic field at a number of different harmonics.

E. P. Kolokolov showed that, applying wedge shaped quartz plates (X-cut) in the transformation of high frequency pulses in liquid and solid media, he could expand the frequency region of transmission and decrease the loss in transduction in comparison with ordinary flat transducers.

Great interest in the problems of physiological acoustics was shown by foreign acousticians who made up almost the entire program at the joint session of the section on electroacoustics and building and architectural acoustics.

In spite of the depth and the number of researches on the investigation of curves of the estimate of loudness, the results of separate authors obtained by different methods has remained uncorrelated to date. Comparing the loudness curves obtained by the step method of Kwiek and the method of Garner with the previous methods of Laird, Taylor, Gamp and others, **J. Fazanowicz**, **D. Grzesik**, **A. Lampkowski** and **E. Porwik** (Poland) came to the conclusion that the results of all works obtained by the step method and method of the division of loudness, agree with one another after some recalculations. But these results cannot be in agreement with data obtained by integration of the function of the minimum distinguishable increase in loudness.

Comparatively recently, interest has arisen in the investigation of the phenomenon of residual masking (sometimes called short-time ear fatigue), that is, masking which is observed after the sensation of the sounding of the masking tone.

C. M. Harris (USA) reported on accurate experimental determination of the threshold of hearing within a second after the cessation of a masking tone at low frequencies in a free field. He also reported frequency characteristics of auditory sensitivity (by the method of Munson and Gardner). It appears that, at low frequency, the peaks of these characteristics are much sharper than the peaks of the frequency characteristics of the amplitudes of vibration of the basilar membrane. The mechanism of the increase in the frequency selectivity in the auditory preception remains unknown.

M. Kwiek, **E. Karaskiewicz**, **A. Sliwinski** and **I. Malecki** (Poland) reported on results of experimental investigation of acceptable echo amplitudes (for which the echo blends with the original signal) for time intervals between the fundamental signal and echo in the range 0 to 60 milliseconds, and for a frequency spectrum of the echo which differs from the spectrum of the original signal.

H. Niese (West Germany) discussed the problem of the perception of loud noises with low periodicity. He showed that the level of loudness lies within the limits of effective and peak values of the sound pressure, being dependent on the frequency of repetition; for very low frequencies, there arise two different judgments on the loudness. The speaker constructed an electrical circuit for the objective measurement of the loudness, employing the subjective data that have been found.

Visualization of speech attracted considerable attention in connection with those aspects which are amenable to objective methods in the investigation of speech and in practical applications. **S. Steinbach** (West Germany) developed an apparatus with the help of which speech is continuously recorded on a magnetic tape. The signal from a probe attachment which rotates at high velocity is analyzed by a system of band filters, the output of which is applied by an electronic commutator to the modulator of the brightness of a cathode-ray tube with a horizontal time sweep. The vertical deflection of the beam corresponds to the band of frequencies switched on at the given instant. Thus a picture appears on the screen of the tube of a traveling recording of the speech. The report was illustrated by pictures of separate sounds and discontinuous speech.

The section on architectural and building acoustics devoted its sessions to problems of the investigation of the spatial acoustics of residences and their sound insulation, and also to problems of the development and measurement of acoustical materials and structures.

V. V. Furduev and **G. A. Gol'dberg** made a detailed review of the methods of artificial reverberation which are applicable for sound recording for architectural-acoustical investigations, for the improvement of room acoustics, etc. Reverberation models were discussed with a different number of measurements, imitating a wave process in the room and a structure with a delay line, imitating a successive reflection at some point in the room. Criteria were given in the paper for the quality of reverberators, and possible ways for their improvement were noted.

Investigating the reverberation of moving picture theatres, **A. N. Kacherovich** explained that it is appropriate to represent the reverberation curve in the form of two parts: the first, which can be constructed on the basis of a geometric approach, and the final, which is computed on the basis of a statistical theory. The mean absorption coefficient connected with reverberations of the second type depends not only on the amount of sound absorption but also on its position. With a corresponding arrangement of absorptions, one can obtain a higher dispersion for relatively larger reverberation times.

B. M. Gardash'yan reported on investigations of the acoustical properties of rooms in acoustic models with a scale factor of 1/40. Reverberation and pulse measurements on the models were carried out at an average frequency of 20 kilocycles. In the case of models of the Moscow Panoramic Movie Theatre, acoustic defects were discovered which were later removed in the final construction.

Applying pulse signals, **O. D. Burkov** investigated the acoustics of several large halls. In addition to obtaining a picture of individual reflections for the reverberation curve in different frequency ranges, reflections which correspond to the initial part of the reverberation, he found the acoustic defects of some of the rooms and the reasons for their appearance.

I. G. Leĭzer compared the reverberation time measured in twelve rooms and auditoria, with volumes from 1000 up to 28,000 m^3, with that computed by the Eyring formula (with use of coefficients of absorption known from the literature) and discovered that in all cases there was a significant increase in the sound absorption. This absorption (which is brought about by an armature with different slots and openings in the walls of the room, with flexural elements in the structure, etc.) can be considered as a certain addition to the ordinarily computed mean coefficient which enters into the Eyring formula.

I. Malecki (Poland) spoke of the acoustical properties of the so-called spatial absorbers, in particular, those completed in the form of freely suspended spheres with perforated transparent walls. With a choice of the corresponding form of internal partitions in the absorber it is possible to produce a multiresonant system with an optimal frequency characteristic of absorption of background noises.

I. V. Lebedeva spoke on results of measurement (in a reverberation chamber) of several types of resonant sound absorbing structures in the form of perforated sheets of plywood or duralumin located at some distance from a solid wall. The measured frequency characteristics of the coefficient of sound absorption in the diffuse field agree well with the calculated values. In comparison with the case of normal incidence, the characteristic of absorption was broader, without sharp maxima and minima.

L. Keibs (East Germany) described a simple method of measurement of impedance on the surface of a closed airspace by means of recording the sound pressure in the space before and after the introduction of the specimen being used. Comparison of the data obtained this way with a normalized set of curves allows the determination of the components of mass rigidity and friction impedance. The method described is simpler than the usual method of Kundt's tube and is suitable for measurement of already prepared wall coverings.

In the paper of **N. I. Naumkina**, **B. D. Tartakovskiĭ** and **M. M. Éfrussi** data were reported on the investigation of vibration absorbing materials, consisting of mixture of asphalt and rubber pellets. Measurements of the elastic modulus and the coefficient of attenuation of different shapes of these materials, differing in composition and in technique of preparation were carried out on a rod undergoing flexural vibrations. Measurements were carried out at room temperature in the frequency range from 30 to 100 cps. It was found that some of these materials, particularly layered materials, can be used for the absorption of vibration of metallic structures.

In the fight against noise, researches have been carried out in the direction of the improvement of metal-rubber vibration reducers. In this connection, systematic analyses were carried out of viscoelastic properties of rubber in a wide temperature and frequency range. The distinctive characteristic of the measuring apparatus applied for this purpose by **I. L. Orem** was the strengthening of a movable system of electromechanical transducers with the aid of several forms of materials which operate on shear.

Measuring the sound isolation of homogeneous plates (20 × 20 cm) **R. P. Golovinskiĭ** discovered that with an increase in the rigidity on the bending of specimens, their sound isolation in the frequency range 100 to 500 cps increased by 10 db.

In view of the crudeness of measurement of sound insulation of natural building elements, **I. G. Leĭzer** carried out investigations on the sound insulation in models in a scale of 1/5. The model and the original were made from identical materials, the coefficient of attenuation of which was frequency dependent. This condition, along with the exact relation of the geometrical dimensions and identical conditions of construction, guaranteed the satisfaction of the condition of similarity.

Acoustical modeling was also applied by **E. Meyer** and **G. Kutruff** (West Germany) for the clarification of measures with whose help it is possible to obtain diffuse reverberation fields in a chamber with an absorber. It was explained that the covering of several walls by rigid half cylinders and the suspension of a scatterer allowed him to increase the weak diffusivity that is characteristic of a rectangular room. Deviation from the rectangular shape of a room also somewhat increases the diffusivity of the reverberation field.

The sound field in a deadened room is usually considered as a free field, while the reflections from the surfaces of the walls are regarded as interference. Going along these lines, **I. A. Bazhina** considered (by the method of simulated sources) the correction applied to the first reflections in the curves of the incidence of a sound field in a rectangular room covered with material having an absorption coefficient independent of the angle of incidence. In the phenomenon of standing waves, the effect of partial reflection lies in a certain displacement of the extremes of sound pressure and vibration velocity relative to each other. The calculations agree very well with experiment.

A. S. Osakchenko approached the problem somewhat differently. He assumed that each point of the surface of the room (sphere, ellipsoid, and parallelopiped) is a source of reflected spherical waves, and considered the formation of a sound field in this fashion. He found the relation between the volume of the chamber and the absorption coefficient for a given deviation of the sound pressure from the value corresponding to an infinite medium. These dependencies can be used in planning rooms.

K. N. Ivanov-Shits reported on the results of the experimental investigation of the sound field of the anechoic chamber of Moscow State University at various frequencies.

M. G. Burle and **A. F. Osakchenko** measured absorbing structures of variable cross section of capron fiber in a tube. Absorbers were prepared in the form of pyramids of different slopes and a layered construction of several plane layers of different thickness and density. The optimum conditions were found for which the coefficient absorption at 60 to 600 cps was larger than 0.985.

I. Tichy (Czechoslovakia) analyzed the conditions of the sound power and spectral composition of noise of typical sources at electrical stations measured by him with the help of objective instruments. Estimating by conversion the audibility of these noises, the author noted the measures for combating noise at electric power stations.

In the paper of **B. Klimes** (Czechoslovakia) consideration was given to the principles involved in planning an airport with the purpose of a universal reduction of the sound of airplanes which have their unpleasant effect on passengers and operating personnel. For calculation of the location of regions with minimum noise level (where one ought to locate the building) it is necessary as a preliminary to measure the noises of the airplanes.

In a final note by Academician **N. N. Andreev**, the great importance of the conference was noted in demonstrating the quantitative and qualitative growth of the scientific investigations in the field of physical and technical acoustics. A valuable result of the conference is the establishment of many friendly contacts between Soviet and foreign acousticians.

The organizing committee of the conference (presided over by L. M. Brekhovskikh) made it possible for all those taking part in the conference to become acquainted with the scientific researches carried out at the Acoustics Institute of the Academy of Sciences, USSR, at the Chair of Acoustics, Moscow State University, and in a number of other Moscow acoustical scientific laboratories. Excursions were also organized for tours of the acoustical laboratories and apparatus of the Sound Recording House of the Telecenter, the Panoramic Movie Theater, and other scientific technical establishments, and also for friendly meetings between Soviet and foreign acousticians.

Translated by R. T. Beyer

Personalia

VLADIMIR ALEKSANDROVICH FOCK ON HIS SIXTIETH BIRTHDAY

M. G. VESELOV

Uspekhi Fiz. Nauk **66**, 695-699 (December, 1958)

DECEMBER 22, 1958, will be the sixtieth birthday of one of our greatest theoretical physicists, academician Vladimir Aleksandrovich Fock. V. A. Fock was born in St. Petersburg, into the family of a surveying engineer, A. A. Fock, well-known among specialists for his work in the science of forestry. After he finished high-school in Petrograd in 1916, Fock entered the mathematical physics faculty of Petrograd University, but then volunteered for the artillery school and after a few short courses there was sent to the front. In 1918 he was demobilized, returned to Petrograd, and resumed his studies at the university.

At that time the state institute of optics had just been organized in Petrograd, and Fock with a group of the best students in the physics faculty was invited by D. S. Rozhdestvenskiĭ to serve as "laborant" at the institute. Among the "laborants," who later formed the nucleus of the scientific division of the optics institute, were many names which are now famous: A. N. Terenin, E. F. Gross, S. É. Frish and others. The exceptional abilities of Fock in theoretical physics and mathematics became obvious during his student years, and in 1922 he had already completed two research papers, one in quantum theory and the other in applied mathematics.[2] These two fields, quantum theory and applied mathematics in the wide sense of the term, together with the theory of relativity, remained Fock's chief scientific interests.

The twenties were years of vigorous post-revolutionary development of research in physics in the newly founded physics institutes. At the beginning of the twenties the theoretical physicists in the Soviet Union could be counted with the fingers, and Fock took an active part in the work of several institutes. In 1924-36 he was working in the theoretical division of the physico-technical institute. In 1928-41 he was director of theoretical research at the state institute of optics. Fock's scientific and teaching activities continued uninterruptedly at Lenigrad university, where he was successively student, aspirant, lecturer and professor. In recent years he has held the chair of theoretical physics. By his own research and by his personal guidance he has played a great part in the development of theoretical physics in the Soviet Union and especially in Leningrad.

Fock's research was strongly influenced by his close relations with the applied mathematicians of the famous St. Petersburg school. In the first years after he finished his studies (1924-26) he was mainly occupied in solving mathematically difficult problems of applied physics. The varied and practical character of these investigations can be seen from a list of some of the titles: calculation of radiation from a surface of arbitrary form, theory of the mixing of optical glass during heating, solution of a diffusion problem by finite differences with applications to the diffusion of light, calculation of the thermal resistance of a cable with multiple core, etc.

In these and later papers in mathematical physics Fock showed his characteristic physical intuition, which allowed him to discard the inessential features of a physical phenomenon and construct mathematical machinery appropriate to its understanding. He showed also his mastery of mathematical manipulations, which allowed him to carry through the solution of the problem and reduce the results to explicit formulae and tables.

Fock's enthusiasm and skill in the creation of general methods and theories found expression also in his solutions of concrete problems of practical physics. For example, in his calculations of the illumination problem,[3] Fock introduced the new concept of an illumination vector, and built upon it a theory of the vector illumination field, which was developed further by other scientists and has been largely instrumental in establishing the high level of lighting technology in the Soviet Union. A second example is Fock's paper in the theory of elasticity,[3] in which he reduced a plane elasticity problem to a Fredholm integral equation. The solution of this problem, which was obtained seven years later by N. I. Muskhelishvili, has been widely exploited by the Georgian school.

Fock's mastery of mathematical technique may be seen particularly clearly in his study of the conformal representation of a zero-angle quadrilateral upon a half-plane.[6] The solution of this problem is needed in calculating the distribution of heat and of electric field in a cable, and was considered by many mathematicians to be practically impossible. Fock obtained it in an analytic form suitable for computation.

These first investigations of Fock in the field of applied mathematics made him well-known among Soviet physicists and mathematicians.

We cannot here mention all the mathematical papers of Fock. His work on integral equations,[2,28] together with G. A. Grinberg, has led to interesting and important results in the theory of the scattering of light and in the theory of shore effect.[32]

After 1926, Fock's main interest was in quantum theory, and until 1941 he turned to problems of applied mathematics only sporadically, stimulated by practical questions which arose from time to time. In 1927 he worked out a thermal theory of electrical break-down in dielectrics.[7] In 1930 he gave an elegant solution to the problem of the skin-effect in a curved conductor.[11] Fock published several papers concerned with electrical methods of ore-prospecting, and in 1935 published a monograph on the theory of electrical core-sampling.[17] In 1935 he solved a difficult problem of internal ballistics, the motion of the gases in the barrel of a gun as it is discharged. In 1940 Fock solved an acoustical problem in the theory of resonant absorption.[27]

Fock's first papers in quantum mechanics appeared in 1926, immediately after the discovery of the wave equation by Schrödinger. His first paper[4] generalized Schrödinger's equation to the case of a magnetic field and deduced a formula for the normal Zeeman effect. In the same paper he applied perturbation theory to obtain the splitting of the levels of the hydrogen atom in an electric field. His second paper[5] generalized the wave equation to the case of a relativistic charged particle in an electromagnetic field, and verified the gauge-invariance of the equation. These papers appeared at the beginning of the creation of wave mechanics, and played an important part in its development. In 1927-1928 Fock lived abroad in Göttingen and Paris. There he completed several major researches in quantum mechanics,[8,9] and became one of the leading theoretical physicists in the world.

In 1929 he published a series of papers, partly in collaboration with D. D. Ivanenko, interpreting geometrically the Dirac equation. He established the peculiar geometrical properties of the Dirac wave-field, and used the notion of parallel displacement of a spinor to generalize the Dirac equation to Riemannian geometry.

Fock's book[13] "Principles of Quantum Mechanics," the first Soviet book on quantum mechanics, was published in 1932. It contains discussions of many problems using Fock's original methods.

Fock devoted a long series of investigations to the theory of many-electron systems, and to the creation of approximate methods in this field. This work began in 1930 with the invention of the method of the self-consistent field with exchange.[12] The basic idea of this method is to apply a variation principle to obtain equations for the one-electron wave functions. In this way he justified the Hartree equations, which had been derived earlier from intuitive considerations, but he found that a correct treatment of the Pauli principle led to a system of equations which differed from the Hartree equations by the presence of additional exchange terms. Fock with his collaborators[19] applied the Hartree-Fock method to calculations of atomic structure. The method was immediately and widely used in the theory of many-electron systems. In a series of papers, Fock studied the effects of the exchange terms, and explained various spectroscopic phenomena, the inversion of doublets, violation of sum-rules for oscillator strengths, and some anomalies in the intensities of series of lines. In the same group of papers he published a semi-classical method for calculating wave-functions[20] and a generalization of the Fock method to situations in which the one-electron approximation can be partially avoided.[25]

In 1940 Fock studied the problem of constructing many-electron wave-functions not containing spin-variables, and he was the first to formulate the cyclic symmetry properties of these wave-functions.[26] In 1935 he published an exquisite paper on the hydrogen atom,[22] in which he showed that the degeneracy of the levels of an electron in a Coulomb field is connected with a symmetry of the field with respect to a group rotations in four dimensions. We may mention also his 1954 paper,[36] in which he obtained an exact solution of the Schrödinger equation for helium-like atoms in the form of a series expansion. This problem has become important in recent years in connection with the verification of the radiative corrections predicted by modern quantum electrodynamics.

In the thirties Fock published his fundamental investigations in quantum field theory. The ideas and methods which he then developed have found an extraordinarily wide application in the post-war development of quantum field theory. His great work of 1932 on second quantization[16] established a method for describing in configuration-space systems with a variable number of particles. This paper was the first to develop clearly and consistently the method of second quantization, and to show its connection with a configuration-space picture. A system with a variable number of particles is described by an infinite set of equations for functions which describe states with a fixed number of particles. This method is now widely used in meson theory, and was generalized by several physicists in the early fifties.

Fock published in 1932, in collaboration with B. Podol'skiĭ, several papers working out a new version of quantum electrodynamics which had been proposed by Dirac in a one-dimensional model. In this version the Coulomb interaction appears as a result of eliminating the longitudinal electromagnetic field. The invariant version of electrodynamics was perfected in a joint paper by Dirac, Fock and Podol'skiĭ. They developed the so-called many-time formalism, in which each particle, as well as the field, has its own time. The many-time theory was further generalized by Tomonaga and Schwinger in 1946-1947, when it became the super-many-time formalism.

Fock's last work in quantum field theory was the creation of a rigorous theory for systems of a variable number of Bose particles, using the so-called method of functionals. Already in 1928,[9] Fock had considered the idea of describing systems with a variable number of particles by means of generating functions, instead of by an infinite set of functions referring to systems with a fixed number of particles. The final form of this idea,[21] with applications to electrodynamics, was worked out in 1934. Fock's method of functionals exhibits the wave-function of the field, or the the state-functional, as the generating function of the probability amplitudes of the field in states with a fixed number of particles.

Fock's original paper, and other papers by his pupils, applied the method successfully to various problems of quantum electrodynamics. The idea of a generating functional has been widely used in quantum field theory in recent years. One may describe the field by a sequence of functions of a different type, for example Feynman amplitudes, instead of probability amplitudes in configuration-space. Each such set of functions has its own generating functional.

Fock's 1937 paper on proper time[23] is a singular and highly original piece of work. He introduced into the Dirac equation a parameter representing the proper-time of the electron, and in this way solved the Cauchy problem for the Dirac equation. The solution of the equation is obtained as a contour integral over the proper-time. Since the proper time is invariant, the proper-time method has been particularly fruitful in problems of quantum field theory where considerations of relativistic invariance play an essential role. It is remarkable that the importance of this method was understood only in the early fifties.

A large part of Fock's scientific work dealt with the theory of relativity. In the twenties he paid his tribute to the prevailing fashion by attempting to construct a unified five-dimensional field theory. But he quickly abandoned this attempt, considering it formalistic and fruitless.

The earliest of his well-known work in relativity theory was a long paper on the motion of extended masses.[24] This paper appeared in 1939, and developed an approximate method for solving the Einstein equations with extended masses of spherical shape, assuming the space to be Euclidean at infinity. He showed that the equations of motion of Newton are obtained as integrability conditions of the Einstein field equations. In this paper he introduced for the first time a harmonic coordinate system, defined uniquely up to a Lorentz transformation, and generalizing the Newtonian inertial system to general relativity.

Afterwards Fock studied various other important problems of gravitation theory: the derivation of equations of motion of bodies taking account of their structure and rotation, the deduction of the ten integrals of the equations of motion, the theory of astronomical aberration, etc. As a result of these investigations, Fock arrived at some general views concerning the physical interpretation of Einstein's theory. He observed that the covariant form of the equations is not related to the idea of relativity. The departure from uniformity of space-time in Riemannian geometry is not a generalization but a specialization of the idea of relativity, since relativity is a consequence of the uniformity of the space-time continuum. Thus the name "general relativity" throws a false light on the theory of gravitation. Fock also emphasized the local character of the equivalence between acceleration and gravitational fields. He recognized the great importance of boundary conditions, and of the introduction of harmonic coordinate systems, in ensuring the uniqueness of solutions.

Fock summarized all these researches in his book "Theory of Space, Time and Gravitation,"[37] which appeared at the end of 1955. His book contains a thorough and original exposition of the special theory of relativity and of gravitation theory. Besides discussing general questions, he also worked out many concrete problems, some of which were published there for the first time.

Fock's work in the theory of diffraction, which occupied him during the war and post-war years, is of importance both theoretically and practically. Here again he worked out powerful mathematical methods, and obtained practical results, which are basic to the whole modern theory of diffraction and its applications. He found an approximate method for calculating the infinite series and integrals which represent the exact solutions of diffraction problems, and hence constructed a rigorous theory of the propagation of radio waves over the surface of the earth, neglecting the effects of the atmosphere.[29] His paper "Propagation of a Plane Wave around the Earth, Including Effects of Diffraction and of Refraction"[33] solved the same problem in an atmosphere with a non-uniform distribution in height. Fock also established the important principle of the locality of the electromagentic field in the penumbral region. He was able to obtain approximate but sufficiently good solutions to the diffraction problem for convex conductors of arbitrary shape,[35] using the well-known conditions of M. A. Leontovich to define the problem. Fock's work in diffraction is receiving greater recognition among scientists and engineers who work in radio-technology, both here and abroad.

This cursory review naturally cannot give an adequate picture of the richness of Fock's scientific work. A large number of highly original and interesting papers cannot even be mentioned. A complete list of Fock's books and papers, including his contributions to general and philosophical discussions of physical theory, contains more than two hundred titles.

Fock's scientific services received manifold recognization. His work in atomic theory was awarded a Mendeleev prize in 1936. His work on the propagation of radio-waves earned him a Stalin prize, first class, in 1946. He received two orders of Lenin and the order of the Workers' Red Banner. In 1932 he was elected a corresponding member, and in 1939 a full member, of the U.S.S.R. Academy of Sciences.

A great quantity of difficult and important research witnesses to Fock's exceptional talent and industry. And this is not all that he did. For many years he directed the theoretical physics faculty of Leningrad university, devoting much time to guiding the work of his staff. For many years he gave courses on quantum mechanics and relativity. Finally, he took an active part in discussions of general problems concerning the physical and philosophical interpretation of the new theories. He many times spoke out against incorrect and unscientific papers on relativity and quantum theory, defending the modern progressive scientific view. His articles and papers on general problems of theoretical physics are always distinguished by the depth and rigor of his arguments, and always contribute to the clarification of the questions and to the correct public understanding of the nature of modern theories. His writings always show his characteristic directness and honesty, those native qualities which, together with his scientific genius, have earned Vladimir Aleksandrovich universal admiration.

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BOOK REVIEW

THE CREATIVE PATH OF M. PLANCK

É. V. SHPOL' SKIĬ

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Max Planck, Physikalische Abhandlunger und Vorträge. Band I. Pp XV + 773; Band II. Pp XI + 716; Band III. Pp XI + 426. — Verlag Friedr. Vieweg und Sohn. Braunschweig, 1958. Preis DM. 150.

On the centennary of the birth of Max Planck, April 23, 1958, the Union of German Physics Societies and the Max Planck Society for the Advancement of Science (Max Planck Gesellschaft zur Förderung der Wissenschaften e.v.) issued a collection of Planck's original papers, published by the well known Vieweg Publishing House in Braunschweig. The editorial committee, which prepared this edition, consisted of Dietrich Hahn, Max Laue, and Wilhelm Westfahl. The foreword to the first volume was signed by M. Laue, a student and friend of Planck, who obviously was also the editor of the publication. As already mentioned, the publication was undertaken by several societies, including the Max Planck Society. It may be appropriate to explain what this society represents. As is known, at the beginning of the 20th century there was organized in Germany the so-called "The Kaiser Wilhelm Society for the Advancement of Sciences" ("Kaiser Willhelm Gesellschaft zur Förderung der Wissenschaften"). Organized with the support of the major industries, this society was something like an unofficial academy. It founded and financed many research institutes. The first president of this society was the historian A. Harnack. The second, from 1930 to 1937, was Max Planck. After the second world war, Planck was again president of this society until 1946, when the society itself was renamed the Max Planck Society.

Returning to a description of the collection of Planck's papers, we note that this collection includes only the published original papers and some of his lectures and reports of general physical and philosophical nature. The textbooks (Thermodynamics, Radiation Theory, the five volumes of the Textbook of Theoretical Physics), and the monographs (Principle of Conservation of Energy, Eight Lectures on Theoretical Physics) are not included in this collection. We can note with satisfaction that all these books by Planck, without exception, are available in Russian, and that his text on thermodynamics was published in Russian at least twice.

To conclude the description of the external aspect of the reviewed collection of Planck's papers, we note that the first two volumes contain, approximately in chronological order, his original research in the field of theoretical physics. The third volume contains articles of general physical, historical, and philosophical nature. In spite of the fact that most articles have been reproduced by offset, the appearance of the publication is very good. Only in those cases when some article was published in large-scale journals (for example, Physikalische Zeitschrift or Naturwissenchaften) did the reduction in the format cause the type size of the articles to become too small.

Using this collection of Planck's papers, we can follow his creative path. In one of his autobiographical articles, Planck writes that from early youth he was interested mostly in the more general laws of physics, which are of importance to all natural phenomena (III, 255*). Since the years of his youth coincided with the time when the fundamental principles of thermodynamics were established, it is no wonder that indeed this branch of physics, characterized exactly by its unusual generality, was the center of his first scientific interests. Even in his school years, a lasting impression was made on him by the law of conservation of energy. In the university, he was no less impressed by the second law of thermodynamics, which he studied independently by reading Clausius. It is therefore quite understandable that Planck's first two papers in the first volume are devoted indeed to the principles of thermodynamics. These two papers — his doctoral dissertation (On the Second Law of the Mechanical Theory of Heat, 1879) and the so-called "habilitation paper ("Habilitations

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*We shall henceforth refer to the reviewed collection of Planck's works: the Roman number denotes the volume, the arabic the page.

Schrift" — The Status of Equilibrium of Isotropic Bodies at Different Temperatures, 1880) presented, in the German custom to the Munich University to gain the right to teach there. These papers can be briefly classified as the foundation and the application of the most important thermodynamic function, entropy. No one before Planck, not even Clausius himself, who introduced the entropy concept in 1866, used this concept to solve specific problems in thermodynamics. The usual method, developed principally by Clapeyron and Clausius, was to construct a reversible cyclic process for each case.

As to thermodynamic quantities, only one, the temperature, was essentially used in the first papers. Considerably later, in an article devoted to Boltzmann (Boltzmann Festschrift, Leipzig 1904; Planck Collection, Vol. II, pp 79-88), Planck formulated very clearly the shortcomings of the temperature concept and the advantages of the entropy: while one can speak of temperature only in the equilibrium state, the concept of entropy in statistical thermodynamics can be defined for all possible processes. In this interesting article Planck analyzes in detail Boltzmann's statistical definition of entropy and Gibbs' three statistical definitions. In his first papers, however, Planck did not use statistics and introduced entropy and other thermodynamic functions purely phenomenologically. As to the application of thermodynamic functions instead of cycles, this method, as is known, was introduced into thermodynamics gradually and by now completely replaced the cycle method. Gibbs' papers, "discovered" by Ostwald at the beginning of the 20th century, were not known to Planck at that time. In his scientific autobiography, Planck expresses his regret of this fact and acknowledges Gibbs' priority (III, 379).

Planck's early scientific work brought him some disappointment, for his papers on the principles of thermodynamics went simply unnoticed. Planck commented that his dissertation was not even understood by the members of the faculty, who granted him a degree, not on the basis of his dissertation, but thanks to the reputation he acquired by participating in special seminars etc. Not being deterred by this, Planck continued working along these lines and applied successfully the thermodynamic method in his well known papers on the theory of weak electrolytes, on electrochemistry, and other physical and chemical problems. These papers occupy more than one half of the first volume of the reviewed collection, and in spite of the fact that all these have been included in his textbook of thermodynamics, familiarity with the original papers is of great interest to students of physical chemistry and thermodynamics.

During that period, the preoccupation with thermodynamics influenced also the general physical viewpoint of Planck. Being a zealous adherent of the phenomenological method of thermodynamics, Planck was skeptical about atomistics during the first half of his scientific activity. His atitude towards these problems, which interested physicists during the end of the last century and the first years of the present century, was characterized by Planck himself in his scientific autobiography. Recalling the struggle between the most ardent exponent of atomistics, L. Boltzmann, and its opponent, W. Ostwald, Planck wrote:

"Understandably, this struggle between Boltzmann and Ostwald was a rather lively one and led to numerous bright sayings, since both opponents were equally strong in their striking ability and in their sharp native wit. I myself... could act here only as a second for Boltzmann, who not only failed to acknowledge my services, but even considered them undesirable. This is because Boltzmann knew very well that my point of view differed substantially from his. He particularly disliked the fact that I was not merely indifferent to atomistics, which served as the basis for his research but opposed it to a certain extent (sogar etwas ablehnend gegenüberstand). The reason for that was that I then considered the principle of entropy increase as inviolate as the principle of conservation of energy, while Boltzmann considered the principle of entropy increase only a probabilistic law, which admits of exceptions as such. The value of H can also sometimes increase.* Boltzmann did not treat this point at all in his derivation of the so-called H theorem, and my talented student E. Zermelo pointed this out and emphasized that his theorem has no rigorous basis. In fact, Boltzmann's calculations contain no mention of the "molecular chaos" premise needed to substantiate his theorem." These objections of Zermelo and Planck are equivalent to stating that, in view of the strict reversibility of purely mechanical processes, it is impossible by means of Newtonian dynamics alone to explain logically, without contradiction, the irreversibility of real processes, called for the second law of thermodynamics. The resultant discussion indeed led to the missing link in the logical chain, in the form of the molecular-chaos postulate.

After the application of thermodynamics to the study of physico-chemical processes proved to be so fruitful in the hands of Planck, he turned to a study of a new problem by thermodynamic means, that of the radiation from a black body. As is well known, this study has led to the discovery that has immortalized his name — the discovery of the elementary quantum of action. The discovery of the quantum laws has played a tremendous role in Planck's personal life since, as we shall see, it was associated on one hand with a crisis in his entire scientific outlook, and on the other hand, it was experienced by him as a real tragedy. The path followed by Planck when solving this problem was as follows. Since, by Kirchhoff's law, thermal radiation is independent of the nature of the radiating body, Planck chose to represent matter in the simplest form, that of a linear harmonic oscillator or resonator which, owing to its electric charge, can exchange energy with the surrounding electromagnetic field. Planck wrote in the above-mentioned article on the history of the establishment of the radiation law bearing his name (III, pp 258 ff): "I had hoped that for any initial state of this system (i.e., linear oscillator — É. Sh.) Maxwell's theory will lead to irreversible radiation processes, which terminate in a stationary state of thermodynamic equilibrium, in which the radiation from the cavity has the sought black-body energy distribution.

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* As is well known, Boltzmann succeeded in defining a certain quantity H, which diminished constantly with time, so that its negative value could be identified with entropy (Remarks by É. Sh).

"I therefore began to investigate first resonant absorption and emission of electromagnetic waves. Here I adhered to the theory that the interaction between an oscillator, excited by an electrodynamic wave that emits and absorbs energy, and the exciting wave is an irreversible process. However, this opinion, expressed in so general a form, is erroneous as was convincingly pointed out by L. Boltzmann.* The entire process could proceed just as well in the opposite direction. It is merely necessary, at a certain instant of time, to reverse the signs of the magnetic field intensities while retaining the previous directions of the electric intensities. Now the oscillator will again absorb the energy, radiated in the form of spherical waves by similar spherical waves, and again emit energy absorbed from the exciting radiation. Consequently, there can be no talk of irreversibility in such a process.

"To be able to progress further in the theory of radiation, it was necessary to introduce a limiting condition that would immediately exclude processes never occurring in nature, such as concentric inwardly-directed sperical waves, and would at the same time exclude the possibility of simultaneous reversal of the signs of all magnetic intensities. I made this step by introducing the hypothesis of "natural radiation," which is tantamount to assuming that the individual partial harmonic waves, which constitute the thermal radiation are fully incoherent."*

Using the "hypothesis of natural radiation" as a base, thereby insuring the irreversibility of the interaction between the oscillators and the radiation, Planck then derived a simple formula, connecting, in the stationary state, the oscillator energy U with the radiation intensity K_ν. This formula is

$$K_\nu = \frac{\nu^2}{c^2} U,$$

where K_ν is the radiation intensity (more accurately, the surface density of radiation), and U is the oscillator energy. The procedure then followed a purely thermodynamic path. In fact, Planck found (obviously by trial and error) an expression for the entropy of a system made up of oscillators and radiation, expressing S as a function of U and ν, which, in addition, contains two universal constants. Using next the thermodynamic relation

$$\frac{dS}{dU} = \frac{1}{T}$$

and eliminating the oscillator energy U from the result of the application of this formula to the foregoing expressions, Planck obtained the Wien formula, i.e., an expression of the form $A\nu^3 \exp(-\beta\nu/T)$, containing the temperature in the exponent. At first this result appeared satisfactory, since the Wien formula, which gave good results for the visible portion of the spectrum and for short waves in general, was considered valid in those days. In an extensive article, summarizing the accomplishments of this stage of the work (I, 614-667) and published in 1900 in Annalen der Physik, Planck wrote the following concerning his formula (p. 661): "This, however, is none other than the law, established by W. Wien for of the energy distribution, which was found to be correct, at least approximately, by the thorough research of F. Paschen, F. Paschen and G. Wanner, O. Lummer, and E. Pringsheim.

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*We omit the literature references cited in the original. An analysis of the absorption and radiation of electromagnetic radiation by an oscillator was reported by Planck in five communications to the Berlin Academy of Sciences, which bear the common title "On Irreversible Processes of Radiation." In the reviewed collection of Planck's papers, these are printed in the first volume, pp 493–601. It is interesting, from the historical point of view, that Boltzmann's objection, raised immediately after the first communication, Planck attempted to attribute in the second communication to a misunderstanding, and that only later on did he understand this objection. (–É. Sh.)

*It is obvious that the "hypothesis of natural radiation" in the statistics of radiation plays exactly the same role as the "hypothesis of molecular chaos" in the statistic of material systems. Thus, in this respect Boltzmann in his criticism avenged the Zermelo-Planck criticism of the proof of his H-theorem. (–É. Sh.)

"Wien derived his law by making a specific assumption concerning the number of radiation centers contained per unit volume and concerning their velocity. In the theory developed here, these quantities do play no role whatever, but the law follows as the necessary consequence of the definition, established in Section 17, of the electromagnetic entropy of radiation. The question of the validity of this law thus coincides with that of the validity of this definition."

Actually, this result could not be considered satisfactory. By the time this paper was published, the was also published the Rayleigh-Jeans formula was made public. The latter, derived by a logically indisputable, exceedingly clever, and perfectly direct application of classical statistics to radiation, it differed substantially from the Wien formula, particularly in that the temperature dependence in this law was not exponential but in the form of a direct proportionality. Although this formula led to the well-known "discrepancies" ("ultraviolet catastrophe"), measurements made in the same year, 1900, by F. Kurlbaum and G. Rubens showed that for very long waves the absolute black-body radiation comes closer and closer, with increasing temperature, to being proportional to the temperature, in sharp contradiction to the Wien law and in agreement with the Rayleigh-Jeans law. The subsequent events developed with dramatic speed.

Planck himself reported on these clearly and with great frankness in the historical article already cited, written shortly before his death.

The work of Kurlbaum and Rubens was reported at a session of the German Physical Society on October 19, 1900. Planck wrote in the cited article: "Since this result (i.e., the proportionality of the radiation intensity to the temperature at long waves and high temperatures — É. Sh.) became known to me from a conversation with the authors a few days prior to the session, I had time, even before the session, to draw conclusions from this result by my own method and to use it to calculate the entropy of resonating oscillators" (III, 262-263). As a result, Planck derived a formula for the energy spectrum which he reported, after transforming the frequency distribution into a wavelength distribution, to the same session during the discussions following the paper of Kurlbaum and Rubens, and which he proposed to check further.

"During the next morning," continues Planck, "my colleague Rubens looked me up and mentioned that after the session (which took place in the evening — É. Sh.) he compared, that very night, my formula with the results of his measurements and obtained satisfactory agreement everywhere. Lummer and Pringsheim, too, who first thought that they found deviations, soon withdrew their objections in view of the fact, as reported to me personally by Pringsheim, it became clear that the deviations they found were due to errors in the calculations. Subsequent measurements again and again confirmed the equation, with an accuracy that increased with the accuracy of the experiments."

Soon afterwards Planck published a brief note, in which he reported his equation (I, 687-689), which became the well-known "Planck formula."

However, history did not stop there. There is no doubt that the Planck formula was obtained by interpolation between those of Wien, valid for short waves and low temperatures, and of Rayleigh-Jeans, which hold for long waves and high temperatures. Were this all, Planck's services would be limited to solving a certain very special problem although of practical importance. However, Planck did not stop with a successful empirical formula. He undertook to explain it theoretically, and at the December 14, 1900 meeting of the Physical Society he reported a proof of his formula, based on the concept of the discrete nature of the resonator energy. This date, December 14, 1900, should indeed be considered the birthday of the quantum theory. In his Nobel-prize address (III, 120-124) Planck remarked that the work performed by him during the few weeks between the sessions of October 19 and December 14 was the most difficult he ever did in his life. The intermediate stages, are not mentioned by Planck anywhere. According to M. Laue [Naturwiss. **45**, 221 (1958)], he never spoke of the intermediate stages of his work. But it can be stated with full assurance that the change from the empirically-derived radiation formula to its theoretical justification was connected with a sharp reversal in his entire scientific viewpoint, from the phenomenologically-thermodynamic method, which Planck used exclusively prior to introducing the quantum hypothesis, to statistics. In particular, this manifested itself in the use of the Boltzmann's premise of the connection between the entropy and probability, i.e., the admission of atomistics. However, an even sharper turn was connected with the introduction of the hypothesis of the discrete nature of the energy, i.e., with a break with one of the principal premises of classical physics in general. Judging from later papers and from evidence of persons who knew him intimately (for example, the same M. Laue), this break tormented Planck for several decades after this paper appeared.

By now the initial derivation of the radiation formula, given by Planck, is dated, in view of Einstein's much simpler and logically more rigorous derivation. However, from the historical point of view, particularly for a correct historical perspective, it is interesting to recall the principal features of Planck's derivation and to cite verbatim that portion of his paper, in which the discreteness of energy is first mentioned.

The derivation leading to the correct radiation formula, is analogous in its general features to that indicated above, except that the expression used for the entropy is somewhat different from the one that led Planck previously to Wien's formula. However, when Planck attempted to justify this expression for the entropy, he was forced to use statistics. Planck himself wrote (III, 264-265): "I myself have not resorted so far to the relation between the entropy and probability; this relation did not attract me, since any probability law admits exceptions, while I believed the second law as admitting of no exceptions. It is only in the course of time that it became clear to me that I could prove the irreversibility of the radiation processes only by admitting the "hypothesis of natural radiation," and that consequently, this restricting hypothesis in the theory of radiation was just as necessary and played exactly the same role as the hypothesis of "molecular chaos" in gas theory. "But since I could find no other way out, I attempted to use the Boltzmann method and wrote a general expression for any state of any physical system

$$S = k \ln W, \tag{9}$$

where W is a suitably calculated probability of the state.

Now to apply (9) to the given case, I postulated a system consisting of a very large number N of identical oscillators and attempted to calculate the probability that such a system would have a specified energy U_N. But since the probability could be obtained only by calculation, it was necessary first to consider the energy U_N as the sum of discrete elements ϵ, all equal, the number of which should be represented by a very large number P. Consequently

$$U_N = NU = P\varepsilon\text{."} \tag{1}$$

In the paper read on December 14, 1900, Planck formulated his basic assumption in the following manner (I, page 698): "It is merely necessary to find, in addition, the energy distribution over the individual resonators within each class of resonators and, above all, the distribution of the energy E over N resonators with frequency ν. Were E to be considered as a quantity that can be subdivided without limit, the distribution would have an infinite number of possibilities. But we consider — and this is the essential point of the entire calculation — E as being composed of a definite number of finite equal parts and use for this purpose a universal constant (Naturkonstante) $h = 6.55 \times 10^{-27}$ erg-sec. This constant, multiplied by the number of oscillations ν of the resonators, gives the energy element ϵ in ergs, by dividing ϵ into E we obtain the number P of the energy elements, which must be distributed over the N resonators." As can be seen, the premise is quite clearly formulated. However, it is not emphasized anywhere that this is a hypothesis that contradicts radically all of classical physics.

Furthermore, much later, in 1909 (i.e., after the publication of Einstein's paper on light quanta and on specific heat), Planck emphasized an opposite idea in lectures delivered at Columbia University. We cite this interesting excerpt from a lecture devoted to the atomistic theory of matter: "If . . . we wish to reduce the entropy of radiant heat to probability, then, as previously, we see that atomistics should play a significant role in radiant heat. Since, however, radiant heat is not connected with matter, atomistics should pertain not to matter but to energy, from which it follows that certain energy elements should play a substantial role in radiant heat. No matter how strange such a conclusion may sound — and the lively protest which is raised against it in many circles even now is quite understandable — physical science cannot fail to accept it, more so since it is quite satisfactorily confirmed experimentally. We shall return to this problem in lectures on radiant heat; here I should like to add also that the introduction of atomistics to the study of radiant heat represents nothing new, nothing that should revolutionize our concepts, as may appear at first glance,* since there is no need, at least in my opinion, of imagining, from the atomistic point of view, radiation phenomena in absolute vacuum, but is is enough to apply the atomistics to the source of the radiation, i.e., to those phenomena that take place in the centers where rays are emitted and absorbed.†

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*Emphasis mine (É. Sh.)

†I cite from the Russian translation: M. Planck, Theoretical Physics, Eight Lectures Delivered at Columbia University in New York. Translated by I. M. Zapchevskiĭ, St. Petersburg, 1911, p. 59.

This tendency to retain somehow the concepts of classical physics, in spite of the fundamental facts that he himself discovered and that call for a break with these concepts, indicates the conflict which Planck felt painfully during the rest of his life.

Let us return again, however, to the historical paper delivered on December 14, 1900. Using the above formula to relate the radiation energy density with the resonator energy and with Wien's law, Planck derives the following formula for the radiation from an absolutely black body.

$$u_\nu \, d\nu = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{\frac{h\nu}{kT}} - 1} .$$

Changing from frequencies to wavelengths, we obtain the Planck formula in the form used for practical radiation calculations:

$$u_\lambda \, d\lambda = \frac{8\pi ch}{\lambda^5} \cdot \frac{d\lambda}{e^{\frac{ch}{k\lambda T}} - 1} .$$

Either form of Planck's formula contains the two universal constants h and k. The constant k is usually called the Boltzmann constant, since it is also the proportionality coefficient in the Boltzmann formula that relates entropy and probability. Planck, however, pays attention to the fact that Boltzmann himself never used this constant (he used the quantity R), and that its numerical value was first calculated by Planck. The constant h is the elementary quantum of action, the discovery of which has immortalized Planck's name Using empirical data for the intensity of radiation from an absolutely black body, Planck, as mentioned above, calculated the numerical values for these constants and obtained (I, 727)

$$k = 1.346 \cdot 10^{-16} \text{ erg-deg}^{-1},$$
$$h = 6.55 \cdot 10^{-27} \text{ erg-sec.}$$

For comparison, we cite the modern exact values:

$$k = 1.3804 \cdot 10^{-16} \text{ erg-deg}^{-1}, \qquad h = 6.625 \cdot 10^{-27} \text{ erg-sec.}$$

With the aid of the value of k obtained by him and of the well known value of the universal gas constant R, Planck calculated Avogadro's number; from this, using the Faraday number, he found the charge of the electron

$$e = 4.69 \cdot 10^{-10} \text{ CGS electrostatic units.}$$

This number was greatly different from the result obtained that time by direct experiment. In fact, J. J. Thomson's measurements (1898), which were deemed the most reliable, yielded e = 6.5×10^{-10}, and the old measurements of F. Richarz (1894) yielded e = 1.29×10^{-10}. The discrepancy between the value of e calculated by Planck and that determined experimentally was used as an argument by the critics of the quantum hypothesis of whom, as always in the case of radically new ideas, there was no shortage. However, we know now that it was Planck who was considerably closer to reality, since the modern most precise value of e is 4.805×10^{-10}.

Further progress in theoretical physics was marked by a literal victory march of quantum theory. Let us recall the principal stages of this development: in 1905 Einstein transferred the quantum concepts from the mechanism of radiation to the nature of radiant energy itself, thereby radically resolving many accumulating substantial contradictions. Somewhat later (1908) he introduced quantum concepts into solid-state physics, to explain the puzzling drop in specific heat near absolute zero. In 1913 N. Bohr generalized the quantum postulates and applied them with brilliant success to the theory of atomic structure; finally, in 1925-1927 W. Heisenberg, M. Born, and P. Jordan on one hand, and L. de Broglie and E. Schrödinger on the other, created and completed the formulation of non-relativistic quantum mechanics, while P. Dirac established the relativistic wave equation of the electron. Planck no longer participated directly in this rapid progress. And although the break with classical physics was painful to him, he understood with the intuition of a great scientist its inevitability and depth. In 1912, in the foreword to his second edition of "Lectures on the Theory of Heat Radiation" he wrote: ". . . anyone who wants to base his relationship to the quantum hypothesis on whether it is possible to explain fully the significance of the quantum of action to elementary physical processes or at least to illustrate it by some simple dynamic model, does not understand, in my opinion, the character and meaning of the quantum hypothesis. A substantial new principle cannot be reproduced by models that obey the laws of the old theory" (emphasis mine — É. Sh.). These clever words should serve as a lesson to those few "lovers of antiquity" who sometimes, without realizing, make attempts to reduce the unique quantum laws to some models that perform in accordance with the laws of classical physics.

It is not our purpose here to present a complete survey of Planck's papers, and we shall dwell here on some of his important works not directly related to the discovery of the quantum of action. Such is his interpretation of the third law of thermodynamics, discovered by W. Nernst, requiring that the entropy vanish at the absolute zero of the temperature (III, 54-64). Such, too, are his investigations on relativity (see particularly II, 176-209), with which he enthusiastically sided from its inception.

The third volume differs greatly in character from the first two. It contains a collection of articles, papers, and lectures of philosophic and popular character, and also articles devoted to outstanding physicists (H. Hertz, H. A. Lorentz, P. Drude and others) in connection with various anniversaries. This volume includes also articles of autobiographic character: "Personal Recollections" (358-363), "On the History of the Discovery of the Physical Quantum of Action" (255-267), and "Scientific Autobiography" (pages 374-401). The last article is known to the readers of our journal (see Usp. Fiz. Nauk **64**, 625, 1958).

All the articles in this volume remain highly interesting to this day. A detailed analysis of Planck's scientific-philosophical views cannot be given in this article, which is a general survey of the collection of Planck's works. Let us note only two points. In the speech "Unity of the Physical Picture of the World" (III, 6-29), which contains the scientific-philosophical "credo" of Planck, he spoke up with vigorous objections to the positivism of E. Mach which, at that time, (1908) was quite popular among the natural-science writers. In his thermodynamic papers he showed the inconsistency of Ostwald's energetics. Nor did Planck stay outside the latest scientific-philosophical discussions, arising in connection with the problem of causality and, like Einstein, he decisively supported classical determinism in many articles and papers (see, for example, "The Concept of Causality in Physics," III, 219-239).

Planck deserves honor for the most important discovery, the existence of an elementary quantum of action of finite (not infinitesimal) value. He gave the first formulation of one of the great ideas that are the basis of modern physics and separate so radically modern from classical physics — the idea of the quantum and of universal discreteness in natural phenomena. This idea was formulated in its time, and as we know, became exceedingly fruitful.

Surveying the development of the quantum theory from its inception in 1900 to the present day, we cannot help but wonder at the tremendous distance covered by physics in approximately 60 years. We have left far behind the first steps of quantum theory, which more than 30 years ago has been converted from a "quantum hypothesis" into an ordered system of quantum mechanics, not inferior to classical mechanics in its completeness and logical consistency. The universal role of discreteness in the laws of nature, exhibited by Planck's discovery, has not merely been incorporated in the "flesh and blood" of modern physics as a fact, but has raised the problem of explaining the origin of this discreteness itself. We are not dealing now with the admission of the existence of elementary particles and their mutual transformations, but with an explanation of why these exist and have precisely these properties. It is therefore understandable that, against the background of the deepest and most difficult problems of modern theory of elementary particles, the discussions that excited the physicists at the beginning of our century in connection with the appearance of quantum theory appear now so simple and almost naive! Yet an acquaintanceship with the many articles contained in the three volumes of Planck's papers is highly interesting from the historical point of view and deeply instructive, particularly for those studying theoretical physics. Therefore the commemoration of Planck's 100th birthday by publishing a collection of his original papers is not only due homage to the activity of this outstanding scientist, but a valuable contribution to the education of a new generation of physicists.

NEW BOOKS IN PHYSICS

V. I. Kalinin and G. M. Gershteĭn.
Введение в радиофизику (Introduction into Radio Physics. Moscow, Gostekhizdat, 1957, 660 pp. 1265 rub.

The authors of the book have set upon producing a textbook on radio physics, containing an exposition of the basic general problems of radio physics and a clarification of the connection between these problems. This is indeed why the book is called "Introduction Into Radio Physics," without pretending at a complete treatment of basic radio-physics problems.

The principal purpose of the authors has been successfully realized. A serious textbook has been produced, in which the basic general problems of radio physics are exposed systematically from a unified point of view.

The book consists of an introduction and seven sections (a total of 38 chapters).

The introduction (Chapters 1 — 3) is devoted to general problems: oscillations in radio circuits (sinusoidal and transient processes, representation of oscillatory processes in the phase plane), the spectra of oscillation processes (the Fourier series and integral, spectral analysis of pulsed processes, response of a linear network to a complex signal), and principles of modulation (amplitude, phase, and frequency modulation). We agree with the tackling of modulation problems in the very introduction, to the book. A concise mathematical formalism is used in the arguments.

The first section (Chapters 4 and 5) is devoted to a study of forced oscillations in a single closed circuit (free and forced oscillations, resonance in series and parallel circuits, spectral properties of a resonant circuit) and also oscillations in coupled circuits (free and forced oscillations in coupled circuits, resonance phenomena in coupled circuits). The problems are treated compactly and with physical clarity. We notice, however, that Figs. 5.11, 5.15, and 5.18 are not successful: it is difficult to see what the axes really represent.

Processes in circuits with distributed parameters are treated in the second section (Chapters 6 — 10). This covers such problems as the fundamental theory of long lines, long lines at microwave frequencies, theory of spherical diagrams in rectangular and polar coordinates, the long line is an oscillating system, and the fundamentals of filter theory. It is well that the end of Chapter 8 contains several examples on the solution of problems in impedance matching with the aid of spherical diagrams, and at the end of Chapter 9 a special paragraph is devoted to coupled systems with distributed parameters. These problems are usually not treated in radio engineering textbooks.

We note that in the theory of long lines the authors did not touch upon the important problems of the distortion of the signal waveform due to the frequency dependence of the phase velocity; nor do they consider the interesting problem of the behavior of the group velocity of the signal in a long line as a function of the frequency. They should have also indicated the connection between the particular solution of the telegraph equations, proportional to $\exp i\omega t$, with the general solution, in which includes the distributions of the current and voltage along the line at $t = t_0$. In the study of line-to-load matching they should have emphasized the existence of two types of impedance matching, direct and conjugate, used in problems of different nature.

The third section (Chapters 11 — 17) is devoted to waveguides and cavity resonators. The section contains the general waveguide concepts and discusses the physical picture of the propagation of waves in waveguides, and studies waves in rectangular and round waveguides and losses of waveguides. The conditions under which a wave guide is replaced by an equivalent line are breiefly discussed. The properties of regular cavity resonators (reactangular parallelopiped, cylinder, sphere) are discussed along with the properties and application of cavity resonators of complex shapes (toroidal etc.) The exposition is compact and clear.

The following remarks can be made concerning this section. In Chapter 11, the boundary conditions on the boundary surface between two media are formulated "in the most general form" as follows: $E_{t_1} = E_{t_2}$ and $H_{t_1} = H_{t_2}$. These conditions are true only if the surface electric or magnetic (fictitious) currents are disregarded. In Chapter 12, devoted to rectangular waveguides, a formula $v_{ph}v_{gr} = c^2$ is derived. This formula is correct for cylindrical waveguides of arbitrary cross section, and should have therefore been contained in the chapter devoted to the general properties of waveguides. Figs. 12.4 and 12.5 are unsuccessful: the electric-field force lines are not normal to the metallic surface.

The important concept of the wave impedance of a rectangular waveguide of sides a and b is discussed very briefly. It is introduced with the aid of the equation $Z_0 = E_y/H_x$. It is known, however, that in many cases Z_0 is defined as $Z_0 = E_yb/H_xa$; this means that the concept of the wave impedance of a waveguide is not unique. The physical significance of the appearance of H^2 in Eqs. (14.4) and (16.8) should have been explained with the aid of the boundary condition $H_t = 4\pi I/c$, where I is the surface current density. In Chapter 17, devoted to cavity resonators of complex shape, the formula $k^2 = k_0^2(1 + \frac{\Delta W_e - \Delta W_H}{W_0})$ should have been supplemented, say, with an illustrative example of a toroidal resonator whose capacitive portion is slightly deformed; indidentally, there is a misprint in this formula. It must also be noted that certain interesting problems (such as the transformation of waves of different modes in waveguides) are not discussed at all.

In the fourth section (Chapters 18 — 25) we consider general problems of generation and amplification of electromagnetic oscillations. The general properties of self-oscillating systems are formulated; the elements of the theory of the vacuum-tube oscillator are given; linear, nonlinear, and quasi-linear methods of the theory of vacuum-tube oscillators are studied, along with relaxation self-oscillations, parametric excitations of electromagnetic oscillations, and frequency stabilization. The end of the section includes an analysis of amplifier circuits, an investigation of amplifier stability, and examples of the use of the Nyquist-Tsypkin criterion.

A large amount of material is treated very concisely in the section. The authors succeeded in so doing because of their economical method of exposition: the general formulas are usually first established at the beginning of the chapter, and are then applied further to individual problems (see for example Chapters 24 and 25). Among the shortcomings of the section is the almost complete lack of treatment of semiconductor devices and semiconductor electronics, which gains in importance from year to year.

The fifth section is devoted to problems of generation and amplification of microwave frequencies (Chapters 26 — 32). The section begins with general problems of interaction between an electric beam and an electric field (the energy effect of the interaction, the induced-current theorem, the behavior of the electron gap at microwave frequencies, etc.). This is followed by a brief description of the properties of generators with static control of the electron beam, and then by a discussion of the principles of dynamic beam control and of properties of microwave devices with intermittent interaction (klystrons and magnetrons). The end of the chapter contains a brief formulation of the properties of slow-wave systems and microwave devices with continuous interaction. The exposition is logical and clear, disclosing the connection between the various problems in generation and amplification of microwaves. We notice that this is apparently the first time that a Russian textbook contains the description of properties of slow-wave systems with anomalous dispersion and of their dispersion characteristics.

The following remarks can be made concerning the section on the generation and amplification of microwaves. In the klystron bunching theory the authors disregarded the important phenomenon of debunching by Coulomb forces. The theory of multi-cavity magnetron contains no discussion of the connection between the parabola of the critical mode, the threshold line, and the synchronism line — the principal lines that characterize the operating conditions of a magnetron in the V_a, B (anode voltage — induction) plane. Formula (31.8) contains a misprint. The example of a coaxi-helix transition (Fig. 32.3) has been unsuccessfully chosen: it is not clear where the electron beam will travel. Another shortcoming of the section is the almost complete lack of data on the electron-wave tube. The bibliography on waveguides, slow-wave systems, and theory of traveling waves omits without justification the papers by L. A. Vaĭnshtein.

In section VI, devoted to nonlinear signal transformations, the authors treat the theoretical principles of detection, regeneration, superregeneration, and frequency transformation.

Chapter VII, the last in the book, discusses briefly problems of noise in electric circuits and vacuum tubes, and also problems of microwave reception.

Summarizing, we must say that the book touches upon almost all fundamental problems of modern radio physics. The exposition is at a high scientific level, logical and systematic, employing a unique mathematical formalism. The book helps in the interpretation of the connection between different fields of radio physics. The authors pay principal attention to the physical picture of the processes, and the mathematical description acquires a subsidiary role. The important role of Russian scientists in the development of the principal problems of radio physics is correctly presented.

The book is written in very good style, clarity, and brevity of exposition; it is easy to read, a circumstance helped by the large number of small well thought-out diagrams, which contain nothing excessive, but include everything necessary for the understanding of the text.

In addition to the specific critical remarks made above, we note that the authors have devoted too little place to the new rapidly-developing trends in radio physics — statistical radio physics and semiconductors. The problems of quantum radio physics, which have a great future, are not discussed at all. Thus, the book summarizes and systematizes well the development of many branches of radio physics, but does not indicate sufficiently fully its further development. We also note that the authors have preferred the Gaussian system of units, although recently the practical system of units is more frequently employed in radio physics.

On the whole, the book by V. I. Kalinin and G. M. Gershteĭn "Introduction to Radio Physics," is an exceedingly valuable textbook for students in universities, teachers' colleges, and in some engineering colleges, It will be widely used by students taking special courses in radio engineering, theory of oscillations, electronics, microwave electronics, and propagation of radio waves.

Representing the long teaching experience of the authors, the book reviewed is an example of a masterful exposition of the fundamentals of radio physics. It will be met with satisfaction by all radio physicists.

—S. D. Gvozdover and V. M. Lopukhin
Usp. Fiz. Nauk **66**, 700-702 (December, 1958)

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Contents: Part 1. Supporting Premises for the Discovery of the Law of Conservation and Transformation of Energy. Chapters: 1. Development of Theory of Conservation of Motion in Mechanics in the 17 and 18th Centuries. 2. Development of the Concepts of the Nature of Heat in the 17th and 18th Centuries. 3. Discoveries and Investigation of the Interactions Between Various Forms of Energy During the First Half of the 19th Century. Part 2. Development of the Theory of Conservation and Transformation of Energy. Chapters: 1. The Research of R. Maier. 2. The Research of Joule and Colding. 3. Helmholtz's "On the Conservation of Force." 4. Discovery of the Directivity of Energy Processes. 5. Discovery of the Laws of Motion in Absorption and Emission of Energy. 6. Law of Conservation and Transformation of Energy in Physics During the 20th Century. Chronological table of the most important discoveries connected with the law of conservation and transformation of energy.

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Contents: From the Editor. Chapters: 1. Fundamental Properties of Liquid Metals. 2. Fields of Applications of Liquid-Metal Coolants. 3. Hydraulic Resistance in Flow of Liquid Metals. 4. Turbulent Heat Transfer in Liquid Metals. 5. Heat Output During Flow in Pipes. 6. Heat Output in Longitudinal Flow Over a Plate. 7. Heat Transfer in Transverse Flow Over Cylinders. 8. Heat Transfer in Compound Convection. 9. Heat Transfer in Vapor Condensation. 10. Heat Transfer in Boiling. 11. Heat Exchangers. 12. Resistance of Refractory Materials in Liquid Metals. 13. Measuring Instruments.

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— T. O. Vreden-Kobetskaya

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